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Original Article

DTM Approach on Certain Differential Equations in Real Life Applications

H. R. Swathi[1], T. C. Sushma[2]

[1,2] *Department of Mathematics, Nitte Meenakshi Institute of Technology, Bengaluru, KA, India.*

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Abstract *- Enormous applications in real life and in engineering motivated this analysis that provides a report on present work. A Method of Differential Transforms (DTM) is applied on certain differential equations to get a semi analytic solution that helps in rectifying the critical gap that allows to improvise the results of differential equation model in numerous fields. The uniqueness of this paper proposes a succinct solution found with certain added features making the equations compatible, and are matched well with analytical, numerical approach and Laplace transform procedure.*

Keywords *- Electric circuit, Population growth, Non-linear, DTM.*

1. Introduction

Due to extraordinary applications to real world situations and ease in solving ODE and PDE, the Differential Transform Method DTM has been a topic of focus by numerous authors across the world. A promising semi-analytic numerical method that uses Taylor's series in finding the solution of various differential equation is a Differential Transform Method (DTM).

Zhou[1] was the first to propose DTM concept in 1986. This method allows to find the solution of complicated differential equations which usually arises in real world various situations which are basically non-linear in nature. DTM eases the solution of non linear differential equations with its procedure of unique kind which does not require discretization and linearization. DTM provides the solution in series of convergent form, because of this unique procedure of working DTM caught the focus of many academicians in solving numerous differential equations.

Tanfer Tanriverdi and Nermin Agiragac[2] presented an article on solving basic ordinary differential equations with the procedure of Differential Transform. Birol Ibis[3] computed solutions for Emden-Fowler differential equation of non linear type with DTM. Javed ali [7]studied the solutions for boundary value problems of higher order in a finite domain using differential transform method for one dimensional System of Differential equations both linear and non linear are analysed by Farshid[8] using DTM. Sutkar[4] investigated solutions for some specific set of differential equations which relays on powerful method of DTM catches the solution of the exact kind to show case the robustness and its capabilities. Also he computed numerical solution and analysis on system of some differential equations. Vedat [5]applied a technique of Differential transformation to get the solution of Lane-Emden differential equations type. DTM is applied to obtain the solution of Differential equation is of the type quadratic Ricatti is studied by Biazar *et al*[6]. Montri *et al*[10] reports the solutions of a system of differential equations by applying certain techniques like differential transform , Laplace transform and Numerical computaions. Hooman *et al*.[9] [11] analytically results on Computation of Laplace transforms by the differential transform method. Ogwumu *et al*. [12] reported a DTM procedure to compute the solution of ordinary linear differential equations upto third order.

This paper addresses the DTM to find the solution of specific applications of differential equations in real life like charge or current in the electric circuit consisting of an inductor and capacitor. In addition, this paper also discuss the solution of differential equation which arises in Model of Population Growth. Also a Non linear differential equation solution by DTM in comparison with numerical method is discussed.

2. Materials and Methods

The differential transform of Y(k) of the derivative $\left(\frac{d^k y(t)}{dt^k}\right)$ is defined by $Y(k) = \frac{1}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=t_o}$

The inverse differential transform of Y(k) is defined by, $y(t) = \overset{k=0}{\underset{\infty}{\Sigma}}\, Y(k)(t-t_0)^k$

Table 1. The fundamental mathematical operations under Differential Transform Method

Function	Differential Transform
$y(t) = f(t) \pm g(t)$	$Y(k) = F(k) \pm G(k)$
$y(t) = \frac{\partial g(t)}{\partial t}$	$Y(k) = (k+1)G(k+1)$
$y(t) = \lambda g(t)$	$Y(k) = \lambda G(k)$
$y(t) = \frac{\partial^m g(t)}{\partial t^m}$	$Y(k) = (k+1).......(k+m).G(k+1)$
$y(t) = t^m$	$Y(k) = \delta(k-m) = \begin{cases} 1 & at \quad k = m \\ 0 & otherwise \end{cases}$
$y(t) = f(t)g(t)$	$Y(k) = \sum_{r=0}^{k} F(r)G(k-r)$
$y(t) = f_1(t) f_2(t)...... f_m(t)$	$Y(k) = \sum_{k_1}^{k} ..... \sum_{k_{m-1}=0}^{k_2} F_1(k_1)F_2(k_2-k_1).....F_m(k-k_{m-1})$
$y(t) = \sin(wt+\beta)$	$Y(k) = \frac{w^k}{k!}\sin\left(\frac{\pi k}{2}+\beta\right)$
$y(t) = cos(wt+\beta)$	$Y(k) = \frac{w^k}{k!}cos\left(\frac{\pi k}{2}+\beta\right)$
$y(t) = [y(t)]^m$	$F(k) = \begin{cases} Y(0)^m & , k = o \\ \frac{1}{Y(0)}\left[\sum_{r=1}^{k} \frac{(m+1)r-k}{k} Y(r)F(K-r)\right] & , k \geq 1 \end{cases}$

3. Results and Discussion

Example 1:

To find the current in the circuit consisting an Inductor L=0.2H, capacitor of C=0.05F with an applied Emf of $E = sin\, t$volts where initial charge and current in the circuit is zero.

This problem can be modelled using Kirchhoff's voltage law. The applied emf in the circuit is equal to the sum of voltage drop across inductor and capacitor.

$\therefore V_L + V_C = E$ Where is the voltage drop across inductor and it is given by $L\frac{dI}{dt}$, V_C is the voltage drop across capacitor and it is given by $\frac{Q}{C}$, $E$is the applied emf.

$I(t)$ is the current in the circuit at any instant of time t, and $I(t) = \frac{dQ}{dt}$, where Q(t) is charge at any time t.

$$L\frac{dI}{dt}+\frac{Q}{C}=E$$

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=E$$

$$\frac{d^2Q}{dt^2}+\frac{Q}{LC}=\frac{E}{L}$$

Using the given data, we get $Q''(t)+100Q(t)=5\,sin\,t\,;Q(0)=0 and Q'(0)=0$ (1)

Solving the 2nd order ODE, we get

$Q(t)=A\,cos\,10t+B\,sin\,10t+\frac{5}{99}sin\,t$ (2)

Using the constraints, we get $A=0 and B=-\frac{5}{990}$

Hence the charge in the circuit at any time t analytically is

$Q(t)=-\frac{5}{990}sin\,10t+\frac{5}{99}sin\,t$ (3)

Using Maclaurin's series expansion we can represent $Q(t)=0.8333t^3-4.2083t^5+10.0208t^7$

DTM Method

Using DTM to the 2nd order differential equation (1)

$$Q''+100Q=5\,sin\,t\,;Q(0)=0 and Q'(0)=0$$

$(k+2)(k+1)\,Q(k+2)+100Q(k)=\frac{5}{k!}sin\left(\frac{\pi k}{2}\right)$

The recurrence relation is given by

$Q(k+2)=-\frac{1}{(k+2)(k+1)}\left[100Q(k)-\frac{5}{k!}sin\left(\frac{\pi k}{2}\right)\right]$ (4)

$$k=0,1,2,3\ldots.$$

$Q(0)=0 and Q(1)=0$

For $k=0$ $Q(2)=-\frac{1}{(0+2)(0+1)}\left[100Q(0)-\frac{5}{k!}sin\left(\frac{\pi 0}{2}\right)\right]=0$

For $k=1 Q(3)=-\frac{1}{(1+2)(1+1)}\left[100Q(1)-\frac{5}{1!}sin\left(\frac{\pi*1}{2}\right)\right]=\frac{5}{6}$

For $k=2 Q(4)=-\frac{1}{(2+2)(2+1)}\left[100Q(2)-\frac{5}{2!}sin\left(\frac{2\pi}{2}\right)\right]=0$

For $k=3 Q(5)=-\frac{1}{(3+2)(3+1)}\left[100Q(3)-\frac{5}{3!}sin\left(\frac{3\pi}{2}\right)\right]=\frac{-101}{24}$

$$Q(k)=0 for k=even$$

$$Q(1) = 0; Q(3) = \frac{5}{6}; Q(5) = \frac{-101}{24}; Q(7) = \frac{5(10101)}{7!}$$

$$Q(t) = \sum_{k=0}^{\infty} Q(k).t^k$$

$$Q(t) = \frac{5}{3!}t^3 - \frac{101}{4!}t^5 + \frac{50505}{7!}t^7 \quad (5)$$

The current I(t) is obtained by differentiating the charge Q(t) [equation (5)] at any time t.

Example 2:

Let N(t) represent the population size at any time t, the population growth model is given by logistic differential equation $\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right)$ where the constant r defines the growth rate and k represents the carrying capacity.

Carrying capacity of an organism in a given environment is defined to be the maximum population of that organism that the environment can sustain indefinitely.

This logistic equation was first published by Pierre Verhulst in the year 1845.

The analytical solution of this differential equation by the method of separation of variables with initial condition $N(0) = N_o$ is given by,

$$N(t) = \frac{N_0 k e^{rt}}{(k-N_0)+N_0 e^{rt}} \quad (6)$$

Consider a problem where population of rabbits in a meadow is observed to be 200 rabbits initially. After a month, the rabbit population is increased by 4. Given the carrying capacity of rabbits is 750 and growth rate is 0.04 find the population at any time t.

Modelling this we get $\frac{dN}{dt} = 0.04N\left(1 - \frac{N}{750}\right); N_o = 0$ (7)

The analytical solution is $N(t) = \frac{3000e^{0.04t}}{11+4e^{0.04t}}$ (8)

Applying DTM to the equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) let \frac{r}{k} = B$$

$$\frac{dN}{dt} = rN - BN^2; N(0) = 200$$

$$N(k+1) = \frac{1}{(k+1)}\left[rN(k) - B\sum_{n=0}^{k} N(n)N(k-n)\right] for k = 0,1,2,3.. \quad (9)$$

Using this recurrence relation, we get

$$N(1) = \frac{1}{(0+1)}\left[rN(0) - B\sum_{n=0}^{0} N(n)N(0-n)\right]$$

$$= rN(0) - BN(0).N(0) = 5.88$$

$$N(2) = \frac{1}{(1+1)}\left[rN(1) - B\sum_{n=0}^{1} N(n)N(1-n)\right]$$

$$= \frac{1}{2}[rN(1) - BN(0)N(1) - BN(1)N(0)] = 0.05527$$

$$N(3) = \frac{1}{(2+1)}\left[rN(2) - B\sum_{n=0}^{2} N(n)N(2-n)\right]$$

$$= \frac{1}{3}[rN(2) - BN(0)N(2) - BN(1)N(1) - BN(2)N(0)] = -0.00026$$

$$N(4) = \frac{1}{(3+1)}\left[rN(3) - B\sum_{n=0}^{3} N(n)N(3-n)\right]$$

$$= \frac{1}{4}[rN(3) - B\{N(0)N(3) + N(1)N(2) + N(2)N(1) + N(3)N(0)\}]$$

$$= \frac{1}{4}[rN(3) - 2B\{N(0)N(3) + N(1)N(2)\}] = -0.0000098$$

Thus, the population of rabbits at any time t (in months) is given by

$$N(t) = \sum_{k=0}^{\infty} N(k).t^k = 200 + 5.88t + 0.05527t^2 - 0.00026t^3 - 0.0000098t^4 \quad (10)$$

Table 2. Comparison of the solution by Analytical and DTM

t (months)	Analytical solution $N(t)$	DTM method $N(t)$
2	211.950027	211.978843
4	224.322853	224.385171
6	237.099674	237.200859
8	250.257778	250.404019
10	263.770571	263.969000
12	277.607684	277.866387

Example 3:

Nonlinear differential equation is difficult to solve analytically. We need to employ numerical methods or Laplace transform methods to solve. DTM method helps to solve nonlinear equations. In this paper we approach to solve one such equation and see comparison with Runge-Kutta 4th order method.

$$\text{Consider } \frac{d^2y}{dx^2} = y + yx^2; y(0) = 1, y'(0) = 0 \quad (11)$$

Applying DTM, we get

$$Y(k+2) = \frac{1}{(k+2)(k+1)}[Y(k) + \sum_{r=0}^{k} Y(k-r)\delta(r-2)]; Y(0) = 1, Y(1) = 0 \quad (12)$$

$$where \delta(r-2) = \begin{cases} 1 if r = 2 \\ 0 if r \neq 2 \end{cases} and k = 0,1,2,3\ldots.$$

$$Y(2) = \frac{1}{2}[Y(0) + Y(0)\delta(-2)] = 0.5$$

$$Y(3) = \frac{1}{(1+2)(1+1)}\left[Y(1) + \sum_{r=0}^{1} Y(1-r)\delta(r-2)\right] = 0$$

$$Y(4) = \frac{1}{(2+2)(2+1)}\left[Y(2) + \sum_{r=0}^{2} Y(2-r)\delta(r-2)\right] = 0.125$$

$$Y(5) = Y(7) = 0 = Y(9) i.e for odd values$$

$$Y(6) = \frac{1}{(4+2)(4+1)}\left[Y(4) + \sum_{r=0}^{4} Y(4-r)\delta(r-2)\right] = 0.020833$$

$$Y(8) = \frac{1}{(6+2)(6+1)}\left[Y(6) + \sum_{r=0}^{6} Y(6-r)\delta(r-2)\right] = 0.002604$$

Solution of equation (11) is $y(x) = \sum_{k=0}^{\infty} Y(k)x^k$

$$\text{Thus } y(x) = 1 + 0.5x^2 + 0.125x^4 + 0.020833x^6 + 0.002604x^8 \quad (13)$$

Table 3. Comparison of the solution with Runge-Kutta method and DTM

x	$y(x)$ **(DTM method)**	$y(x)$ **(RK method)**
0.1	1.0050	1.0050
0.2	1.0202	1.0202
0.3	1.0460	1.0460
0.4	1.0833	1.0833

4. Conclusion

DTM (differential transform method) is used to solve the differential equations of first and second order arised in the practical physical situations like electric circuits as in Example 1 consisting an inductor and capacitor, in population growth model narrated in Example 2. The results are compared with the analytical solutions and are in good agreement.

DTM method helps in analyzing non linear differential equations provided in Example 3 easily compared with numerical solutions obtained by fourth order Runge-Kutta method. As analytical approach is quite tedious in analyzing ODE which is non linear, this method is advantageous.

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