

Original Article

Addition of Two Equal and Opposite Integers: A Learning Process

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Abstract - At school level, it is seen that learners face in troubles to achieve the learning competencies on the topic 'addition of two equal and opposite integers' in algebra for improper teaching-learning process. So, proper activity is needed to help the learners in developing their natural process of reasoning on it. In view of this, in this paper, four activities on 'addition of two equal and opposite integers' have been developed and presented sequentially for the target group, i.e. the learners, considering their ages, grades and ability levels. The collection of text books, analysis of text books/text book scanning, developing the activity, sequencing of steps of the activity and experts' opinions are the materials and methods which have been adopted for this study. Three activities out of four have not yet been considered in the prescribed text books of mathematics in both West Bengal Board of Secondary Education (W.B.B.S.E.) and National Council of Educational Research and Training (N.C.E.R.T.). These activities will definitely help the learners to get the relish for learning the 'addition of two equal and opposite integers'.

Keywords - Addition, Opposite integers, Text book scanning, Developing the activity, Activity-oriented learning, Upper primary at school level.

2010 Mathematics Subject Classification: 97A99, 97B50, 97C70, 97D40, 97H20.

1. Introduction

Integers, a fundamental concept of numbers for which algebra and some other branches of mathematics have been developed. It has limitless role in science and technology. Considering its significant roles in modern civilization, it has been introduced in the beginning of upper primary at school level i.e., class-VI, age group 11-12 years in our country.

Now, the research documentation and comments of mathematicians express that a large number of learners' performance in mathematics at school level is not cheering up [1-6, 22-23, 41-42]. As a secondary level school teacher, the researcher has the similar experience. He has got a worrisome finding from his application of an achievement-cum-diagnostic test on algebra on school learners. The weakness in algebra, specially in integers, of learners started from their early stages [25]. There are many reasons which are responsible for this unwanted situation. Improper teaching-learning approach is one of them. As a result, they do not get the logical beauty and the taste of its abstraction. So, it is an urgent need to emphasize for removing this unwanted situation. The presentation of the concept should be meaningful among the learners playfully. At upper primary level as well as early stage of algebra, the presentation of content should be activity based using concrete materials. It may be done through real life situation/real life activity i.e., the meaningful and logical activity which will help the learners to catch the abstractness of integers and they will be able to explain several facts which are related to sense of integers as well as negative number from daily life activities and natural facts and phenomena. The peer group learning mode is more preferable in this learning approach. The development of natural process of reasoning of learners will be enriched through achieving the learning competencies on integers by activity-oriented learning in peer group mode.

The author has already done some works on integers and some other major concepts in this direction [30-40, 44]. Now, the 'addition of two equal and opposite integers is zero' is a crucial concept of integers. Although, few activities on it have been presented in the prescribed text books of mathematics in both N.C.E.R.T & W.B.B.S.E. But these are not appeared properly for the target group i.e., considering the learners and their ages, grades and ability levels. So, the researcher has developed four activities on 'addition of two equal and opposite integers' from the known materials and facts considering the target group.



Therefore, in this paper, four new developed activities on ‘addition of two equal and opposite integers’ have been presented which will help the learners to get the relish for learning the same.

2. Objective of the Study

The aim of this study is to develop various types of learning activities on ‘addition of two equal and opposite integers’ in the ambience of known materials and natural facts and activities.

3. Materials and Methods

Materials and methods have been discussed below:

3.1. Collection of Text Books

The prescribed text books of mathematics from class-VI to class-VIII standard of W.B.B.S.E., N.C.E.R.T. and other available books from the market were collected at first. [7-9, 13-15, 43]

3.2. Analysis of Text Books/Text Book Scanning

These books were analysed to identify the activities of the ‘addition of two equal and opposite integers’ in algebra.

3.3 Developing the Activities

The activities of the ‘addition of two equal and opposite integers’ were developed considering the psychological order of learners and the learners’ ability level.

3.4. Sequencing the Steps of the Activities

The several steps of the developed activities were sequenced keeping in view the logical order of the subject matter and the psychological order of learners. Several steps of the developed activities are sequenced in stage-I and stage-II for the cognitive development of the learners.

3.5. Experts’ Opinions

Experts’ opinions were taken on the developed activities.

Finally, the sequential form of the activities on ‘addition of two equal and opposite integers’ incorporating the experts’ opinion was developed.

4. Salient Points of the Study

Here, total four activities of addition of two equal and opposite integers have been developed using calibrated beaker by the researcher. These activities and their several steps have been presented sequentially below.

Activity-1: Activity for addition of two equal and opposite integers is always zero using calibrated beaker.

Requirements: Cup, calibrated beaker, water filled bucket.

Mode: Pair group. **Strategy:** Learning through activities.

Objective of the development: Cognitive development.

Activity Follows

Stage-I: The facilitator will do the following activity involving the learners.

1. The facilitator will show a cup (Figure 1) and a calibrated beaker (Figure 2).



Fig. 1 Cup

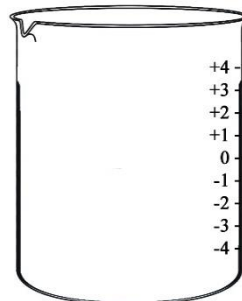


Fig. 2 Calibrated beaker

Here, one cup of water is equal to the one-unit height of the beaker.

Stage-II: The learners will do the following activities with the help of facilitator, if needed.

Each pair group

1. Takes one cup, two calibrated beakers and a water filled bucket.
2. Pours both the beakers with water up to normal level i.e. 0 level.
3. Takes then one cup of water from the first beaker and then pours the same water into the second beaker.
4. Writes the height of the water level of each beaker from normal surface level.
5. Again, takes one cup of water from the second beaker and then pours the same water into the first beaker.
6. Tells now the height of the water level of each beaker.
7. Writes it in mathematical form for the first beaker.
8. Writes it in mathematical form for the second beaker.
9. Takes then two cups of water from the first beaker and then pours the same water into the second beaker.
10. Writes the height of the water level of each beaker from normal surface level.
11. Again, takes two cups of water from the second beaker and then pours the same water into the first beaker.
12. Writes the height of the water level of each beaker from normal surface level.
13. Writes them in mathematical form for both beakers.
14. Takes then three cups of water from the first beaker and then pours the same water into the second beaker.
15. Repeats the works like the steps 12, 13 and 14.
16. Finally concludes about the result of 'addition of two equal and opposite integers'.

The work is illustrated below

Each pair group will take the following cup (Figure 3), two calibrated beakers (Figure 4) and a water filled bucket (Figure 5) .



Fig. 3 Cup

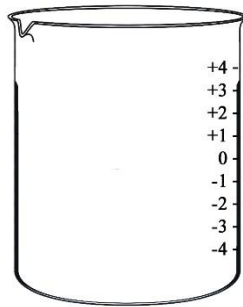


Fig. 4 Calibrated beaker



Fig. 5 Water filled bucket

Then, they will pour the calibrated beakers with water up to normal level i.e. 0 level which are shown in Figure 6 & 7.

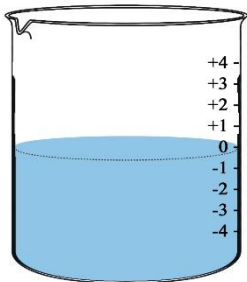


Fig. 6 Calibrated beaker

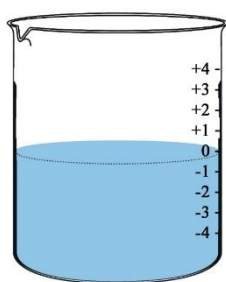


Fig. 7 Calibrated beaker

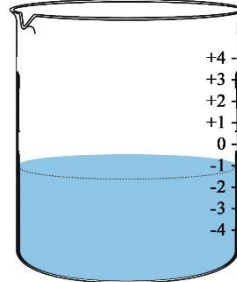


Fig. 8 Calibrated beaker

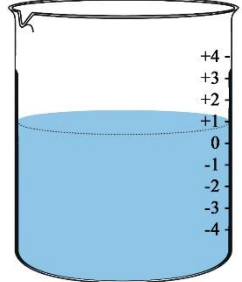


Fig. 9 Calibrated beaker

They will take one cup of water from the first beaker (Figure 7) and pour the same into the second beaker (Figure 8) which are shown in Figure 8 & 9.

In the first beaker (Figure 8), the height of the water level= (-1)
and in the second beaker (Figure 9), the height of the water level= (+1)

Again, they will take one cup of water from the second beaker (Figure 9) and pour the same into the first beaker (Figure 8) which are shown in Figure 10 & 11.

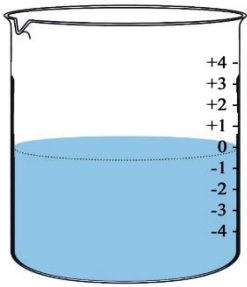


Fig. 10 Calibrated beaker

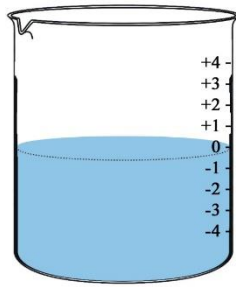


Fig. 11 Calibrated beaker

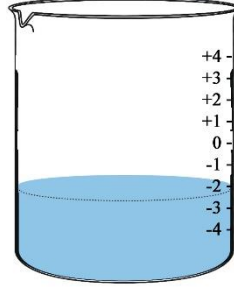


Fig. 12 Calibrated beaker

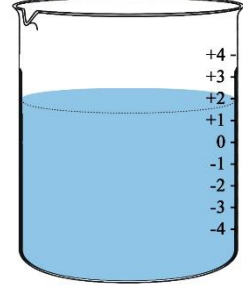


Fig. 13 Calibrated beaker

Now, the height of the water level of each beaker (Figure 10 & 11) = 0

The resulting operations can be written for the first beaker (Figure 10) is $(-1) + (+1) = 0$ and for the second beaker (Figure 11), it is $(+1) + (-1) = 0$

They will take two cups of water from the first beaker (Figure 10) and pour the same into the second beaker (Figure 11) which are shown in Figure 12 & 13.

In the first beaker, the height of the water level (Figure 12) = (-2)

and in the second beaker, the height of the water level (Figure 13) = $(+2)$

Again, they will take two cups of water from the second beaker (Figure 13) and pour the same into the first beaker (Figure 12) which are shown in Figure 14 & 15.

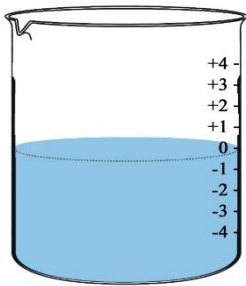


Fig. 14 Calibrated beaker

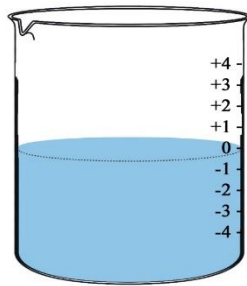


Fig. 15 Calibrated beaker

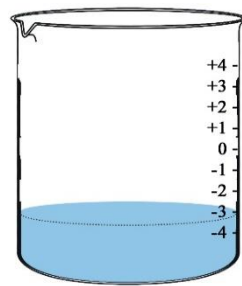


Fig. 16 Calibrated beaker

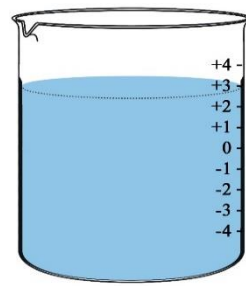


Fig. 17 Calibrated beaker

Now, the height of the water level of each beaker (Figure 14 & 15) = 0

The resulting operations can be written for the first beaker (Figure 14) is $(-2) + (+2) = 0$ and for the second beaker (Figure-15), it is $(+2) + (-2) = 0$

They will take three cups of water from the first beaker (Figure 14) and pour the same into the second beaker (Figure 15) which are shown in Figure 16 & 17.

In the first beaker (Figure 16), the height of the water level = (-3)

and in the second beaker (Figure 17), the height of the water level = $(+3)$

Again, they will take three cups of water from the second beaker (Figure 17) and pour the same into the first beaker (Figure 16) which are shown in Figure 18 & 19.

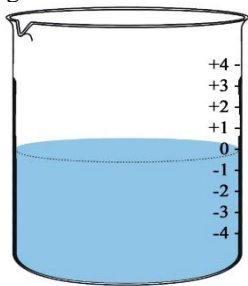


Fig. 18 Calibrated beaker

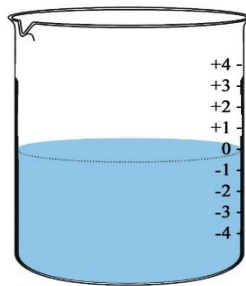


Fig. 19 Calibrated beaker

Now, the height of the water level of each beaker (Figure 18 & 19) = 0

The resulting operations can be written for the first beaker (Figure 18) is $(-3) + (+3) = 0$ and for the second beaker (Figure-19), it is $(+3) + (-3) = 0$

Therefore, addition of any two equal and opposite integers is always zero.

Activity-2: Activity for addition of two equal and opposite integers/numbers is always zero using an instrument made of wooden scale of numbers and an indicator running over an attached wire.

Requirements: An instrument made of wooden scale of numbers and an indicator running over an attached wire, exercise book, pen/pencil.

Mode: Pair group. **Strategy:** Learning through activities.

Objective of the development: Cognitive development.

Activity Follows

Stage-I: The facilitator will do the following activities involving the learners.

1. Identify the following integers on wooden scale of number line using indicator.

+6, +3, 0, -3, -6

2. Add (+5) and (-3).

Stage-II: The learners will do the following activities with the help of facilitator, if needed.

Each pair group

1. Takes a wooden scale of number line including indicator.

2. Adds (+5) and (-5) on this number line.

3. Tells the result.

4. Writes the result in mathematical form.

The work is illustrated below

Each pair group will add (+5) and (-5) using a wooden scale of number line including indicator which is shown in Figure 20.

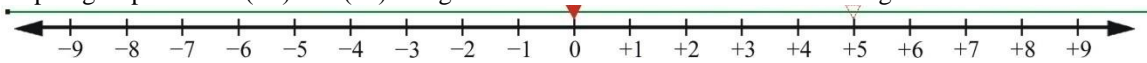


Fig. 20 Addition of (+5) and (-5)

The addition of (+5) and (-5) is 0.

$\therefore (+5) + (-5) = 0$

Activity-3: Activity for addition of two equal and opposite integers is always zero using two wooden/plastic scale of number line.

Requirements: Two wooden/plastic scale of number line, exercise book, pen/pencil.

Mode: Pair group. **Strategy:** Learning through activities.

Objective of the development: Cognitive development.

Activity Follows

Stage-I: The facilitator will do the following activities involving the learners.

1. Identify the following integers on wooden scale of number line.

+6, +3, 0, -3, -6

2. Add (+5) and (-3).

Stage-II: The learners will do the following activities with the help of facilitator, if needed.

Each pair group

1. Takes two wooden scale of numbers.

2. Adds (+5) and (-5) using these two scales and following these directions.

i. Fix the first scale.

ii. Set '0' of the second scale at (+5) of the first scale.

iii. Find the number of the first scale at (-5) of the second scale.

iv. Tells the result.

3. Writes the result in mathematical form.

The work is illustrated below

Each pair group sets '0' of the second scale at (+5) of the first scale. Then they find the number '0' of the first scale at (-5) of the second scale which is the required result. It is shown by the Figure 21.

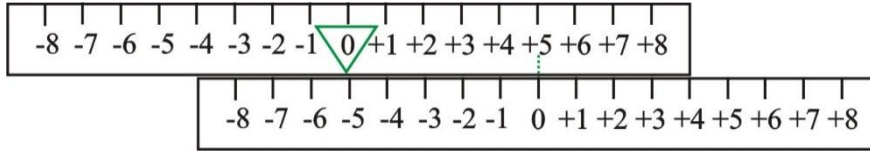


Fig. 21 Addition of (+5) and (-5) on the number line

The addition of (+5) and (-5) is 0.

$$\therefore (+5) + (-5) = 0$$

Activity-4: Activity for addition of two equal and opposite integers is always zero using number line on exercise book.

Requirements: Instrument of geometrical box, exercise book, pen/pencil.

Mode: Pair group. **Strategy:** Learning through activities.

Objective of the development: Cognitive development.

Activity Follows

Stage-I: The facilitator will do the following activities involving the learners.

1. Identify the following integers after drawing a number line.

+6, +3, 0, -3, -6

2. Add (+5) and (-3).

Stage-II: The learners will do the following activities with the help of facilitator, if needed.

Each pair group

1. Draws a number line on exercise book.

2. Adds (+5) and (-5) on this number line.

3. Tells the result.

4. Writes the result in mathematical form.

The work is illustrated below

After drawing a number line, each pair group will add (+5) and (-5) on it which is furnished in Figure 22.

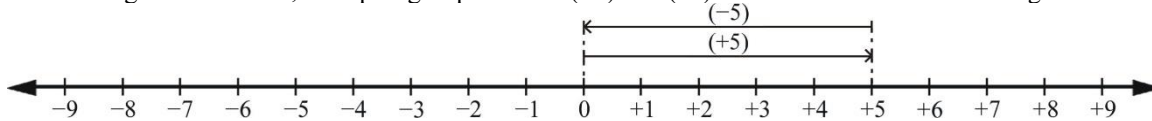


Fig. 22 Addition of (+5) and (-5) on the number line

The addition of (+5) and (-5) is 0.

$$\therefore (+5) + (-5) = 0$$

5. Conclusion

This paper contains four activities on ‘addition of two equal and opposite integers’ in a new look and these activities have been presented sequentially. All these activities except activity-4 have not been considered in the prescribed text books of mathematics in both W.B.B.S.E. and N.C.E.R.T. But the presentation of activity-4 is more meaningful and logical than the text books of mathematics in both W.B.B.S.E. and N.C.E.R.T.

In the chapter of Integer in the NCERT’s Mathematics Text Book for class-VI, there is an activity ‘Do this’ which is started like “Take two different coloured buttons like white and black. Let us denote one white button by (+1) and one black button by (-1). A pair of one white button (+1) and one black button (-1) will denote zero i.e. $[1+(-1) = 0]$.” [p-24]

In order to introduce negative integer (-1) by a black button, a white button is considered as (+1) and pair of buttons, one white and one black is considered as zero, the additive identity. In the case of composition, identity is introduced first and with the help of identity the inverse is defined. Here how does the concept of identity (additive) is conceived by a pair of different coloured buttons (i.e., two buttons) to a child? Similar question maybe raised for the similar type of activity adopted in the prescribed text books of mathematics of WBBSE. Any kind of deviation of the ‘identity’ concept will result in the same to the inverse concept with cumulative effect. It seems, in deed, a deeper problem related to maturity of perception of ‘abstract concept’. So much thoughts and experiments are required to cope up the problem.

This study will help the facilitator and the learners to understand about ‘addition of two equal and opposite integers’ through activity-oriented learning.

Collection of multiple numbers of activities for addition of two equal and opposite integers' will help the facilitator to choose the appropriate activity for the learners considering the learners' ability levels, time limits, availability of working materials and class room ambience etc.

This study will also help to prepare a proper syllabus, to develop a good text book and to improve the quality of teaching-learning process on addition of two equal and opposite integers.

These types of activities will help the children to enjoy learning mathematics so that the phobia in mathematics will be reduced and stop the drop out of learners who leave from institution on account of anxiety for mathematics learning.

Special interest towards mathematics can be enhanced which will be helpful for entire science education.

As these activities are presented step by step i.e., in an iterative sequential form. This can be appropriate in preparing text material through computer-based learning.

6. Implementation

This study may be implemented in the text books of mathematics at school level. It could also be used in the teaching-learning process. It would be used in preparing text material through computer-based learning and in on-line teaching-learning process.

7. Further Study

These activities for addition of two equal and opposite integers may be applied on large number of samples of class-VII.

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