International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V69I2P511

Volume 69 Issue 2, 80-90, February 2023

On Degree Cordial Labelings of Certain Families of Graphs

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Received: 03 January 2023 Revised: 04 February 2023 Accepted: 16 February 2023 Published: 28 February 2023

Abstract. *In this paper, we define a new graph labeling called Degree Cordial labeling and call a graph Degree Cordial if it admits a Degree Cordial labeling. Let* $G = (V, E)$ *be a graph and let* $f : E \to \{0, 1\}$ *be a function. We call* f *a Degree Cordial labeling if for each vertex* $v \in V$*, the number of edges incident on* v *with edge label* 0 *and* 1 *differ by atmost* 1*. Here we discuss Degree Cordial labeling for the following graphs: Paths, Cycles, Wheel graphs, Helms, Closed Helms, Generalised Webs, Flower, Sunflower, Combs, Complete Bipartite graph and some Complete graphs.*

Keywords: *Paths, Cycles, Wheels, Helms, Webs, Flower Graphs, Combs, Complete Bipartite, New Graph Labeling.*

Mathematics Subject Classification [2020] Primary 05C78

1. Introduction

The origins of Graph Labeling methods go back to the pioneering work of Rosa [22] in 1967 or to the method given by Graham and Sloane [15] in 1980. Rosa introduced the concept of β (beta) valuation of a graph G with q edges as an injection f from the vertex set $V(G)$ of G to $\{1, 2, \ldots, q\}$ such that when each edge uv in G is assigned the label $|f(u) - f(v)|$, the edge labels are distinct. Such a labeling was called Graceful by Golomb [14]. Cahit [10] introduced a variation of graceful labeling by defining a function f from the vertex set of G to $\{0, 1\}$ which assigns to each edge uv of G the label $|f(u) - f(v)|$. The function f is called a Cordial labeling of G if the number of vertices labeled 0 and 1 viz. $v_f(0)$ and $v_f(1)$, differ by atmost 1 and the number of edges labeled 0 and 1 viz. $e_f(0)$ and $e_f(1)$, differ by atmost 1. A graph G is called a cordial graph if it admits a cordial labeling. A large number of graphs have been shown to be cordial by a host of authors and variations of cordial labelings have been studied. A complete literature survey can be found on this in [13]. For expository work in graph labelings you may refer to work by [10], [13],

[1], [2], [3], [4], [5], [6], [9], [8]). For variations of graph labelings refer to work of [13], [18],[19], [20], [11], [12], [7], [23], [14], [15], [17], [24] Ng and Lee [21] defined edge cordial labeling in 1988. Yilmaz and Cahit [25] in 1997 renamed edge cordial labeling as e-cordial labeling.

Definition 1.1. Let $G = (V, E)$ be a graph with $|E| = e$. Let $f : E \to \{0, 1\}$ be a function and the induced vertex labels for this function be $f(v) = \sum_{\forall u} f(u, v)$ (mod 2), where $\{u, v\} \in E$. Let $v_f(i)$, $e_f(i)$, for $i = 0, 1$ denote the number of vertices and edges labeled 0 and 1 respectively. f is called an e-cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. The graph G is called e-cordial if it admits an e-cordial labeling.

In this paper we introduce Degree Cordial labeling, another variation of e-cordial labeling and describe various graphs which admit such a labeling. All our graphs will be finite, connected and undirected. For basic definitions in graph theory refer to [16].

Definition 1.2. Let $G = (V(G), E(G))$ be a graph and let $f : E(G) \to \{0, 1\}$ be a function. For each vertex v in G, let $f_0(v)$ = number of edges incident on v with the label 0, that is $f_0(v) = |\{u \in V(G) \mid uv \in E(G) \text{ and } f(uv) = 0\}|$ and $f_1(v)$ = number of edges incident on v with the label 1, that is $f_1(v) = |\{u \in V(G) \mid uv \in E(G) \text{ and } f(uv) = 1\}|$. We say that f is a Degree Cordial labeling if for every $v \in V(G)$, $|f_0(v) - f_1(v)| \leq 1$, i.e. the number of edges incident on v with the label 0 and 1, differ by at most 1. The graph G will be called Degree Cordial if it admits a Degree Cordial labeling.

2. Degree Cordial labelings of certain graphs

Theorem 2.1. *All Paths are Degree Cordial.*

Proof. Let P_n be a path with vertex set $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(P_n) = \{v_i v_{i+1} \mid 1 \leq i \leq n-1\}$. Let $f : E(P_n) \to \{0, 1\}$ be defined as:

$$\begin{aligned} f(v_i v_{i+1}) &= 0, \text{ if } i \text{ is odd;} \\ &= 1, \text{if } i \text{ is even.} \end{aligned}$$

Then $f_0(v_1) = 1,\ f_1(v_1) = 0$;
$f_0(v_i) = f_1(v_i) = 1,\ \forall i,\ 2 \leq i \leq n-1$.
Finally, for the vertex v_n, we have
$f_0(v_n) = 1,\ f_1(v_n) = 0$, if n is even
$f_0(v_n) = 0,\ f_1(v_n) = 1$, if n is odd.
Thus f is a Degree Cordial labeling and every Path is a Degree Cordial graph. □

Theorem 2.2. *A cycle of length n is Degree Cordial iff n is even.*

Proof. Let C_n be a cycle with vertex set $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(C_n) = \{v_i v_{i+1} \mid 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.
Case 1: n is even
Let $f : E(C_n) \rightarrow \{0, 1\}$ be defined as:
for $1 \leq i \leq n-1$,

$$\begin{aligned} f(v_i v_{i+1}) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even} \\ f(v_n v_1) &= 1. \end{aligned}$$

Basically we label the edges on the cycle, alternately by $0's$ and $1's$. Then $f_0(v_i) = f_1(v_i) = 1, \forall i, 1 \leq i \leq n$. Thus f is a Degree Cordial labeling and every cycle with an even number of vertices is a Degree Cordial graph.
Case 2: n is odd
Since there are exactly 2 edges incident on every vertex, one edge will have to receive the label 0 and one the label 1, hence alternately the edges will have to be labeled as $0, 1, 0, 1, \ldots$ or $1, 0, 1, 0, \ldots$; however this forces both edges incident on v_n to receive the label 0 in the first case and the label 1 in the latter. Hence C_n cannot be Degree Cordial when n is odd. □

Theorem 2.3. *All Wheels are Degree Cordial.*

Proof. Let W_n be a wheel with vertex set $V(W_n) = \{v, v_1, v_2, \ldots, v_n\}$ and edge set $E(W_n) = \{v v_i \mid 1 \leq i \leq n\} \cup \{v_i v_{i+1} \mid 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.
Case 1: n is even
Let $f : E(W_n) \rightarrow \{0, 1\}$ be defined as:

$$\begin{aligned} f(v v_i) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even.} \end{aligned}$$

For $1 \leq i \leq n-1$,

$$\begin{aligned} f(v_i v_{i+1}) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even} \\ f(v_n v_1) &= 1. \end{aligned}$$

Then $f_0(v) = f_1(v) = \frac{n}{2}$, and $\forall i, 1 \leq i \leq n$,
$f_0(v_i) = 2,\ f_1(v_i) = 1$, if i is odd and
$f_0(v_i) = 1,\ f_1(v_i) = 2$, if i is even.
Thus f is a Degree Cordial labeling.

Case 2: n is odd
Let $f : E(W_n) \to \{0, 1\}$ be defined as:

$$\begin{aligned} f(vv_i) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even.} \end{aligned}$$

For $1 \le i \le n - 1$,

$$\begin{aligned} f(v_iv_{i+1}) &= 1, \text{if } i \text{ is odd} \\ &= 0, \text{if } i \text{ is even} \\ f(v_nv_1) &= 1. \end{aligned}$$

Then $f_0(v) = \frac{n+1}{2}$, $f_1(v) = \frac{n-1}{2}$, and $\forall i, 1 \le i \le n$,
$f_0(v_i) = 1$, $f_1(v_i) = 2$, if i is odd and $f_0(v_i) = 2$, $f_1(v_i) = 1$, if i is even.
Thus f is a Degree Cordial labeling. □

Theorem 2.4. *All Helms are Degree Cordial.*

Proof. Let H_n be a helm with vertex set $V(H_n) = \{v, v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$ and edge set
$E(H_n) = \{vv_i \mid 1 \le i \le n\} \cup \{v_iv_{i+1} \mid 1 \le i \le n - 1\} \cup \{v_nv_1\} \cup \{v_iw_i \mid 1 \le i \le n\}$.
Case 1: n is even
Let $f : E(H_n) \to \{0, 1\}$ be defined as:
for $1 \le i \le n$,

$$\begin{aligned} f(vv_i) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even} \\ f(v_iw_i) &= 1, \text{if } i \text{ is odd} \\ &= 0, \text{if } i \text{ is even.} \end{aligned}$$

For $1 \le i \le n - 1$,

$$\begin{aligned} f(v_iv_{i+1}) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even} \\ f(v_nv_1) &= 1. \end{aligned}$$

Then $f_0(v) = f_1(v) = \frac{n}{2}$,
and $\forall i, 1 \le i \le n, f_0(v_i) = f_1(v_i) = 2$, and $f_0(w_i) + f_1(w_i) = 1$.
Thus f is a Degree Cordial labeling.
Case 2: n is odd
Let $f : E(H_n) \to \{0, 1\}$ be defined as:

for $1 \leq i \leq n$,

$$f(vv_i) = 0, \text{if } i \text{ is odd}$$
$$= 1, \text{if } i \text{ is even.}$$

For $1 \leq i \leq n-1$,

$$f(v_iv_{i+1}) = 1, \text{if } i \text{ is odd}$$
$$= 0, \text{if } i \text{ is even}$$
$$f(v_nv_1) = 1.$$

For $2 \leq i \leq n$,

$$f(v_iw_i) = 0, \text{if } i \text{ is even}$$
$$= 1, \text{if } i \text{ is odd}$$
$$f(v_1w_1) = 0.$$

Then $f_0(v) = \frac{n+1}{2}$, $f_1(v) = \frac{n-1}{2}$,
and $\forall i, 1 \leq i \leq n, f_0(v_i) = f_1(v_i) = 2,\ f_0(w_i) + f_1(w_i) = 1$.
Thus f is a Degree Cordial labeling. □

Theorem 2.5. *All Closed Helms are Degree Cordial.*

Proof. Let CH_n be a closed helm with vertex set
$V(CH_n) = \{v, v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$ and edge set
$E(CH_n) = \{vv_i \mid 1 \leq i \leq n\} \cup \{v_iv_{i+1}, w_iw_{i+1} \mid 1 \leq i \leq n-1\} \cup \{v_nv_1, w_nw_1\} \cup \{v_iw_i \mid 1 \leq i \leq n\}$.
Case 1: n is even
Let $f : E(CH_n) \to \{0, 1\}$ be defined as:
for $1 \leq i \leq n$,

$$f(vv_i) = 0, \text{if } i \text{ is odd}$$
$$= 1, \text{if } i \text{ is even}$$
$$f(v_iw_i) = 1, \text{if } i \text{ is odd}$$
$$= 0, \text{if } i \text{ is even.}$$

For $1 \leq i \leq n-1$,

$$f(v_iv_{i+1}) = f(w_iw_{i+1}) = 0, \text{if } i \text{ is odd}$$
$$= 1, \text{if } i \text{ is even}$$
$$\text{and} f(v_nv_1) = f(w_nw_1) = 1.$$

It can be verified that f is a Degree Cordial labeling.
Case 2: n is odd

Let $f : E(CH_n) \to \{0,1\}$ be defined as:
for $1 \le i \le n$,

$$\begin{aligned} f(vv_i) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even.} \end{aligned}$$

For $1 \le i \le n-1$,

$$\begin{aligned} f(v_i v_{i+1}) = f(w_i w_{i+1}) &= 1, \text{if } i \text{ is odd} \\ &= 0, \text{if } i \text{ is even} \\ \text{and } f(v_n v_1) = f(w_n w_1) &= 1. \end{aligned}$$

For $2 \le i \le n$,

$$\begin{aligned} f(v_i w_i) &= 0, \text{if } i \text{ is even} \\ &= 1, \text{if } i \text{ is odd} \\ f(v_1 w_1) &= 0. \end{aligned}$$

Thus f is a Degree Cordial labeling. □

Theorem 2.6. *All Generalised Webs are Degree Cordial.*

Proof. Let $W(t,n)$ be a generalised web with t cycles of length n each. Let the vertices on the k^{th} cycle be denoted by $v_{k1}, v_{k2}, \ldots, v_{kn}$; the apex vertex or the central vertex be denoted by v and the pendant vertices attached to the outermost cycle be denoted by $w_1, w_2, \ldots, w_n$. Then the edge set
$E(W(t,n)) = \bigcup_{k=1}^{t} (\{v_{ki}v_{ki+1} \mid 1 \le i \le n-1\} \cup \{v_{kn}v_{kn+1}\}) \cup \{vv_{1i}, v_{ti}w_i \mid 1 \le i \le n\}$.
Case 1: n is even
Let $f : E(W(t,n)) \to \{0,1\}$ be defined as:
for $1 \le k \le t$,

$$\begin{aligned} f(v_{ki}v_{ki+1}) &= 0, \text{if } i \text{ is odd} \\ &= 1, \text{if } i \text{ is even, where } 1 \le i \le n-1 \\ \text{and } f(v_{kn}v_{kn+1}) &= 1; \end{aligned}$$

i.e. on each cycle we label the edges as $0,1,0,1,\ldots,0,1$. Along any radiating segment starting from the vertex to the pendant vertex i.e. along $vv_{1i}, v_{1i}v_{2i}, \ldots, v_{ti}w_i$, we label the edges as 0, 1, 0, 1, … if i is odd and $1,0,1,0,\ldots$ if i is even.
It can be verified that f is a Degree Cordial labeling.
Case 2: n is odd
Let $f : E(W(t,n)) \to \{0,1\}$ be defined as:

for $1 \le k \le t$,

$$
\begin{aligned}
f(v_{ki}v_{ki+1}) &= 1, \text{if } i \text{ is odd} \\
&= 0, \text{if } i \text{ is even, where } 1 \le i \le n-1 \\
\text{and } f(v_{kn}v_{kn+1}) &= 1;
\end{aligned}
$$

i.e. on each cycle we label the edges as $1, 0, 1, 0, \ldots, 0, 1$. Along any radiating segment starting from the vertex to the pendant vertex i.e. along $vv_{1i}, v_{1i}v_{2i}, \ldots, v_{ti}w_i$, we label the edges as 0, 1, 0, 1, ... if i is odd, $i \neq 1$ and $1, 0, 1, 0, \ldots$ if i is even; for $i = 1$, we label all edges as 0 i.e. $f(vv_{11}) = f(v_{11}v_{21}) = \ldots = f(v_{t1}w_1) = 0$. Then f is a Degree Cordial labeling. □

Theorem 2.7. *All Combs are Degree Cordial.*

Proof. Let $P_n \odot K_1$ be a comb with vertex set $V(P_n \odot K_1) = \{u_i, v_i \mid 1 \le i \le n\}$ and edge set
$E(P_n \odot K_1) = \{u_iu_{i+1} \mid 1 \le i \le n-1\} \cup \{u_iv_i \mid 1 \le i \le n\}$.
Let $f : E(P_n \odot K_1) \to \{0, 1\}$ be defined as:
for $1 \le i \le n-1$, $f(u_iu_{i+1}) = 1$ and for $1 \le i \le n, f(u_iv_i) = 0$.
Thus f is clearly a Degree Cordial labeling. □

Theorem 2.8. *All Flower Graphs are Degree Cordial.*

Proof. Let Fl_n be a flower graph with vertex set
$V(Fl_n) = \{v, v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$ and edge set
$E(Fl_n) = \{vv_i, v_iw_i, vw_i \mid 1 \le i \le n\} \cup \{v_iv_{i+1} \mid 1 \le i \le n-1\} \cup \{v_nv_1\}$.
Let $f : E(Fl_n) \to \{0, 1\}$ be defined as:
for $1 \le i \le n$,
$f(vv_i) = f(v_iw_i) = 0$, $f(vw_i) = 1$,
for $1 \le i \le n-1$,
$f(v_iv_{i+1}) = 1$, $f(v_nv_1) = 1$.
Then $f_0(v) = f_1(v) = n$ and for $1 \le i \le n$, $f_0(v_i) = f_1(v_i) = 2$, and
$f_0(w_i) = f_1(w_i) = 1$. Thus f is a Degree Cordial labeling. □

Theorem 2.9. *All Sunflower graphs are Degree Cordial.*

Proof. Let SF_n be a sunflower graph with vertex set
$V(SF_n) = \{v, v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$ and edge set
$E(SF_n) = \{vv_i, v_iw_i \mid 1 \le i \le n\} \cup \{v_iv_{i+1}, w_iv_{i+1} \mid 1 \le i \le n-1\} \cup \{v_nv_1, w_nv_1\}$.
Case 1: n is even
Let $f : E(SF_n) \to \{0, 1\}$ be defined as:

for $1 \leq i \leq n$,

$$f(vv_i) = 0, \text{if } i \text{ is odd}$$
$$= 1, \text{if } i \text{ is even}$$
$$\text{and } f(v_iw_i) = 0.$$

For $1 \leq i \leq n-1$,

$$f(v_iv_{i+1}) = 0, \text{if } i \text{ is odd}$$
$$= 1, \text{if } i \text{ is even}$$
$$f(v_nv_1) = 1,$$
$$f(w_iv_{i+1}) = f(w_nv_1) = 1.$$

Then $f_0(v) = f_1(v) = \frac{n}{2}$, and $\forall i, 1 \leq i \leq n$,
$f_0(v_i) = 2,\ f_1(v_i) = 3$, if i is even $f_0(v_i) = 3,\ f_1(v_i) = 2$ if i is odd;
finally $f_0(w_i) = f_1(w_i) = 1$. Thus f is a Degree Cordial labeling.
Case 2: n is odd
$f : E(SF_n) \to \{0, 1\}$ be defined as:
for $1 \leq i \leq n$,

$$f(vv_i) = 0, \text{if } i \text{ is odd}$$
$$= 1, \text{if } i \text{ is even}$$
$$\text{and} f(v_iw_i) = 0.$$

For $1 \leq i \leq n-1$,

$$f(v_iv_{i+1}) = 1, \text{if } i \text{ is odd}$$
$$= 0, \text{if } i \text{ is even}$$
$$f(v_nv_1) = 1,$$
$$f(w_iv_{i+1}) = f(w_nv_1) = 1.$$

Then $f_0(v) = \frac{n+1}{2},\ f_1(v) = \frac{n-1}{2}$,
and $\forall i, 1 \leq i \leq n, i \neq 1$,
$f_0(v_i) = 2,\ f_1(v_i) = 3$, if i is even
$f_0(v_i) = 3,\ f_1(v_i) = 2$ if i is odd,
$f_0(v_1) = 2,\ f_1(v_1) = 3$;
finally $f_0(w_i) = f_1(w_i) = 1$. Thus f is a Degree Cordial labeling. □

Theorem 2.10. *All Complete Bipartite graphs are Degree Cordial.*

Proof. Let $K_{m,n}$ be a complete bipartite graph with vertex set
$V(K_{m,n}) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and edge set

$E(K_{m,n}) = \{u_i v_j \mid 1 \leq i \leq m, 1 \leq j \leq n\}$.
Let $f : E(K_{m,n}) \to \{0,1\}$ be defined as:

$$f(u_i v_j) = 0, \text{if } i+j \text{ is even}$$
$$= 1, \text{if } i+j \text{ is odd.}$$

Case 1: m and n are both even
Here $f_0(u_i) = f_1(u_i) = \frac{n}{2}$, for all i, $1 \leq i \leq m$
and $f_0(v_j) = f_1(v_j) = \frac{m}{2}$, for all j, $1 \leq j \leq n$.
Case 2: m is even, n is odd
Now, for $1 \leq i \leq m$,
$f_0(u_i) = \frac{n+1}{2}$, $f_1(u_i) = \frac{n-1}{2}$, if i is odd
and $f_0(u_i) = \frac{n-1}{2}$, $f_1(u_i) = \frac{n+1}{2}$, if i is even.
For all j, $1 \leq j \leq n$, $f_0(v_j) = 1, f_1(v_j) = \frac{m}{2}$.
Case 3: m and n are both odd
Now, for $1 \leq i \leq m$,
$f_0(u_i) = \frac{n-1}{2}$, $f_1(u_i) = \frac{n+1}{2}$, if i is odd
and $f_0(u_i) = \frac{n+1}{2}$, $f_1(u_i) = \frac{n-1}{2}$, if i is even.
For $1 \leq j \leq n$, $f_0(v_j) = \frac{m-1}{2}$, $f_1(v_j) = \frac{m+1}{2}$, if j is odd
and $f_0(v_j) = \frac{m+1}{2}$, $f_1(v_j) = \frac{m-1}{2}$, if j is even. □

Theorem 2.11. *The Complete graph K_n is Degree Cordial, for even values of n.*

Proof. Let K_n be a complete graph with $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ and n be even.
Let $f : E(K_n) \to \{0,1\}$ be defined as:

$$f(v_i v_j) = 0, \text{if } i+j \text{ is odd}$$
$$= 1, \text{if } i+j \text{ is even; for } 1 \leq i,j \leq n,\ i \neq j.$$

Then $f_0(v_i) = \frac{n+1}{2}$, $f_1(v_i) = \frac{n-1}{2}$, for $1 \leq i \leq n$.
Hence f is a Degree Cordial labeling on K_n. □

Theorem 2.12. *The Complete graphs K_5 and K_9 are Degree Cordial.*

Proof. Let K_n be a complete graph with $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ and let e_{ij} denote the edge $\{v_i, v_j\}$.
Let $f : E(K_5) \to \{0,1\}$ be defined as:
$f(e_{12}) = 0$, $f(e_{13}) = 1$, $f(e_{14}) = 0$, $f(e_{15}) = 1$, $f(e_{23}) = 0$, $f(e_{24}) = 1$, $f(e_{25}) = 1$, $f(e_{34}) = 1$, $f(e_{35}) = 0$, $f(e_{45}) = 0$.
Then $f_0(v_i) = f_1(v_i) = 2$, for $1 \leq i \leq 5$.
Hence f is a Degree Cordial labeling on K_5.
Next, let $f : E(K_9) \to \{0,1\}$ be defined as:
$f(e_{12}) = 0$, $f(e_{13}) = 1$, $f(e_{14}) = 0$, $f(e_{15}) = 1$, $f(e_{16}) = 0$, $f(e_{17}) = 1$, $f(e_{18}) = 0$,

$f(e_{19}) = 1,\ f(e_{23}) = 1,\ f(e_{24}) = 0,\ f(e_{25}) = 1,\ f(e_{26}) = 0,\ f(e_{27}) = 1,\ f(e_{28}) = 0,$
$f(e_{29}) = 1,\ f(e_{34}) = 0,\ f(e_{35}) = 0,\ f(e_{36}) = 1,\ f(e_{37}) = 0,\ f(e_{38}) = 1,\ f(e_{39}) = 0,$
$f(e_{45}) = 0,\ f(e_{46}) = 1,\ f(e_{47}) = 1,\ f(e_{48}) = 1,\ f(e_{49}) = 1,\ f(e_{56}) = 1,\ f(e_{57}) = 0,$
$f(e_{58}) = 1,\ f(e_{59}) = 0,\ f(e_{67}) = 0,\ f(e_{68}) = 0,\ f(e_{69}) = 1,\ f(e_{78}) = 1,\ f(e_{79}) = 0,$
$f(e_{89}) = 0.$
Then $f_0(v_i) = f_1(v_i) = 4$, for $1 \leq i \leq 9$.
Hence f is a Degree Cordial labeling on K_9. □

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