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*Original Article*

# Solving Fuzzy Game Theory Problem using Pentagonal Fuzzy Numbers and Hexagonal Fuzzy Number

Amit Nalvade[1], Ashok Mhaske[2], Sagar Waghmare[3], Smt. Shilpa Todmal[4]

[1,2,3]*Department of Mathematics, Dada Patil Mahavidyalaya, Karjat, Ahmednagar, Savitribai Phule Pune University, Pune (Maharashtra) India*
[4]*Department of Electronics, Dada Patil Mahavidyalaya, Karjat, Ahmednagar, Savitribai Phule Pune University, Pune(Maharashtra) India*

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***Abstract -*** *In this research article a new approach is proposed to solve fuzzy game theory problem. The crisp fuzzy game theory problem is converted to fuzzy game theory problem using Pentagonal and Hexagonal fuzzy numbers. New ranking method based on the area of membership function of Pentagonal and Hexagonal fuzzy numbers. This new ranking method is used to find the best approximate solution to the fuzzy game theory problem.*

***Keywords -*** *Fuzzy, Pentagonal, Hexagonal, Ranking, Crisp, Strategy, Membership.*

## 1. Introduction

Fuzzy Mathematics and fuzzy logic are used to process natural language and are widely used In decision making In Artificial Intelligence. In real-life, game theory, analysis is used in economic competition, economic conditions such as negotiation, auctions, voting theory etc. However, in real life situations, the information available for decision making to select an optimum strategy is imprecise. In this article, the crisp game theory problem is transformed into a fuzzy game theory problem by using triangular and trapezoidal fuzzy numbers. To order any two fuzzy numbers, a new and simple method invented which is based on the area of membership function. A computer program was written in Python which is given in this article to make calculations easier and simpler. Although the modern world is undergoing major changes in science and technology, there are unavoidable uncertainties in any field of science, engineering, medicine, or government. It is well known that an important factor in the development of the modern concept of uncertainty was the publication of a seminar paper by Loft A. Zadeh in year 1965. In his article Zadeh transformed the probability theory and which is based on two value logic i.e. true or false. If ‘A’ is a fuzzy set and x is two valued logic, but it may be true to central degree to which x is realistically a member of A. The degree of membership lies between the interval [0,1]. new method of Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number is introduced by (Mohamed A. H., Dec- 2020). (S. Salahshour S. Abbasbandy T., July 2011 ) developed new techniques for ranking fuzzy numbers using fuzzy maximizing-minimizing points. (Savitha M T, 2017) make known to new methods for ranking of trapezoidal fuzzy nummbers. (Ganesan, 2018) used A new approach for the solution of fuzzy games using fuzzy numbers. A Fuzzy Approach to Strategic Games is intorduced (Qian Song .,1999). An application of fuzzy game theory to industrial decision making is introduced by (M.D. Khedeka) The game is a decision situation with many players, each with conflicting goals. The the players involved in the game usually decide according to conditions of risk or uncertainty. In this article, the fuzzy approach is designed to solve a strategy game problem in which the the net strategy set for each player is already defined. Based on concepts of fuzzy set theory, this approach will use multicriteria decision-making method to obtain an optimal strategy v game, a method that shows more advantages than the classic one game methods. In addition, some useful conclusions regarding the famous "prisoner's dilemma" can be reached with this approach problem in game theory.

Logical decision-makers game theory generally refers to the study of mathematical models. It is extensively used in many fields such as engineering, economics, political science, politics, and computer science, and can be used to model many real-world scenarios. Game theory is a theoretical framework for conceiving social situations among competing players. Generally, a game refers to a situation involving a set of players who each have a set of possible choices, in which the outcome for any individual player depends partially on the choices made by other players.

### *1.1. Some Basic Definition*

#### *1.1.1. Fuzzy Set*

If X is a universe of discourse and x be any particular element of X, then a fuzzy set $\tilde{A}$ defined on X may be written as a collection of ordered pairs $\tilde{A} = \{(x,\mu_{\tilde{A}}(x)) : x \in X\}$. Where each pair $(x,\mu_{\tilde{A}}(x))$ is called a singleton and $\mu_{\tilde{A}}(x)$ is membership function which maps X to [0,1]

*1.1.2. Support of a Fuzzy Set*

The support of a fuzzy set $\tilde{A}$ of the set X is a classical set defined as

$$\text{Sup}(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\}$$

*1.1.3. Fuzzy Number*

A Fuzzy set $\tilde{A}$ is a Fuzzy set on the real line R must be satisfy the following conditions

a. There exist at least one $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0)=1$.
b. $\mu_{\tilde{A}}(x)$ is piecewise continuous.
c. $\tilde{A}$ must be normal and convex.

*1.1.4. Crisp Set*

A Crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

*1.1.5. Pure Strategy*

Pure strategy is a decision making rule in which one particular course of action is selected.

*1.1.6. Mixed Strategy*

A set of strategies that a player chooses on a particular move of the game with some fixed probability are called mixed strategies.

*1.1.7. Saddle Point*

If the maximin value equals to the minimax value, then the game is said to have a saddle point and the corresponding strategies which give the saddle point are called optimal strategies. The amount of payoff at an equilibrium point is called the crisp game value of the game matrix.

*1.1.8. Value of the Game*

This is the expected payoff at the end of the game, when each player uses his optimal strategy.

*1.1.9. Solution of all 2× 2 Matrix Game*

Consider the general 2x2 game matrix A=$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ To solve this game we proceed as follows:

a. Test for a saddle point
b. If there is no saddle point, solve by finding equalizing strategies. The optimal mixed strategies for player A=$(p_1, p_2)$ $and\ for\ player\ B = (q_1, q_2)$
   Where $p_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$ , $p_2 = 1 - p_1$ and
   $$q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, q_2 = 1 - q_1$$
   Also Value of the game $V = \frac{a_{11}a_{22}-a_{21}a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

***1.2. Pentagonal Fuzzy Number***

A fuzzy number $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5)$ is called a pentagonal fuzzy number when the membership function has the form. The middle point $a_3$ has grade of membership 1 and $r, s$ are the grades of points $a_2, a_4$. Its membership function is defined as follows

$$\mu_{\tilde{A}}(x; r, s) = \begin{cases} 0 & , \quad x < a_1 \\ \dfrac{r(x - a_1)}{a_2 - a_1} & , \quad a_1 \le x \le a_2 \\ 1 - \dfrac{(1 - r)(x - a_2)}{a_3 - a_2} & , \quad a_2 \le x \le a_3 \\ 1 & , \quad x = a_3 \\ 1 - \dfrac{(1 - s)(x - a_3)}{a_4 - a_3} & , \quad a_3 \le x \le a_4 \\ \dfrac{s(x - a_5)}{a_4 - a_5} & , \quad a_4 \le x \le a_5 \\ 0 & , \quad x > a_5 \end{cases}$$

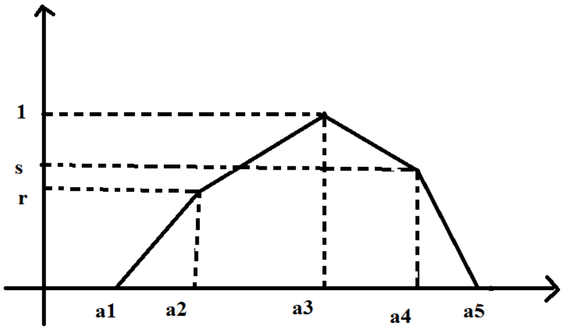


**Fig. 1 Pentagonal Fuzzy Number ($a_1$,$a_2$,$a_3$,$a_4$,$a_5$)**

### *1.3. Hexagonal Fuzzy Number*

A fuzzy number $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5, a_6)$ is called a Hexagonal fuzzy number when the membership function has the form Where the middle point $a_3$ has the grade of membership 1 and $r, s$ are the grades of points $a_2, a_4$. Its membership function is defined as follows

$$\mu_{\tilde{A}}(x; r, s) = \begin{cases} 0 & , \quad x < a_1 \\ \dfrac{r(x-a_1)}{a_2-a_1} & , \quad a_1 \le x \le a_2 \\ 1 - \dfrac{(1-r)(x-a_2)}{a_3-a_2} & , \quad a_2 \le x \le a_3 \\ 1 & , \quad a_3 \le x \le a_4 \\ 1 - \dfrac{(1-s)(x-a_3)}{a_4-a_3} & , \quad a_4 \le x \le a_5 \\ \dfrac{s(x-a_5)}{a_4-a_5} & , \quad a_5 \le x \le a_6 \\ 0 & , \quad x > a_6 \end{cases}$$

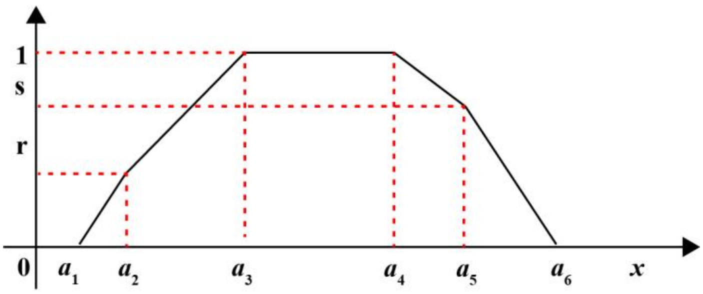


**Fig. 2 Hexagonal Fuzzy Number ($a_1$,$a_2$,$a_3$,$a_4$,$a_5$, $a_6$)**

### *1.4. Ranking of Fuzzy Number*

Let $\tilde{A}$ be a fuzzy number with $\mu_{\tilde{A}}$(x) is membership function which maps R to [0,1] and $Sup(\tilde{A}) = (a, b)$ is subset of R. The measure of $\tilde{A}$ is denoted by $R(\tilde{A})$ and defined as

$$R(\tilde{A}) = (a+b)\left[\frac{1}{b-a} * \ Area\ of\ membership\ function\ \mu_{\tilde{A}}(x) over\ [a,b]\right]$$

$$i.e.\ R(\tilde{A}) = (a+b)\left[\frac{1}{b-a}\int_a^b \mu_{\tilde{A}}(x)\ dx\right]$$

### *1.5. Ranking of Pentagonal Fuzzy Number*

Let $\tilde{A}_{r,s} = (a_1, a_2, a_3, a_4, a_5)$ be a Pentagonal fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1, a_5)$

$$Area\ of\ membership\ function\ \mu_{\tilde{A}}(x) over\ [a_1\ , a_5]$$
$$= \frac{1}{2}(a_2 - a_1)r + r(a_3 - a_2) + \frac{1}{2}(a_3 - a_2)(1 - r) + \frac{1}{2}(a_4 - a_3)(1 - s) + s(a_4 - a_3) + \frac{1}{2}(a_5 - a_4)s$$
$$\therefore Area\ of\ membership\ function\ \mu_{\tilde{A}}(x)\ over\ [a_1, a_5] = \frac{1}{2}(a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4)$$
$$\therefore R(\tilde{A}) = (a_1 + a_5)\left[\frac{1}{a_5 - a_1} \times \frac{1}{2}(a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4)\right]$$
$$\therefore R(\tilde{A}) = \frac{a_1 + a_5}{2(a_5 - a_1)}(a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4)$$

***1.6. Ranking of Hexagonal Fuzzy Number***

Let $\tilde{A}_{r,s} = (a_1, a_2, a_3, a_4, a_5, a_6)$ be a Hexagonal fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1\ , a_6)$

$$Area\ of\ membership\ function\ \mu_{\tilde{A}}(x) over\ [a_1\ , a_6]$$
$$= \frac{1}{2}(a_2 - a_1)r + r(a_3 - a_2) + \frac{1}{2}(a_3 - a_2)(1 - r) + (a_4 - a_3) + \frac{1}{2}(a_5 - a_4)(1 - s) + s(a_5 - a_4) + \frac{1}{2}(a_6 - a_5)s$$
$$\therefore Area\ of\ membership\ function\ \mu_{\tilde{A}}(x)\ over\ [a_1, a_6]$$
$$= \frac{1}{2}(a_1(-r) + a_3(r - 1) - a_4(s - 1) + a_6 s - a_2 + a_5)$$
$$\therefore R(\tilde{A}) = (a_1 + a_6)\left[\frac{1}{a_6 - a_1} \times \frac{1}{2}(a_1(-w_1) + a_3(w_1 - 1) - a_4(w_2 - 1) + a_6 w_2 - a_2 + a_5)\right]$$
$$\therefore R(\tilde{A}) = \frac{a_1 + a_6}{2(a_6 - a_1)}(a_1(-r) + a_3(r - 1) - a_4(s - 1) + a_6 s - a_2 + a_5)$$

***1.7. Numerical Examples***

1. Consider the following fuzzy game problem

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (1,2,4,6,9) & (8,9,11,12,14) \\ (-2,-1,0,3,5) & (-5,-3,-1,0,1) \end{bmatrix} \end{array}$$

Take $r = 0.3, s = 0.6$

Solution: By definition of Ranking of Pentagonal fuzzy number

Let $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5)$ be a triangular fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1\ , a_5)$

$$R(\tilde{A}) = \frac{a_1 + a_5}{2(a_5 - a_1)}(a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4)$$

Step 1: Convert the given fuzzy problem into a crisp value problem

| Fuzzy Number | Crisp value |
|---|---|
| $a_{11} = (1,2,4,6,9)$ | $R(a_{11}) = 4.9375$ |
| $a_{12} = (8,9,11,12,14)$ | $R(a_{12}) = 10.45$ |
| $a_{21} = (-2,-1,0,3,5)$ | $R(a_{21}) = \frac{57}{35}$ |
| $a_{22} = (-5,-3,-1,0,1)$ | $R(a_{22}) = -1.8$ |

Step 2: The pay-off matrix is

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4.9375 & 10.45 \\ \frac{57}{35} & -1.8 \end{bmatrix} \end{array}$$

Minimum of $1^{st}$ row = 4.9375 and Minimum of $2^{st}$ row = -1.8

Maximum of $1^{st}$ column = 4.9375 and Maximum of $2^{st}$ column = 10.45

$$\therefore Maximin = 4.9375 \; and \; Minimax = 4.9375$$

It has saddle point

∴ Strategy for player A=$A_1$ and Strategy $for\ player\ B = B_1$ .

Value of the game $V = 4.9375$

2. Consider the following fuzzy game problem

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (0,2,4,5,6,9) & (-5,-3,-2,1,0,1) \\ (-4,-3,-1,0,1,2) & (5,8,9,11,12,13) \end{bmatrix} \end{array}$$

Take $r = 0.4, s = 0.5$

Solution: By definition of Ranking of Hexagonal fuzzy number

Let $\tilde{A}_{r,s}$ = $(a_1, a_2, a_3, a_4, a_5, a_6)$ be a Hexagonal fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1\ , a_6)$

$$R(\tilde{A}) = \frac{a_1 + a_6}{2(a_6 - a_1)}\left(a_1(-r) + a_3(r-1) - a_4(s-1) + a_6 s - a_2 + a_5\right)$$

Step 1: Convert the given fuzzy problem into a crisp value problem

| Fuzzy Number | Crisp value |
|---|---|
| $a_{11} = (0,2,4,5,6,9)$ | $R(a_{11})$ =4.3 |
| $a_{12} = (-5,-3,-2,1,0,1)$ | $R(a_{12}) = -2.4$ |
| $a_{21} = (-4,-3,-1,0,1,2)$ | $R(a_{21}) = -1.2$ |
| $a_{22} = (5,8,9,11,12,13)$ | $R(a_{22}) = 9.675$ |

Step 2: The pay-off matrix is

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4.3 & -2.4 \\ -1.2 & 9.675 \end{bmatrix} \end{array}$$

Minimum of $1^{st}$ row = $-2.4$ and Minimum of $2^{st}$ row = -1.2

Maximum of $1^{st}$ column = 4.3 and Maximum of $2^{st}$ column = 9.675

$$\therefore Maximin = -1.2 \; and \; Minimax = 4.3$$

$$\therefore -1.2 \neq 4.3$$

It has no saddle point.

Step 3: To find Optimum mixed strategy and value of the game

$$Here\ a_{11} = 4.3, a_{12} = -2.4, a_{21} = -1.2, a_{22} = 9.675$$

$p_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{9.675-(-1.2)}{(4.3+9.675)-(-2.4-1.2)} = \frac{10.875}{17.575} = \frac{435}{703};\ \ p_2 = 1 - \frac{435}{703} = \frac{268}{703}$

$q_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{9.675-(-2.4)}{(4.3+9.675)-(-2.4-1.2)} = \frac{12.075}{17.575} = \frac{483}{703};\ \ q_2 = 1 - \frac{483}{703} = \frac{220}{703}$

∴ Strategy for player A= $(p_1, p_2) = \left(\frac{435}{703}, \frac{268}{703}\right)$

∴ Strategy for player B= $(q_1, q_2) = \left(\frac{483}{703}, \frac{220}{703}\right)$

Also Value of the game $V = \frac{a_{11}a_{22}-a_{21}a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

$$V = \frac{(4.3 \times 9.675) - ((-1.2) \times (-2.4))}{(4.3 + 9.675) - (-2.4 - 1.2)} = \frac{38.7225}{17.575} \cong 2.20327$$

∴ Value of the game $V = 2.20327$

## 2. Conclusion

In this paper we have obtained the optimum solution of fuzzy game theory problems using Pentagonal and Hexagonal fuzzy numbers. New ranking is used to order any two Pentagonal and Hexagonal fuzzy numbers. Through a numerical example, we can conclude that using proposed method the value obtained from fuzzy game theory is optimum.

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