# The Riemann Hypothesis: The Vision and How We Proceed 

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#### Abstract

The work in this document advances concepts and procedures that bridge the gap between the prime enumeration and the zeta zero location in three main segments. The first segment is based on the Egyptian enumeration system which is used in this work to design the Construction and Detachment Principle, with the goal of establishing an extension field where the zeta zero location resides side-by-side with the integer under study. In the absence of a reliable formulae, a limit point can be established using a set of heuristics from the Golden Ratio and the Egyptian Continuous Fraction Algorithm. The latter formed the genus of a method that identified a unique zeta zero for a given $x$ value. This is done in accordance with Dimension.al Analysis to test the validity of the zeta zero location distribution with respect to the prime distribution. The second segment establishes two formulas to test the relevancies of the zeta location with respect to the volume of the knapsack boxes containing prime numbers up to a given $x$ value. This document also provides a method to test whether the formula produces an accurate counting of the prime for the given value of $x$ by using the Taylor Approximation Polynomial at the slope point. This slope is the closest point to where the zeta zero resides. In the third segment of the work, the results will show the distribution of the zeta zero's location and, in doing so, lessen the need of finding the largest gap between prime numbers. Finally, Complex analysis as well as modal logic are explored to establish the parameters of a dynamic algorithm for the general solution of the $\pi(x)$ conjecture.


Keywords - Riemann Hypothesis, Fraction Algorithm, Prime enumeration, Zeta zero location.

## 1. Introduction

Creation is the source of all knowledge that is known. Science does not escape this axiom. Thus, this universal truth permeates all aspects of what we do. The universal language of mathematics and the characteristics of Dimensional Analysis offer a variable relationship that is explored in this document to extrapolate the implicit knowledge that exists as we know it.

Dimensional Analysis begins with an initial guess of the solution that is envisioned. There are many ways to tackle Dimensional Analysis; one aspect is to consider formulas that are universal and consistent over time. However, since prime numbers have been elusive for many years, this document describes a new approach to this problem. Thus, to borrow an expression of a well-known mathematician, Terence Tao, "we know the size of the box that contains prime numbers, but we do not know the content of the box." To solve this and other issues, universally known variables must be utilized in search of a specific result, i.e., the Fibonacci Ratio. The latter offers a unique tool in identifying prime numbers, although it is a scissor relationship. Yet, it offers a way forward. Thus, combined with current knowledge of prime numbers, we can delineate the closest heuristics on a specific aspect of the Riemann Hypothesis. Namely $\pi(x) \simeq x /(\ln (x))$, which is a global asymptotic estimate meaning the approximation of the number of prime up to a specific $x$ value. Since the approach of $\pi(x)$ is local, the formula needs to be modified to a specific number under studies, which implies utilizing a step function that is embedded in the prime sequence to reformulate $\pi(x)$. This is necessary because we are looking at the $\pi(x)$ as an initial value solution to the study of the volume of the knapsack containing prime number up to an arbitrary x value. The missing component is the general solution. This is a wellknown structure in differential equations. This type of problem has a name, it is called the initial value problem.

### 1.1. Formula Outline

The formula presented in this document contains the following main points Knapsack Problem: The Knapsack data candidate has a limit point.

- Fibonacci Egyptian Fraction
- The Golden Ratio
- The investigation of the zeta zero location distribution as it is related to prime numbers.
- The essence of the work
- The zeta location
- The counting of zero
- The calculation: The Torus Reality


## 2. The Knapsack Data Candidate Has a Limit Point

- The prime sequence behaves like a step function and each step is of a different size, which implies that each x value must provide its own rate of descent toward a plateau point with a universal limit point. Two issues come to mind when we want to delineate the knapsacks problem:
- First: The Fibonacci Egyptian Fraction Algorithm.
- Second: The Golden Ratio.
- Each of these concepts complement each other toward the same goal - Achieving an exact volume for the sides of the knapsack, using the Golden Ratio and the Fibonacci Egyptian Fraction concept to establish a limit point.


## 3. Fibonacci Egyptian Fraction

The Fibonacci Egyptian fraction is a "greedy" algorithm design for an optimal solution. In this case, we want to establish the rate of descent of a fraction "by being greedy," i.e., the largest portion of the rational will be used as a step function. The remaining segments are insignificant by design.

- Greedy Algorithm: gcd $(\mathrm{a}, \mathrm{b})=\mathrm{d}$
- If $d(a, b)=1$, then we say $a$ and $b$ are relatively prime
- The best way to begin this idea is to work with the asymptotic nature of the prime conjecture. $\Pi(\mathrm{x}) \simeq \frac{x}{\ln (x)}$

Using this, we can begin the work and adjust as we see progress. This will be elaborated on later in the document.

- The following formulation is the fundamental of the Egyptian Fraction:
$1 / a=(1 /(a+1)+1 / a(a+1)$
For example, $d$ is not 1 thus, i.e., $(14,6)=2$

$$
3 / 7=1 / 3+1 / 11+1 /(11 * 3 * 7) \quad \mathrm{c} 33=1 / 2+1 / 3+1 / 5+1 / 7+1 / 11+1 / 13
$$

With the size of the step function known, the formula of $\pi(\mathrm{x})$ can now be adjusted, pending the limit point derivation.

## 4. The Golden Ratio

The Golden Ratio is a relation that is universal, and Dimensional Analysis is referring to that reality. This is a proportion well known in nature. Granted, it is also a scissor relationship that needs to be adjusted. The two values used for the purposes of this document are either 1.61803369 or 0.61803369 . These are the ratios of the Fibonacci sequence i.e., 11235813...

The following is the equation associated with this sequence:

$$
x^{2}+x-1=0 \text { thus } x=\frac{1 \pm \sqrt{5}}{2}
$$

The sequence can be found by: $\mathrm{X}=\left(\frac{1 \pm \sqrt{5}}{2}\right)^{n}$ i.e $11235813 \ldots$, $\mathrm{n}=0,1,2 \ldots$
Now, let $\emptyset=\frac{1 \pm \sqrt{5}}{2}=1.61803$
At the end of each step size is a prime. This document only focuses on the gap between two prime numbers. So, the relevant interval lies in manipulating this reality to our

### 4.1. Zeta Zero Location

The Riemann Hypothesis is a conjecture that has two sets of zeros. One is considered trivial, which is the negative even numbers. The other is the non-trivial zero, which is part of the complex plane with a real part of $1 / 2$. This is of interest since it implies results on the distribution of prime numbers, making the gap point between primes become manageable, implying that this distribution is more stable. However, the following issues arise:

- Where are the zero-locations coming from?
- Is there a formula?
- What is its relation to prime numbers?

Currently, using Dimensional Analysis, we can modify the Golden Ratio value to set a limit point to find the zero's location associated to the x value under study. This extension can then be folded into a torus. This process acts on the rho method and will delineate a limit point that will address the gap points between prime numbers.

An algebraic construction is used in an extension field to find a point of intersection in the complex field. This is how the strip of conversion will be understood. However, we have a lack of basic, fundamental knowledge and the challenge is to fill in the gap in this knowledge base.

The Riemann zeta function $\varsigma(s)$ is defined in the half plane:
$\{\mathrm{R}(\mathrm{s})>1\}$ by the formula:
$C(\mathrm{~s})=1^{\mathrm{s}}+2^{\mathrm{s}}+1 / 3^{\mathrm{s}}+1 / 5^{\mathrm{s}} \ldots$
Because of the fundamental of arithmetic as well as the geometric series formula:
$\left(1-1 / p^{s}\right)^{-1}=\sum 1 / p^{s}=1 / p^{2 s}+1 / p^{s} \ldots \ldots$
One has the Euler product formula:
$\varsigma(\mathrm{s})=\prod_{p}\left(1-\frac{1}{p}\right)^{-1}$
Where the product is over primes. This links the zeta function analytic to number theory. The Euler product formula shows that $\varsigma(s) \pm 0$ whenever $R(s)>1$.

### 4.2. The Essence of Work



Hieroglyphic Numbers


Fig. 1 Model of Enumeration
The Golden Ratio provides two values that can be considered limit points. However, we need to adjust the value to find the zeta location associated with the x value under studies. Namely, $.61 / 5^{\wedge} .5=.2728002933$. To set a universal limit, we will use the Egyptian system, namely, the Model of Enumeration (Figure. 1).

Using the process of construction and detachment principle, we can work on each segment of the numeral under consideration as well as the decimal extension of the numeral that we identified as the zeta location for that number. For our purposes, we will begin construction for 1234567 followed by detachment. Using 0.273000 as a limit point in order to lock in the best zeta location extension.

Note: An integer can be represented with its natural extension. Using " 1234567 " as an example: 1/0.273/1/4026650/0.00000000000073685; Representation of the number:1234567.201

This format is the same as the Egyptian enumeration system; and is the preferred method because it allows detachment of elements in the enumeration of numbers such that a limit can be easily established. We achieve this by using the Golden Ratio and the Fibonacci Egyptian Fraction individual expansion to extrapolate what is needed for the limit point.

Note: The non-trivial zeta zero locations are unique.
The value of $1 / 4026670$ comes out of the Egyptian fraction algorithm.
We begin searching for an appropriate expansion base on $\pi(x)=\frac{x}{\ln (x)}$, then adjust according to the input value x .
The decimal detachment to find the limit point is as follows:

$$
=1-(1234567-1 / 0.273 / 1 / 4026650 / 0.00000000000073685) ; .201
$$

Must change $201 \ldots$ until .27300 is found.
ABS(1/0.35051002/1/4026650/0.00000000000073685-1234567+1)/1000000 DONE.
Here, using the zeta location as a rate of decent, the knapsack volume plateaus when the error term is used as a limit point. In folding the input x and the associated zeta location into itself, a torus is formed that eliminates the need to find the size of the gap between two primes. A torus is defined as two non-intersecting circles embedded in the same plane:

Since a torus is parallel circles that are not sitting in the same plane, they are, by structure, looking at concentric circles. Thus, we need to reduce the last circle to a point. This can be done using Cauchy's Integral Theorem of Imaginary Field. The $\pi$ result after integration is 2 . Now, using the latter as a limit point, we can reduce our outer circle to establish the area of Cauchy's expectation. In doing so, we find the delineate (i.e., the last prime in the knapsack volume we searched for).

### 4.3. The Calculation: The Torus Reality

The second step is the Domain. Applying the error equation stipulate earlier: $\frac{x}{\ln (x)^{2}}$; we are now adding $\pi(\mathrm{x})$
$1234567 / \mathrm{LN}(1234567)+1234567 / \mathrm{LN}(1234567)^{\wedge} 21234567 / \mathrm{LN}(1234567)+1234567 / \mathrm{LN}(1234567)^{\wedge} 2$; i3 Error Growth (I2)^I4/(1-0.3505002)/ ( $2 * \mathrm{~A} 3$ ) ; I2 contains, $\mathrm{a} 3=2 \pi$ formula 1
$\mathrm{LN}(\mathrm{I} 3) / \mathrm{LN}(\mathrm{I} 2) ; \mathrm{I} 4$ Contains a view of the process.
A function $f(z)$ has a period $C$ if and only if: $F(z+C)=f(z)$ for all $z$. In the complex plane it is fine to consider a value for $C$.
$e^{2 \pi i}=1$ by IF
$e^{z+2 \pi i}=e^{z} e^{2 \pi i}=e^{z}$
So, the exponent function is periodic with imaginary period $\mathrm{C}=2 \pi \mathrm{i}$. The method used is analogous to dealing with an angle greater than 360 degrees, by dividing by $360^{\circ}$ to make the domain smaller by imposing a limit $1-\mathrm{L}$ into the domain of the $\ln (\mathrm{x})$. The working assumption is the zeta location and the x value counting the number of zeros is close to each other. To delineate this reality, the bases need to change to the e function and proceed accordingly:
(I2)^I4/(1-0.3505002/ $e^{L}$.Formula 2 limit reach is near $y^{\wedge}$ thus a tori approaches the x value is: $95,359.71$
For these two values, respectively: 85360 VS 95358.045 , round up: ((I2)^I4/(1-0.3505002/A4^3.44590 02),0)

- Left
Zeta L
RIGHT
- $2.04917 \mathrm{E}-05$
1.419777544
1.14105932

Here, we have the structure of a torus not centered at the origin. The effect of applying multiples of $1 \pi, 2 \pi, \ldots \mathrm{k} \pi$ reduce the outer layer of a torus structure to the limit of the knapsack volume - only if we perform a base change to e. Under this scenario, the desired limit will be reached for any input value of $x$ under study. This implies that we are dealing with an embedded torus. If L is the center of the torus, $1-\mathrm{L}$ is the neighborhood to L . This geometric series is related to the zeta location i.e., 1-L.

## John Nash

Here we need to talk about John Nash the father of embedded geometry. He shows that the distance in a two-dimensional plane is equivalent to any embedded sphere of a three-dimensional plane or higher. Surely enough, a torus was one of the classical ways to prove it. The PAC-MAN game outlines the complexity of the problem since it is a visual paradox.

By visual paradox we mean either travelling from the center going up and down or left and right. If this is a square piece of paper, the distance in any direction is equal. However, if we fold the paper to create a torus, the distance left to right seems to be shorter. Thus, a problem - Hence the paradox. This issue occupied young minds for a considerate amount of time and was finally given to John Nash as a dare. This Virginian native was a challenge for his peers. He solved the problem by crunching the torus into smaller rate of changes until it became flat. Hence the name flat torus. He used multiple rates of change, smaller and smaller, to achieve this task. He called it Non-Smooth Embedded. The second he called it smooth embedded to achieve the same thing through rate of change with more dimensions, in the case of a torus six dimensions, this solution used derivative and differential analysis. More can be said but I digress here.

Going back to John Nash, using derivative and differential analysis as a tool, he arrived at a non-trivial answer that shows that by applying a wave to a torus plane, the distance between a flat plane (called intrinsic) and a torus plane is identical. The most important takeaway here is you are changing the position, but not necessarily the distance. Hence in The Riemann Hypothesis strip, the prime is to the right of the strip meaning the position of the prime. The distance is the same in any dimension, extrapolating further if the distance between the zeta location distribution and the prime distribution is closed. In this case, it will have the same property of closeness in any embedded space or dimension. Therefore, it is necessary to know about the gap between prime, but it is not sufficient. This is the strength of Modal Logic: The Possible Worlds of Saul Kripke modal theory.

### 4.4. The Gap Between Prime

Now this is the time to talk about the gap between prime. Three issues are related to the gap point between prime. The first two issues are first finding the minimum gap and second finding the maximum gap. The first concern has been addressed fully. The second concern is still an open question. In this work, we simply look for an alternative way to resolve the gap problem in a procedural form. The latter is shaped through dimensional analysis. This universal concept has been treated earlier in the paper. However, the third issue is how the gap is related to the Euler formula. The latter is so great that we will spend some time here to address its relevancy.

### 4.4.1. The Euler Formula 1707-1783

It is paramount that we talk about Euler the man. One of his enduring quality is that the majority of his work is achieved while he was blind. The circumstance about his blindness is not hard to construct in those days.

Infection in one eye and cataract in the other. Issues now a day are common cases However, this is the condition he was in and producing the most beautiful result in math. With a phenomenon memory he was called the Beethoven of mathematics. The quantity and quality of his work will never be surpassed.

Now we are free to expose some of his celebrated work. The best way we can achieve this is by demonstrating where is work is contributing to our own.

Euler celebrated formula $e^{x}=\cos (\mathrm{x})+\mathrm{i} \sin (\mathrm{x})$ has been treated in many books throughout the years. However, today we are going to stay closed to our own formula presented here in this work so that the intuitive nature in our formulas can be understood.

We need to begin with a power series, although this series usually concentrates on where this series is defined. For now, we will stay with the form of derivation to extrapolate other formulas.
$1 /(1-x)=\sum x^{n}=1+a_{1} x+a_{2} x^{2}+a_{3} x^{3} \ldots$
This geometric sequence can be transformed into an $e^{x}$ function by applying the derivative to the right of the equation.
Nth derivative of each term in the sequence
d/da(1)=0
$\mathrm{d} / \mathrm{dx}(\mathrm{x})=1$
$\left(d / d^{2} x\right)=2$
$\mathrm{d} / \mathrm{d}^{3}\left(\mathrm{x}^{3}\right)=6$
Adjusted for the coefficient of $e^{x}$ which is always 1 we get from the original sequence at $x=0$
$1+a_{1} x+a_{2} x^{2}+a_{3} x^{3} \ldots \rightarrow a_{0}=0, a_{1}=1, a_{2}=1 / 2, a_{3}=1 / 6$ so $a_{n}=n(n-1) f(n-2) \ldots$ commonly known as $n!$

$$
e^{x}=1+x+\frac{x^{2}}{2} \ldots
$$

Now let $x$ become ix then:
$\mathrm{e}^{\mathrm{ix}}=1+\mathrm{ix}+(\mathrm{ix})^{2} / 2!+(\mathrm{ix})^{3} / 3!\ldots$
$\cos x=1-x^{2} / 2!+x^{4} / 4!\ldots$
$i \sin x=x-x^{3} / 3!+x^{5} / 5!\ldots$
The point is to separate the real part from the imaginary part we get the Taylor expansion of $\cos x$ which is real follow by $i \sin x$ the imaginary part. Hence the right side of the equation establish the Euler formula namely:

$$
e^{x i}=\cos (\mathrm{x})+\mathrm{i} \sin (\mathrm{x}) \text { let } \mathrm{x}=\pi \text { then }
$$

Now, going back to the power series we can say further we can work with the left side of the power series to center the zeta zero location. The domain consists of $\pi(x)$ and the error term:
$\mathrm{D}=\mathrm{x} / \ln (\mathrm{x})+\frac{x}{\ln (x)^{2}}$ domain combine with the left-hand side of the power series.

> Formula 1: $\left(\mathrm{x} / \ln (\mathrm{x})+\frac{x}{\ln (x)^{2}}\right) /(1-\mathrm{a}) / 2 \mathrm{k} \pi$
> Formula 2: $\left(\mathrm{x} / \ln (\mathrm{x})+\frac{x}{\ln (x)^{2}}\right) /(1-\mathrm{a}) / \mathrm{e}^{\mathrm{L}}$

Formula 1 is used to shift the domain of the prime conjecture closer to the zeta location.

Formula 2 is used to embrace the contour of the e - function to be positioned in the Cauchy Reimann region of convergence within the boundaries of the singularities.

When the shift reaches the limit point of the singularity, we proceed with the location where the saddle point of the graph can be found. Finally, the latter is used by the Taylor Approximation Polynomial to find the slope of the e-curve for that particular $x$ value.

It is time that we introduce Bernard Riemann as a young man, when he spent intensive time studying complex numbers and applied his view in the distribution of prime numbers.

### 4.4.2. Bernhard Riemann

In 1859 a reserved German mathematician, Bernhard Reimann, under the auspice of his advisor, presented a paper on the prime number distribution with a specific hypothesis on the zeta zero location which he believed could unlock the distribution of prime yet could not prove it. His paper is outlined here:
$\varsigma(s)=\sum 1 / n^{s}=1 / 1^{s}+1 / 2^{s}+1 / 3^{s} \ldots 1 / n^{s} s$ is the imaginary numbers sq 1
Here again Euler prime product formula could be derived before we engage further in our own work.

$$
=\left(2^{s} / 1-2^{s}\right)\left(3^{s} / 1-3^{s}\right)\left(5^{s} / 1-5^{s}\right) \ldots
$$

How he gets to this product he multiplied sq 1 by $1 / 2$
$1 / 2\left(\varsigma(s)=1 / 2^{s}+1 / 4^{s}+1 / 6^{\mathrm{s}} \ldots\right.$
Here is where Euler great idea surface. He took this sequence and subtracted from the original sequence 1 and get
$\varsigma(\mathrm{s})-1 / 2^{\mathrm{s}} \zeta(\mathrm{s})=1+1 / 3^{\mathrm{s}}+1 / 5^{\mathrm{s}}+\ldots$ this continue with a set of odd number
Cleaning this work by factoring
$\varsigma(s)(1-1 / 2)=1+1 / 3^{s}+1 / 5^{s}+1 / 7^{\mathrm{s}} \ldots$ now he multiplied the last sequence by $1 / 3$
$1 / 3^{\mathrm{s}}\left(\varsigma(\mathrm{s})(1-1 / 2)=1 / 3^{\mathrm{s}}+1.9^{\mathrm{s}}+1 / 15^{\mathrm{s}}+\ldots\right.$ subtract last sq from next to last
$\varsigma(\mathrm{s})\left(1-1 / 2^{\mathrm{s}}\right)-1 / 3^{\mathrm{s}}\left(\varsigma(\mathrm{s})(1-1 / / 2)=1+1 / 5^{\mathrm{s}}+1 / 7^{\mathrm{s}}+1 / 11^{\mathrm{s}}+1 / 13+1 / 17 \ldots\right.$ The number that are not prime are subtracted $1 / 5^{\mathrm{s}} \varsigma(\mathrm{s})\left(1-1 / 2^{\mathrm{s}}\right)\left(1-1 / 3^{\mathrm{s}}\right)=1 / 5^{\mathrm{s}}+1 / 25^{\mathrm{s}}+1 / 35^{\mathrm{s}}+1 / 55^{\mathrm{s}}+\ldots$
$\mathrm{C}(\mathrm{s})\left(1-1 / 2^{\mathrm{s}}\right)\left(1-1 / 3^{\mathrm{s}}-1 / 5^{\mathrm{s}} \varsigma(\mathrm{s})\left(1-1 / 2^{\mathrm{s}}\right)\left(1-1 / 3^{\mathrm{s}}\right) \rightarrow\right.$ factoring the left-hand side
$\varsigma(\mathrm{s})\left(1-1 / 2^{\mathrm{s}}\left(1-1 / 3^{\mathrm{s}}\right)\left(1-1 / 5^{\mathrm{s}}\right) \rightarrow\right.$ we can see the prime product appearing clearly thus we can conjecture
$=\left(2^{s} / 1-2^{s}\right)\left(3^{s} / 1-3^{s}\right)\left(5^{s} / 1-5^{s}\right) \ldots$

### 4.4.3. The Argument

By the same stroke, Euler proved that the sequence of prime is infinite. This proof is done by contradiction, a simple assumption is needed here..

$$
\zeta(s)=\sum 1 / n^{s}=\zeta(s)=\prod_{p}\left(1-\frac{1}{p}\right)^{-1}
$$

Euler attached a great significance to above representation he noted on the left that $s \rightarrow 1, \Gamma(s) \rightarrow \alpha$, whereas on the right a product over prime occurred. Hence there must be an infinite number of primes; otherwise $\varsigma(s) \rightarrow \prod_{p}\left(1-\frac{1}{p-1}\right)$ (over a finite set) or $\zeta(\mathrm{s}) \rightarrow \alpha$
Now going back to our original sequence
$\varsigma(\mathrm{s})=\sum 1 / n^{s}=1 / 1^{\mathrm{s}}+1 / 2^{\mathrm{s}}+1 / 3^{\mathrm{s}} \ldots 1 / \mathrm{n}^{\mathrm{s}} \mathrm{s}$ is the imaginary
This series does not have a closed formed, thus:
The Reimann Hypotheses states that all non-trivial zeros of the zeta function have the form:
S=1/2+bi

This forms a strip of convergence where $\varsigma(s)>1$.
Now, we have three sectors of the zeta zeros:
$\mathrm{C}(\mathrm{s})=\sum 1 / n^{s} \quad \mathrm{R}(\mathrm{s})>1$
$\varsigma(\mathrm{s})=1 /(\mathrm{s}-1)+\sum_{n=1}^{\alpha} \int_{n}^{n+1}\left(1 / n^{s}-1 / x^{s}\right) \quad 0<\mathrm{R}(\mathrm{s})<1$ critical except the point $\mathrm{s}=1$,
$\varsigma(\mathrm{s})=2^{s} \pi^{s} \cos \frac{\pi(1-s)}{2} \Gamma(1-\mathrm{s}) \zeta(1-\mathrm{s}) \quad \mathrm{R}(\mathrm{s})<0$
Going back to the choice of using the e-curve beyond the point of its natural properties, one of the qualities of the zeta zero is that vertically, the zeros attract each other, whereas horizontally they repel each other. A significant gain when we are counting zeros. We are interested in the area below the slope where the final high of the curve is under study at L. This ability of attraction simplifies our work significantly. The zeta zeros are well behaved and dense as we move to larger values of x for consideration. This last piece of information is due to Terence Tao in his YouTube video lecture on vaporization (ref. "Terence Tao: Vaporization and freezing the Reimann zeta function").

The second reality of the strip is that it is symmetric about $R(s)=1 / 2$
The hope is that the gap size will be served well by tackling the critical strip after all the domain of the left side of the power series (i.e., $1 / 1-x$ ) is identical to the strip of convergence. We simply need to be rigorous in our research to delineate the new reality. We must take a detour here and talk about $\mathrm{H}_{0}$.

What do we know about $\mathrm{H}_{0}$ ?
It is an entire function of order 1, obeying the functional equations
$\mathrm{H}_{0}(-\mathrm{z})=\mathrm{H}_{0}(\mathrm{z})$ and $\mathrm{H}_{0}(\mathrm{z})^{\prime}=\left(\mathrm{H}_{0}\right)^{\prime}$ thus the zeros of $\mathrm{H}_{0}$ are symmetry around the real and imaginary 0 axes.
All the zeroes of $\mathrm{H}_{0}$ are contained in the strip:
$\{x+i y: y<1\}$
Riemann Hypothesis is all $\mathrm{H}_{0}$ are real.
Riemann-Von Mangoldt formula: for $\Gamma \geq 2$ the number $\mathrm{N}_{0}(\mathrm{~T})$ of zeroes in the rectangle
$\{\mathrm{x}+\mathrm{iy}: 0 \leq \mathrm{x} \leq \mathrm{T}:|\mathrm{y}| \leq 1\}$ is $\mathrm{T} / 4 \pi \log \mathrm{~T} / 4 \pi+\mathrm{O}(\log \mathrm{T})$
Proven using upper and lower bounds on $\mathrm{H}_{0}$ outside of the strip:
$\{(x+i y:|y|<1\}$, and upper bounds inside the strip)
A variant (due to Littlewood), if the Riemann hypothesis is true, then for fixed $\alpha$, one has:
$\mathrm{N}_{0}(\mathrm{~T}+\alpha)-\mathrm{N}_{0}(\mathrm{~T})=\alpha \log \mathrm{T}+\mathrm{O}(\log \mathrm{T})$ as $\mathrm{T} \rightarrow \alpha$ of
In particular, on RH mean spacing between zeroes of $H_{0}$ in $[T, 2 T]$ is roughly $1 / \log T$, and one has equidistributional of the zeroes at scales $\geq \eta(T)$ for some $\eta(T)=\alpha(1)$.

These parameters are talking about the zeroes on a large scale. Under that scenario it seems that the distribution is evenly spread. This fits the narrative that we are expecting. This I believe is what we all wanted in the search for additional knowledge of prime numbers. Obviously, more advances must take root, especially in the arena of Dynamic Programming. Presently, there are still issues in the field of decimal accuracy. Alfa Beta Pruning, commonly known as backtracking, is a necessary but insufficient tool to deal with the complexity we face with regard to the decimal field.

The real is a two-dimensional plane which is called the input plane and the imaginary is a projection to another twodimensional plane called the output plane (i.e., four moves are required). We are dealing with positioning (i.e., angles). It is fitting to see that under the complex system the real issue is indeed positioning. This is what should have been the concern all along. A concept worth mentioning is Analytical Continuity.

### 4.4.4. Analytical Continuity

Any complex number function $\mathrm{F}: \mathrm{C} \rightarrow \mathrm{C}$ that is analytic on a given region, can be extended uniquely to an analytic function defined for almost all real numbers, except for numbers that are called pol or singularities.
The left-hand side of the power series:
The rational function $1 /(1-x)$ extends the geometric series $f((x)$, the power series from the interval $-1<x<1$ to all real numbers except 1. Due to this interval, a strip is established: A region $s=1 / 2+$ bi bounded from $0<x<1$; extrapolating further, a symmetry is found to the strip around the value $1 / 2$.

This asymptotic behavior is similar to all rational equations. However, since it is the left side of a power series, this rationale is adding new knowledge to its behavior in the complex plane. The domain of the rational function is setting up multiple
singularities. The latter behaves like a boundary of a dead-end sector where the orientation is similar to the cot x graphically. If x is replaced by Formula 2, the dead-end region will be evident.

Going back to the fundamentals, we're now going to explain the zero's calculation of base 10 numeral and the log significance.
The Counting of Zeros
If we begin with zeros:

| - 10 | 1 |
| :--- | :--- |
| - 100 | 2 |
| - 1000 | 3 |

What happens halfway between 10 and 100 ?

- 1002
- X so using the following step: 3.1622776602(10)=31.622776602
- 31.6227766021 .5

This is the concept of the log work for base 10 . However, a better base is e. $\operatorname{Ln}(x)$ is what we used for the average size of the gap between prime numbers. The probability is $1 / \ln (\mathrm{x})$ and the expected value for an arbitrary x is $\mathrm{x} / \ln (\mathrm{x})$. This is only relatively accurate for large numbers; a limit must be imposed.

### 4.4.5. Average Number of Prime Factors

The model that best suits our needs is the $\mathrm{e}^{\mathrm{x}}$-curve. However, the basic curve that is used for discussing prime is $\ln \ln (\mathrm{x})$. The latter was a derivation of the gap found between primes. $\mathrm{e}^{\mathrm{x}}$ has properties that are unique and will be discussed and elaborated on graphically within this section (Figure 2 and Figure 3).

The link to the Riemann hypotheses begins in some small ways with the average factorization of numbers into prime factors. The interest in this document is the significance of the slope and its relation to the $\ln (\mathrm{x})$. Figure 2 and Figure 3 illustrate the basics, before we look at the subset of prime number factors in terms of Figures 2 and 3, refer to the YouTube video "The hidden link between Prime Numbers and Euler's Number."

### 4.4.6. The Way Forward to the Proof

Now, the properties of $e^{x}$ and the Taylor Approximation Polynomial can be used to find the high of the volume of the region related to the knapsack of prime numbers up to a given $x$ value. However, before reaching the goal, we must begin with the radius of convergence.

The radius of convergence established the least upper bound of our CR region. The maximum upper bound is at the singularity of the dead-end sector within the two boundaries we can find the saddle point.


Fig. 2 Number of Divisors up to 10 Using Slope


Fig. 3 Number of Divisors Using e

### 4.4.7. Proof the Beginning



Fig. 4 Function: $x=14, p=6$
According to CR , the natural boundary is $\cot (\mathrm{x})$ :

$$
\sum a_{n}\left(\mathrm{Z}-\mathrm{z}_{0}\right)^{n} \lim _{b=0 \rightarrow \alpha} R=\frac{a_{n+1}}{a_{n}}=n^{\frac{1}{x}}
$$

Given the radius of convergence of the Cauchy Reimann region, we can use the Tailor Polynomial Approximation to measure the closeness of the zeros of the zeta location. Finding the Taylor polynomial approximation at a specific location: $\mathrm{P}(\mathrm{x})=1+(\mathrm{x}-$ L)...

### 4.4.8. Taylor Approximation Polynomial

Hence for the number 14:
$\mathrm{p}(\mathrm{x})=1+(.13549-1.2074420274)=-.0719520274$
$1+(.13549-1.2074420274)=-.0719520274$
The slope $p(x)+1+x=L$ predicts the number of prime up to $n$, thus the number of zeros. Hence: $p(x)$ is a good estimation for the slope. The working assumption is the zeta location and the x value counting the number of zeros is close to each other. To delineate this reality, we need to change bases to the e function and proceed accordingly.


Fig. 5 The Point of Tangency
$e^{\wedge} z-e^{\wedge}|p(x)|=.0794939535$ is the closest point according to the original assumption (Figure 5).
Using Euler's formula, we can show the region of the dead-end region. Below the x axis is not our concern. The behavior of figure 5 mimics the behavior of figure 4 and the actual example.
$e^{x i} \cos (\mathrm{x})+\mathrm{i} \sin (\mathrm{x})$
Thus: $\cot (\theta)=\frac{\cos (\theta)}{\sin \theta)}$


Fig. 6 Cot Example
Reiterating what was stated earlier, Figure 6 displays the same nature as Figure 4, asymptoticly. This behaves like a deadend street controlled by the radius of convergence. There exists a saddle point where the final prime lies. The $2^{\text {nd }}$ property for large numbers greater than or equal to 13 digits, the radius of convergence remains at 1 .

Now, going back to the choice of using the e-curve beyond the point of its natural properties, one of the nice qualities of the zeta zeros is that vertically the zeros attract each other, whereas horizontally they repel each other. A significant gain when counting zeros. Needless to say, we are interested in the area below the slope where the final high of the curve is under study at L. This ability of attraction simplifies our work significantly. The zeta zeros are well behaved and dense as we move to larger values of $x$ for consideration. This last piece of information is due to Terence Tao in his lecture on vaporization (ref. "Terence Tao: Vaporization and freezing the Reimann zeta function").

The base conversion is required so that we can use contour principles to embrace the $\mathrm{e}^{\mathrm{x}}$ curve. This implies that we are going to find the saddle point of the curve.
Note: The saddle point is within the range imposed by the dead-end curve. Once you make the change to the e-base search for the L. of the Saddle Point and Singularity
A singularity in an imaginary plane is a point you should not cross. It happens that each x value under study will reach a saddle point close to a region bounded by a singularity.
Taylor polynomial slope on the following numbers:
Analysis of conversion in search of saddle pt
17329647.8606202000

Analysis of conversion in search of saddle pt
17329647.8606202000

### 4.4.9. Lagrange Error Bound of Taylor Polynomial

Hence, for the number 14:

$$
\begin{aligned}
& \mathrm{T}(\mathrm{a})=\mathrm{f}(\mathrm{a})+\mathrm{f}^{\prime}(\mathrm{a})(\mathrm{x}-\mathrm{a})+\mathrm{f}^{\prime}(x-a)^{2} / 2 \\
& \text { ERROR: } \leq \mathrm{M} \frac{(c-a)^{n+1}}{n+1!}
\end{aligned}
$$

- $\quad \mathrm{C}$ is the value we substitute in
- a is where the poly is centered in.
- n is the higher derivative used for poly
- $\quad \mathrm{M}$ is the max pf abs value of $\mathrm{F}^{\mathrm{n}+1}(\mathrm{x})$ on the interval with a and c as endpoints, or a convenient chosen point.

For example:

$$
e^{x}=1+x+\frac{x^{2}}{2} \ldots
$$

Finding the Taylor Polynomial Approximation at a specific location:
$\mathrm{P}(\mathrm{x})=1+(\mathrm{x}-\mathrm{L}) \ldots$
$1+(.13549-1.2074420274)=-.0719520274$ the slope.
Testing the error bound: $=$ r. 0025885471 error

### 4.4.10. The Error Analysis

We are familiar with the concept of error analysis in a procedural method. The world of mathematics is now combined with algorithmic programming. This is due to the increasing power that a computer can bring to a calculation. This marriage is preferred in the long run since the latter could put to rest the concern of accuracy. However, we are not out of the woods yet, we do need to be aware of machine language error potential and so a way to assure that we found the closest point to the high of the $\mathrm{e}^{\mathrm{x}}$ curve is to do robust programming.

Table 1. of convergence from $\boldsymbol{\pi}$ to e and Location of $L$

| $\begin{aligned} & \text { large } \\ & x \rightarrow \end{aligned}$ | 324543343 | $0.272999998 \rightarrow$ | $\begin{aligned} & \rightarrow \text { limit } \quad \mathrm{pt} \\ & \text { Fibo } \end{aligned}$ | $\downarrow$ e base,curve L |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 324543343 |  | 17506978.05 | 17506978.71 | 140108732.8 |

Here we identify two issues that could surface. We will focus closely to rounding up or down errors in each extension field. The error may lie in not carrying all decimal digits required, or in certain cases too many digits in consideration with others. This may render the result less accurate. This scissor relationship is common. In this paper, we relied on the closeness of the boundary of the singularities in the Riemann Cauchy dead-end sector of convergence as it is related to distance. However, an old technique used in the past called Alpha Beta Pruning, commonly known as backtracking, can be used to resolve this issue. It is a wellknown algorithm We then used continuous analysis to extrapolate further. However, more needs to be done in this arena. We must be mindful of Gödel's incompleteness Theorem because we must be humble in our approaches. Below we will attempt to present a complete view of the Gödel incompleteness theorem as a starting point. However, we will extrapolate further the reality presented here in the second chapter

### 4.4.11. Gödel's Incompleteness Theorem

The Gödel's incompleteness theorem is reproduced here for your convenience courtesy of google.
The first incompleteness theorem states that no consistent system of axioms whose theorems can be listed by an effective procedure (i.e., an algorithm) can prove all truths about the arithmetic of natural numbers. For any such consistent formal system, there will always be statements about natural numbers that are true, but that are unprovable within the system. The second incompleteness theorem, an extension of the first, shows that the system cannot demonstrate on its own consistency.

In our case, since this problem involved a limit point, this is not purely an arithmetic issue. We are bound to face this kind of issue indirectly. Thus, a concatenate idea needs to be part of the whole vision for a proper solution. Here we provide you with the essential elements for a mathematic-algorithmic solution. But additional work is needed to delineate all the main concern dealing with accuracy.

- The basic outline here is for an algorithm.
- Embedded Geometry
- Modal logic platform for dynamic programming
- Analytical Continuity and error Analysis
- Reiterating what we state earlier. Complex analysis as well as modal logic are explored to establish the parameters of a dynamic algorithm for the general solution of the $\pi(\mathrm{x})$ conjecture.


## 5. John Nash and Embedded Geometric Formally States

The John Nash embedded theorem states that every Riemannian manifold can be isometrically embedded into some Euclidean space. Isometric means preserving the length of every path. For instance, bending but neither stretching nor tearing a page of paper gives an isometric embedding of the page into Euclidean space because curves drawn on the page retain the same arclength however the page is bent.

The first theorem is for continuously differentiable ( $\mathrm{C}^{\prime}$ ) embedding and the second is for embeddings that are analytic or smooth class $C^{k}, 3 \leq \leq \alpha$.

The third aspect we need to envision is what we already mentioned earlier in the work: Modal Logic. For the purposes of this work, Modal Logic is a collection of formal systems developed to represent statements about necessity and possibility. In math it is a Kripke model of possible world (1990's viewpoint).

## 6. Taylor Series and Its Relation to Modal Logic

$$
\sum_{n=1}^{\alpha} \frac{f^{n}(a)(x-a)^{n}}{n!}
$$

Pursuing the work that I have done on the Riemann hypothesis has identified many instances where the Taylor formula played a critical role in the solution of the problem. I can't remember how many times as a professor many students wondered on the reasoning to learn a particular subject or section in mathematics. Today we intend to illustrate in the different parts of the research how many sectors are affected by the Taylor formula. The most obvious one is in the arena of power series. The latter exposes the link between $e^{x}$ series and the great Euler formula. Also, in this reformulation of the Taylor series as a power sequence. We were able to set an identity equation that permits us to proceed to the center of the zeta location. By establishing the center of orientation of zeta location, we were able to take advantage of the latter to investigate the nature of its behavior with
respect to the counting principle. It happens that the zeta zeroes repel themselves horizontally and attract themselves vertically. This is significant since this knowledge reinforces our choice of the used of the $e^{x}$ curve as the necessary and sufficient platform to work with.

The third aspect we need to envision is what we already mentioned earlier in the work: Modal Logic. For the purposes of this work, Modal Logic is a collection of formal systems developed to represent statements about necessity and possibility. In math it is a Kripke model of possible world (1990's viewpoint).

### 6.1. Taylor Versus Differential Equation

Taylor series also replaces the need to do differential equation and analysis. Case in point, John Nash discusses the concept of geometric embedding of $n$ dimension. We are discussing complex number planes of four dimensions. With this in mind, we need to project the input plane into the output plane by dynamic programming to achieve the desired outcome. Differential equations are in the mix here; however, they can be avoided using Taylor approximation.

Furthermore, approximation uses derivative as a tool to arrive at forming the necessary bent on a curve at a certain location. This tool of analysis used by Jon Nash to solve the embedded problem is useful in many fields of study. Doing analysis on a problem can change the conclusion of a problem. we need to understand the tool we have and know how to use it.

## 7. Analytical Continuity

In the complex field, analytical continuity is paramount. The behaviour of all elements in the field are studied. This is done to delineate where possible singularities lie, which is necessary when graphing is needed. The singularities play the role of boundaries that cannot be crossed. The singular rotation is done in the neighbourhoods of a singularity, not through it. Another useful technique is to use the singularity as limit correctness. This gives you a good read where a rate of change is giving false readings due to machine error. This is significant in decision making. The complex field graph is done in four dimensions. The common wisdom is to split the plane into two fields, the input vs the output, usually there are variable projection changes to transform the input to the desire result than sent the latter to the output. In this scenario, knowing where the pols are located is crucial. When a sector in a field is out of range, continuity may be used. We can extrapolate further and come out with a good understanding of the entire sector.

## 8. Analytical Continuity Versus Singularity Versus Asymptote of Real Graph

This analytical continuity does not depend on the singularity when we consider the strip of convergence of the zeta zeroes at $R(s)=1$. The entire field has a covering even though it is not defined at $R(s)=1$, only in the complex field this is possible. However, in the real field we encounter, on rare occasions, a rational equation when graphs are forced to cross the vertical asymptote. Hence, the need of a testing point closed to the asymptote.

## 9. Analytical Continuity and the Error Term

Even if in the real world, crossing an asymptote is taboo, in the complex world, this is a common occurrence. The latter may expose hidden knowledge that cannot be assessed by any means. These tools and others may help in decision making. Remember, graphing is in four dimensions, a difficult task to undertake. Analytical continuity extends the domain of a function for example the $\log$ function.

Now combining analytical continuity and the singularity concept we can explore the latter to minimize the likelihood that an error occurs in the dead-end region of conversion. Once the properties of the saddle point are located, the latter lays at a delta distance from the singularity. A useful property. However, the case of several complex variables is rather different since singularities need not to be isolated points and its investigation was a major reason for sheaf cohomology. Broadly speaking sheaf cohomology describes the obstructions to solving a geometric problem globally when it can be solved locally. The central work for the study of sheaf cosmology is Grothendieck's 1957 Tohoku papers.

## 10. Conclusion

We begin our work by assuming that the zeta location is the closest distribution where all the prime numbers are stable. This means the slope of the Taylor Approximation Polynomial is the highest of the graph. It is located to the right of the strip of the zeta zero location. The point reveals the area of the prime number, the Asymptotic estimate value for the number of prime up to a given value of $x$. It is conceivable to use the base e formula center close to the zeta zero location to estimate the number of zeros that pass through our periodic domain. The graph which compares the two, estimates one linear vs one exponential, which leads to the same knapsack volume.

Between data and result:
Linear $\pi k . . n$ Vs $e^{L}$ where L or 1-L is the neighborhood of $Y^{\text {" }}$ the zeta zero" for a specific $x$ under study
We can say more expending the left side of Euler product we get

$$
\frac{1}{1-\Upsilon} \zeta(s)=\sum 1 / n^{s}=\zeta(s)=\prod_{p}\left(1-\frac{1}{p}\right)^{-1}
$$

For a specific $x$

$$
\frac{D}{1-Y} \zeta(s)=\sum 1 / n^{s}=\zeta(s)=\prod_{p}\left(1-\frac{1}{p}\right)^{-1}
$$

Positioning $D$ closer to the center by reduction

$$
\frac{D}{\frac{1-Y}{2 \pi k}} \varsigma(s)=\sum 1 / n^{s}=\zeta(s)=\prod_{p}\left(1-\frac{1}{p}\right)^{-}
$$

Making a change of base and Taylor approximation to get the final high of Li.e., the saddle point

$$
\frac{D}{\frac{1-Y}{e^{L}}} \varsigma(s)=\sum 1 / n^{s}=\zeta(s)=\prod_{p}\left(1-\frac{1}{p}\right)
$$

A complete excel table is available at the following location on OneDrive: https://bit.ly/3F2wap9

## 11. Implication for Future Research

We wanted to lessen the need to find the maximum size gap. By establishing the zeta zero location as an alternative form of counting. Now withing that world we can study the maximum gap size up to the given x value. Here space is finite. The criticism is that the size of the gap seem to increase as we consider larger and larger numbers. Also as always for the gap size a pattern is elusive at best. Furthermore, another possible alliance is in the arena of dynamic programming, the latter is on the rise lately. It is the cornerstone of artificial intelligence programing thus merging these two fields, mathematics, computer science is a natural evolution. They complement each other. After all in artificial intelligence information is a sequence of events knowledge is a limit. Whether the latter converges or diverges a decision is made. This ability is what dynamic programming is all about.

$$
C(\mathrm{~s})=\sum 1 / n^{s}=1 / 1^{\mathrm{s}}+1 / 2^{\mathrm{s}}+1 / 3^{\mathrm{s}} \ldots 1 / \mathrm{n}^{\mathrm{s}} \mathrm{~s} \text { is the imaginary numbers sq } 1
$$

It is worth repeating what Terence Tao stated earlier. We know the size of the box but we don't know what is inside. Now we will be able to fill in the box. The latter will open unexpected doors of knowledge and wisdom.

Finally according to Donald E. Knuth in his article" Algorithm thinking and Mathematic thinking. He concludes in his survey that there is no such thing as" mathematical thinking" as a single isolated concept; mathematicians use a variety of mode of though not just one. We in this paper share the same philosophy, and we conclude that the root of thinking is in the heuristic of problem solving and the latter is not the property of mathematicians, it is rather part of the consciousness of all human beings.

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