

Original Article

# The Riemann Hypothesis: Given an Arbitrary X Number the Volume of a Knapsack Containing Prime Numbers Can be Found

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**Abstract** - The work in this document advances concepts and procedures that bridge the gap between the prime enumeration and the zeta zero location in three main segments. The first segment is based on the Egyptian enumeration system which is used in this work to design the Construction and Detachment Principle, with the goal of establishing an extension field where the zeta zero location resides side-by-side with the integer under study. In the absence of a reliable formulae, a limit point can be established using a set of heuristics from the Golden Ratio and the Egyptian Continuous Fraction Algorithm. The latter formed the genus of a method that identified a unique zeta zero for a given x value. This is done in accordance with Dimensional Analysis in order to test the validity of the zeta zero location distribution with respect to the prime distribution. The second segment establishes two formulas to test the relevancies of the zeta location with respect to the volume of the knapsack boxes containing prime numbers up to a given x value. This document also provides a method to test whether the formula produces an accurate counting of the prime for the given value of x by using the Taylor Approximation Polynomial at the slope point. This slope is the closest point to where the zeta zero resides. In the third segment of the work, the results will show the distribution of the zeta zero's location and, in doing so, lessen the need of finding the largest gap between prime numbers.

**Keywords** - Knapsack problem, Prime number gap, Riemann Hypothesis, The content of the box, Zeta zero location.

## 1. Introduction

Creation is the source of all knowledge that is known. Science does not escape this axiom. Thus, this universal truth permeates all aspects of what we do. The universal language of mathematics and the characteristics of Dimensional Analysis offer a variable relationship that is explored in this document to extrapolate the implicit knowledge that exists as we know it.

Dimensional Analysis begins with an initial guess of the solution that is envisioned. There are many ways to tackle Dimensional Analysis; one aspect is to consider formulas that are universal and consistent over time. However, since prime numbers have been elusive for many years, this document describes a new approach to this problem. Thus, to borrow an expression of a well-known mathematician, Terence Tao, "we know the size of the box that contains prime numbers, but we do not know the content of the box." To solve this and other issues, universally known variables must be utilized in search of a specific result, i.e., the Fibonacci Ratio. The latter offers a unique tool in identifying prime numbers, although it is a scissor relationship. Yet, it offers a way forward. Thus, combined with current knowledge of prime numbers, we can delineate the closest heuristics on a specific aspect of the Riemann Hypothesis. Namely  $\pi(x) \approx x/(\ln(x))$ , which is a global asymptotic estimate meaning the approximation of the number of prime up to a specific x value. Since the approach of  $\pi(x)$  is local, the formula needs to be modified to a specific number under studies, which implies utilizing a step function that is embedded in the prime sequence to reformulate  $\pi(x)$ . This is necessary because we are looking at the  $\pi(x)$  as an initial value solution to the study of the volume of the knapsack containing prime number up to an arbitrary x value. The missing component is the general solution. This is a well-known structure in differential equations. This type of problem has a name, it is called the initial value problem.

## 2. Formula Outline

The formula presented in this document contains the following main points:

- Knapsack Problem: The Knapsack data candidate has a limit point.
- Fibonacci Egyptian Fraction
- The Golden Ratio



- The investigation of the zeta zero location distribution as it is related to prime numbers.
- The essence of the work
- The zeta location
- The counting of zero
- The calculation: The Torus Reality

### 3. The Knapsack Data Candidate Has a Limit Point

The prime sequence behaves like a step function and each step is of a different size, which implies that each x value must provide its own rate of descent toward a plateau point with a universal limit point. Two issues come to mind when we want to delineate the knapsacks problem: First: The Fibonacci Egyptian Fraction Algorithm. Second: The Golden Ratio.

Each of these concepts complement each other toward the same goal – Achieving an exact volume for the sides of the knapsack, using the Golden Ratio and the Fibonacci Egyptian Fraction concept to establish a limit point.

### 4. Fibonacci Egyptian Fraction

The Fibonacci Egyptian fraction is a “greedy” algorithm design for an optimal solution. In this case, we want to establish the rate of descent of a fraction “by being greedy,” i.e., the largest portion of the rational will be used as a step function. The remaining segments are insignificant by design.

- Greedy Algorithm:  $\gcd(a,b)=d$
- If  $d(a,b)=1$ , then we say a and b are relatively prime to each other.
- The best way to begin this idea is to work with the asymptotic nature of the prime conjecture.  $\Pi(x) \simeq \frac{x}{\ln(x)}$   
Using this, we can begin the work and adjust as we see progress. This will be elaborated on later in the document.
- The following formulation is the fundamental of the Egyptian Fraction:

$$1/a = (1/(a+1) + 1/a(a+1))$$

For example, d is not 1 thus, i.e.,  $(14,6)=2$

$$3/7 = 1/3 + 1/11 + 1/(11*3*7) \qquad c33 = 1/2 + 1/3 + 1/5 + 1/7 + 1/11 + 1/13$$

With the size of the step function known, the formula of  $\pi(x)$  can now be adjusted, pending the limit point derivation.

### 5. The Golden Ratio

The Golden Ratio is a relation that is universal, and Dimensional Analysis is referring to that reality. This is a proportion well known in nature. Granted, it is also a scissor relationship that needs to be adjusted. The two values used for the purposes of this document are either 1.61803369 or 0.61803369. These are the ratios of the Fibonacci sequence i.e., 11235813...

The following is the equation associated with this sequence:

$$x^2+x-1=0 \text{ thus } x = \frac{1 \pm \sqrt{5}}{2}$$

The sequence is given by:  $X = \left(\frac{1 \pm \sqrt{5}}{2}\right)^n$ . e 1 1 2 3 5 8 13 ...; n=0,1,2...

Now, let  $\phi = \frac{1 + \sqrt{5}}{2} = 1.61803$

At the end of each step size is a prime. This document only focuses on the gap between two prime numbers. So, the relevant interval lies in manipulating this reality to our advantage. Since we can tell when a number is prime, we should be able to significantly narrow the choice of elements that get in the Knapsack volume.

## 6. The Investigation of the Zeta Zero Location Distribution in Relation to Prime Numbers

Given the number 8226, the number of prime up to this x is 1031, whereas the number of prime for the zeta location 8226.680169 up to this distribution is 1031. Without using the zeta zero location table, we need to ascertain which zeta location is associated with which value of x under study. There is no known transformation to achieve this process at random. Granted, we know in an orderly fashion many zeta locations, however, this is not sufficient; a process is required. For example, in the 1970s, there were table values to find the angle of trigonometric functions. This was not an attractive option. With the invention of portable calculators, and then computers, we quickly embrace the latter due to its precision and accessibility.

Here, we outline some of the known work. We will do our own findings in a comprehensive manner later. This is a sample of formulae in the public domain “Google,” and is given here to stress a view. The zeta location formulations are not stable. A table is required. However, to make progress we need to find the zeta zero location for each x under study without a table.

$$Z(1)=1/2[2+\gamma-\ln(4\pi)] =0.0230957;\gamma=.577215664901$$

$$Z(1)=\sum_{k=1}^n \frac{4}{1+4k^2}$$

$$Z(2)=1+\gamma^2 - \frac{1}{8}\pi^2 + \gamma_1=.0994773737$$

$$Z(2)=-\sum_{k=1}^n \frac{8((4t_k^2-1)}{(4k^2+1)^2}$$

$$Z(3)= )=-\sum_{k=1}^n \frac{16((12t_k^2-1)}{(4k^2+1)^3}$$

$$Z(4)=-\sum_{k=1}^n \frac{32((16t_k^4-24t_k^2+1)}{(4k^2+1)^4}$$

Following the fact that the zeta location is a better set of numbers, not all zeta extensions can be used in this work as the universal rate that was established earlier in this document using the Golden Ratio must be satisfied.

### 6.1. Zeta Zero Location

The Riemann Hypothesis is a conjecture that has two sets of zeros. One is considered trivial, which is the negative even numbers. The other is the non-trivial zero, which is part of the complex plane with a real part of 1/2. This is of interest since it implies results on the distribution of prime numbers, making the gap point between primes become manageable, implying that this distribution is more stable. However, the following issues arise:

- Where are the zero-locations coming from?
- Is there a formula?
- What is its relation to prime numbers?

An algebraic construction is used in an extension field to find a point of intersection in the complex field. This is how the strip of conversion will be understood. However, we have a lack of basic, fundamental knowledge and the challenge is to fill in the gap in this knowledge base.

The Riemann zeta function  $\zeta(s)$  is defined in the half plane:

$$\{R(s)>1\}$$

By the formula:

$$\zeta(s)=1^s+2^s+1/3^s+1/5^s\dots$$

Because of the fundamentals of arithmetic, as well as the geometric series formula:

$$(1-1/p^s)^{-1}=\sum 1/p^s = 1/p^{2s} + 1/p^s \dots \dots$$

One has the Euler product formula:

$$\zeta(s)=\prod_p (1 - \frac{1}{p})^{-1}$$

Where the product is over primes. This links the zeta function analytic to the number theory. The Euler product formula shows that  $\zeta(s) \neq 0$  whenever  $R(s)>1$ .

6.2. The Essence of Work

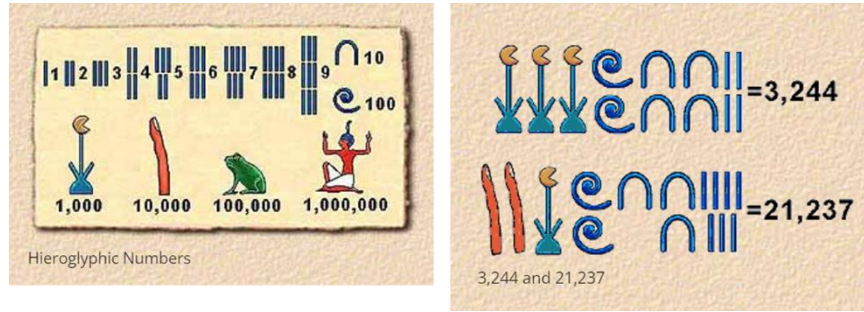


Fig. 1 Model of Enumeration

The Golden Ratio provides two values that can be considered for a limit points. However, we need to adjust the value to find the zeta location associated with the x value under studies. Namely,  $.61/5^{.5}=.2728002933$ . To set a universal limit, we will use the Egyptian system, namely, the Model of Enumeration (**Figure. 1**).

Using the process of construction and detachment principle, we can work on each segment of the numeral under consideration as well as the decimal extension of the numeral that we identified as the zeta location for that number. For our purposes, we will begin construction for 1234567 followed by detachment. Using 0.273000 as a limit point in order to lock in the best zeta location extension.

**Note:** An integer can be represented with its natural extension. Using 1234567 as an example:  
 $1/0.273/1/4026650/0.00000000000073685$ ; Representation of the number: 1234567.201

This format is the same as the Egyptian enumeration system; and is the preferred method because it allows detachment of elements in the enumeration of numbers such that a limit can be easily established. We achieve this by using the Golden Ratio and the Fibonacci Egyptian Fraction individual expansion to extrapolate what is needed for the limit point.

**Note:** The non-trivial zeta zero locations are unique.

The value of  $1/4026670$  comes out of the Egyptian fraction algorithm.

We begin searching for an appropriate expansion base on  $\pi(x)=\frac{x}{\ln(x)}$ , then adjust according to the input value x.

The decimal detachment to find the limit point is as follows:

$$=1-(1234567-1/0.273/1/4026650/0.00000000000073685); .201$$

The limit point to find zeta location needs: .27300

$$ABS(1/0.201294065/1/4026650/0.00000000000073685-1234567+1)/1000000 \text{ found } 0.439783$$

Must change .201 ... until .27300 is found.

$$ABS(1/0.35051002/1/4026650/0.00000000000073685-1234567+1)/1000000 \text{ DONE}$$

Here, using the zeta location as a rate of decent, the knapsack volume plateaus when the error term is used as a limit point. In folding the input x and the associated zeta location into itself, a torus is formed that eliminates the need to find the size of the gap between two primes. A torus is defined as two non-intersecting circles embedded in the same plane:

Since a torus is parallel circles that are not sitting in the same plane, they are, by structure, looking at concentric circles. Thus, we need to reduce the last circle to a point. This can be done using Cauchy's Integral Theorem of Imaginary Field. The  $\pi$  result after integration is  $2\pi$ . Now, using the latter as a limit point, we can reduce our outer circle to establish the area of Cauchy's expectation. In doing so, we find the delineate, i.e., the last prime in the knapsack volume we searched for.

**6.3. The Calculation: The Torus Reality**

The second step is the Domain. Applying the error equation stipulate earlier:  $\frac{x}{\ln(x)^2}$ ; we are now adding  $\pi(x)$

1234567/LN(1234567)+1234567/LN(1234567)^2+1234567/LN(1234567)^3 Error Growth  
 (0.3505002\*1234567)/(2\*A3); I2 contains, a3=2π  
 LN(I3)/LN(I2); I4 Contains a view of the process.

A function f(z) has a period C if and only if: F(z + C)=f(z) for all z. In the complex plane it is fine to consider a value for C.

$$e^{2\pi i} = 1 \text{ by IF}$$

$$e^{z+2\pi i} = e^z e^{2\pi i} = e^z$$

So, the exponent function is periodic with imaginary period C=2πi. The method used is analogous to dealing with an angle greater than 360 degrees, by dividing by 360° to make the domain smaller by imposing a limit 1-L into the domain of the ln(x). The working assumption is the zeta location and the x value counting the number of zeros are close to each other. To delineate this reality, the bases need to change to the e function and proceed accordingly:

$$(I2)^{I4}/(1-0.3505002/e^L). \text{ limit reach is near } \gamma \text{ thus a tori approaches the x value is: } 95,359.71$$

For these two values, respectively: 85360 VS 95358.045, round up: ((I2)^I4/(1-0.3505002/A4^3.4459002),0)

- |               |             |            |
|---------------|-------------|------------|
| • Left        | Zeta L      | RIGHT      |
| • 2.04917E-05 | 1.419777544 | 1.14105932 |

Here, we have the structure of a torus not centered at the origin. The effect of applying multiples of  $1\pi, 2\pi, \dots, k\pi$  reduce the outer layer of a torus structure to the limit of the knapsack volume - only if we perform a base change to e. Under this scenario, the desired limit will be reached for any input value of x under study. This implies that we are dealing with an embedded torus. If L is the center of the torus, 1-L is the neighborhood to L. This geometric series is related to the zeta location i.e., 1-L.

*John Nash*

Here we need to talk about John Nash, the father of Embedded Geometry. He shows that the distance in a two-dimensional plane is equivalent to any embedded sphere of a three-dimensional plane or higher. Surely enough, a torus was one of the classical ways to prove it. The PAC-MAN game outlines the complexity of the problem since it is a visual paradox.

Going back to John Nash, using derivative and differential analysis as a tool he arrived at a non-trivial answer that shows that by applying a wave to a torus plane, the distance between a flat plane (called intrinsic) and a torus plane is identical. The most important takeaway here is you are changing the position, but not necessarily the distance. Hence, in The Riemann Hypothesis strip, the prime is to the right of the strip meaning the position of the prime. The distance is the same in any dimension, extrapolating further if the distance between the zeta location distribution and the prime distribution is closed. In this case, it will have the same property of closeness in any embedded space or dimension. Therefore, it is necessary to know about the gap between prime, but it is not sufficient. This is the strength of Modal Logic: The Possible Worlds of Saul Kripke model theory.

**6.4. The Gap Between Prime Numbers**

Now, this is the time to talk about the gap between prime. Three issues are related to the gap point between prime. The first two issues are first finding the minimum gap, second finding the maximum gap. The first concern has been addressed fully. The second concern is still an open question. In this work we simply look for an alternative way to resolve the gap problem in a procedural form. The latter is shaped through dimensional analysis. This universal concept has been treated earlier in the paper. However, the third issue is how the gap is related to the Euler formula. The latter is so great that we will spend time here addressing its relevancy.

The Euler Formula 1707-1783

This celebrated formula has been treated in many books through the years. However, today we are going to stay closed to our own formula presented here in this work so that the intuitive nature in our formulas can be understood.

We need to begin with a power series, although this series usually concentrates on where this series is defined. For now, we will stay with the form of derivation to extrapolate other formulas.

$$1/(1-x) = \sum x^n = 1 + a_1x + a_2 x^2 + a_3x^3 \dots$$

This geometric sequence can be transformed into an  $e^x$  function by applying the derivative to the right of the equation.

Nth derivative of each term in the sequence

$$d/da(1) = 0$$

$$d/dx(x) = 1$$

$$(d/d^2x) = 2$$

$$d/d^3(x^3) = 6$$

Adjusted for the coefficient of  $e^x$  which is always 1 we get from the original sequence at  $x=0$

$$1 + a_1x + a_2 x^2 + a_3x^3 \dots \rightarrow a_0=0, a_1=1, a_2=1/2, a_3=1/6 \text{ so } a_n = n(n-1)f(n-2) \dots \text{ commonly known as } n!$$

$$e^x = 1 + x + \frac{x^2}{2} \dots$$

Now let  $x$  become  $ix$ , then:

$$e^{ix} = 1 + ix + (ix)^2/2! + (ix)^3/3! \dots$$

$$\cos x = 1 - x^2/2! + x^4/4! \dots$$

$$i \sin x = x - x^3/3! + x^5/5! \dots$$

The point is to separate the real part from the imaginary part — we get the Taylor expansion of  $\cos x$  which is real, followed by  $i \sin x$  the imaginary part. Hence, the right side of the equation establishes the Euler formula, namely:

$$e^{ix} = \cos(x) + i \sin(x)$$

Another useful work by Euler is using the prime number in a sequence and actually showing that it diverges, as follows:

$$\text{Given } \sum 1/p = 1 + 1/2 + 1/3 + 1/5 \dots 1/p$$

We need to do a comparison test by choosing a bigger set, thus if the larger set diverge, the smaller set will also diverge.

Let's pick the harmonic series:

$$\sum 1/n = 1 + 1/2 + 1/3 + 1/4 + 1/5 \dots 1/n$$

Let's also modify the harmonic series to render the decision easier:

$$1 + 1/2 + 1/3 + 1/4 + 1/4 + 1/6 + 1/6 + 1/8 \dots$$

By grouping like terms, we are only adding  $1/2$ . Each time, the partial sum will end-up each time with a different value, which means this series diverges and so does the harmonic series. In doing so, the latter forces the prime number series to also diverge.

*The Argument*

By the same stroke, Euler proved that the sequence of prime is infinite. This proof is done by contradiction, a simple assumption is needed here.

$$\zeta(s) = \sum 1/n^s = \zeta(s) = \prod_p (1 - \frac{1}{p})^{-s}$$

Euler attached a great significance to above representation he noted on the left that  $s \rightarrow 1, \zeta(s) \rightarrow \alpha$ , whereas on the right a product over prime occurred. Hence there must be an infinite number of primes; otherwise  $\zeta(s) \rightarrow \prod_p (1 - \frac{1}{p-1})$  (over a finite set) or  $\zeta(s) \rightarrow \alpha$

Now, going back to the power series, we can further say that we can work with the left side of the power series to center the zeta zero location. The domain consists of  $\pi(x)$  and the error term:

$D=x/\ln(x)+\frac{x}{\ln(x)^2}$  domain combined with the left-hand side of the power series:

**Formula 1:**  $(x/\ln(x)+\frac{x}{\ln(x)^2})/(1-a)/2k\pi$  formula 1

**Formula 2:**  $(x/\ln(x)+\frac{x}{\ln(x)^2})/(1-a)/e^L$  formula 2

Formula 1 is used to shift the domain of the prime conjecture closer to the zeta location.

Formula 2 is used to embrace the contour of the e – function to be positioned in the Cauchy Reimann region of convergence within the boundaries of singularities.

When the shift reaches the limit point of the singularity, we proceed with the location where the saddle point of the graph can be found. Finally, the latter is used by the Taylor Approximation Polynomial to find the slope of the e-curve for that particular x value. It is time that we introduce Bernhard Riemann as a young man, when he spent intensive time studying complex numbers and applied his view in the distribution of prime numbers.

*Bernhard Riemann*

In 1859 a reserved German mathematician, Bernhard Reimann, under the auspice of his advisor, presented a paper on the prime number distribution with a specific hypothesis on the zeta zero location which he believed could unlock the distribution of prime, yet could not prove it. His paper is outlined here:

$\zeta(s)=\sum 1/n^s=1/1^s + 1/2^s +1/3^s \dots 1/n^s$  s is the imaginary numbers

This series does not have a closed form, thus:

The Reimann Hypothesis states that all non-trivial zeros of the zeta function have the form:

$S=1/2+bi$

This forms a strip of convergence where  $\zeta(s) >1$ .

Now, we have three sectors of the zeta zeros:

$\zeta(s)=\sum 1/n^s \quad R(s)>1$

$\zeta(s)=1/(s-1)+\sum_{n=1}^{\alpha} \int_n^{n+1} (1/n^s - 1/x^s) \quad 0<R(s)<1$  critical except the point  $s=1$ ,

$\zeta(s)=2^s \pi^{-s} \cos\frac{\pi(1-s)}{2} \Gamma(1-s) \zeta(1-s) \quad R(s)<0$

Going back to the choice of using the e-curve beyond the point of its natural properties, one of the qualities of the zeta zero is that vertically, the zeros attract each other, whereas horizontally they repel each other. A significant gain when we are counting zeros. We are interested in the area below the slope where the final high of the curve is under study at L. This ability of attraction simplifies our work significantly. The zeta zeros are well behaved and dense as we move to larger values of x for consideration. This last piece of information is due to Terence Tao in his YouTube video lecture on vaporization (ref. “Terence Tao: Vaporization and freezing the Reimann zeta function”).

The second reality of the strip is that it is symmetric about  $R(s)=1/2$

The hope is that the gap size will be served well by tackling the critical strip after all the domain of the left side of the power series (i.e.,  $1/1-x$ ) is identical to the strip of convergence. We simply need to be rigorous in our research to delineate the new reality. We must take a detour here and talk about  $H_0$ .

*What do we know about  $H_0$ ?*

It is an entire function of order 1, obeying the functional equations  $H_0(-z)=H_0(z)$  and  $H_0(z)'=(H_0)'$  thus the zeros of  $H_0$  are symmetry around the real and imaginary 0 axes.

All the zeroes of  $H_0$  are contained in the strip:

$\{x + iy : y < 1\}$

Riemann Hypothesis all  $H_0$  are real.

Riemann-Von Mangoldt formula: for  $\Gamma \geq 2$  the number  $N_0(T)$  of zeroes in the rectangle

$\{x+iy: 0 \leq x \leq T: |y| \leq 1\}$  is  $T/4\pi \log T/4\pi + O(\log T)$

Proven using upper and lower bounds on  $H_0$  outside of the strip:

$\{(x + iy: |y| < 1)\}$ , and upper bounds inside the strip)

A variant (due to Littlewood), if the Riemann hypothesis is true, then for fixed  $\alpha$  one has:

$N_0(T+\alpha) - N_0(T) = \alpha \log T + O(\log T)$  as  $T \rightarrow \alpha$  of

In particular, on RH, mean spacing between zeroes of  $H_0$  in  $[T, 2T]$  is roughly  $1/\log T$ , and one has equidistributional of the zeroes at scales  $\geq \eta(T)$  for some  $\eta(T) = \alpha(1)$ .

These parameters are talking about the zeroes on a large scale. Under that scenario it seems that the distribution is evenly spread. This fits the narrative that we are expecting and is what we all wanted in the search for additional knowledge of the prime numbers. Obviously, more advances must take roots, especially in the arena of Dynamic Programming. Presently, there are still issues in the field of decimal accuracy. Alfa Beta Pruning, commonly known as backtracking, is a necessary but insufficient tool to deal with the complexity faced with regard to the decimal field.

The concept, advanced by model logic, is a viable alternative. In the article “Model Logic Should Say More Than it Does,” Melvin Fitting suggests an integration of all the math in the search for a breakthrough in difficult subjects. There is no modality in thinking whether you are doing math or designing an algorithm. We are entering the complex plane which has some unique properties. In the real world there is no square root of negative numbers, thus  $i^2$  was invented to replace negative numbers so that the operation can take place. Graphically placing a point on a grid is circular. This is done as a challenge in four dimensions. The real is a two-dimensional plane which is called the *input plane* and the imaginary is a projection to another two-dimensional plane called the *output plane* (i.e., four moves are required). We are dealing with positioning (i.e., angles). It is fitting to see that under the complex system the real issue is indeed positioning. This is what should have been the concern all along. It is necessary to mention that there are many algebraic polynomials that have closed form, as well as trigonometric function, but not a series, although some polynomials can be expressed as power series. Another concept worthwhile to mention is Analytical Continuity.

#### Analytical Continuity

Any complex number function  $F: \mathbb{C} \rightarrow \mathbb{C}$  that is analytic on a given region can be uniquely extended to an analytic function defined for almost all real numbers, except for numbers that are called pol or singularities.

The left-hand side of the power series:

The rational function  $1/(1-x)$  extends the geometric series  $f(x)$ , the power series from the interval  $-1 < x < 1$ , to all real numbers except 1. Due to this interval, a strip is established: A region  $s = 1/2 + bi$  is bounded from  $0 < x < 1$ ; extrapolating further a symmetry is found to the strip around the value  $1/2$ .

This asymptotic behavior is similar to all rational equations. However, since it is the left side of a power series, this rationale is adding new knowledge to its behavior in the complex plane. The domain of the rational function is setting up multiple singularities. The latter behaves like a boundary of a dead-end sector where the orientation is similar to the  $\cot x$  graphically. If  $x$  is replaced by Formula 2, the dead-end region will be evident.

Going back to the fundamentals, we're now going to explain the zero's calculation of base 10 numerals and the log significance.

#### 6.4. Average Number of Prime Factors

The model that best suits our needs is the  $e^x$  curve. However, the basic curve that is used for discussing prime is the  $\ln(x)$ . The latter was a derivation of the gap found between primes.  $e^x$  has properties that are unique and will be discussed and elaborated on graphically within this section (Figure 2 and Figure 3).

The link to the Riemann hypotheses begins in some small ways with the average factorization of numbers into prime factors. The interest in this document is the significance of the slope and its relation to the  $\ln(x)$ . Figure 2 and Figure 3 illustrate the basics, before we look at the subset of prime number factors in terms of Figures 2 and 3, refer to the YouTube video “The hidden link between Prime Numbers and Euler’s Number.”



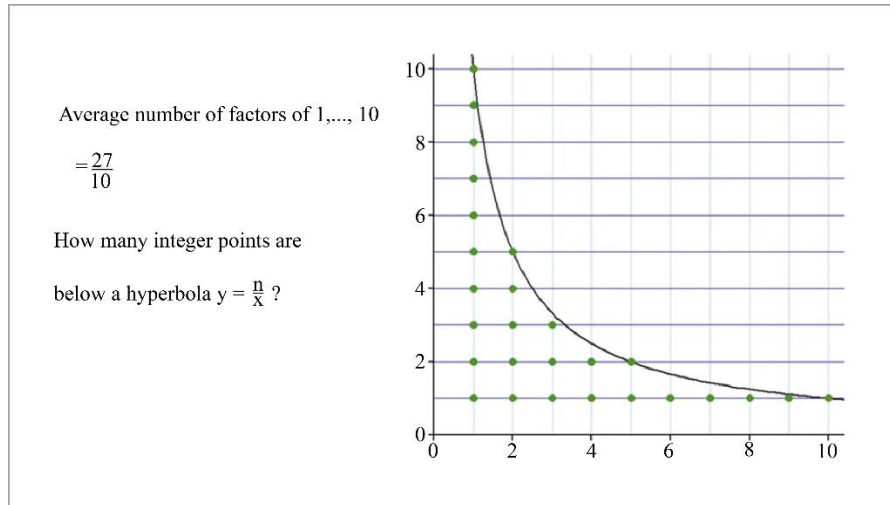


Fig. 2 Number of Divisors up to 10 Using Slope

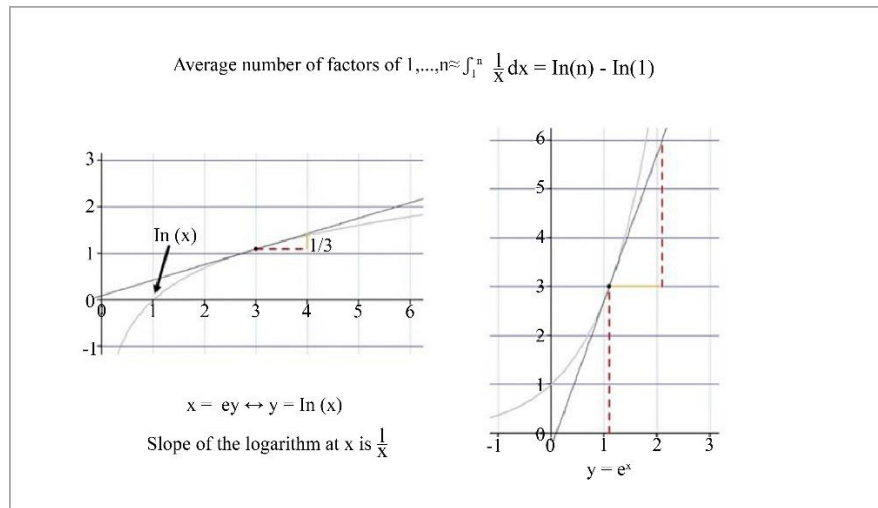


Fig. 3 Number of Divisors Using e

## 7. The Way Forward to the Proof

Now, the properties of  $e^x$  and the Taylor Approximation Polynomial can be used to find the high of the volume of the region. The radius of convergence established the least upper bound of our CR region. The maximum upper bound is at the singularity of the dead-end sector within the two boundaries we can find the saddle point.

### 7.1. Proof the Beginning

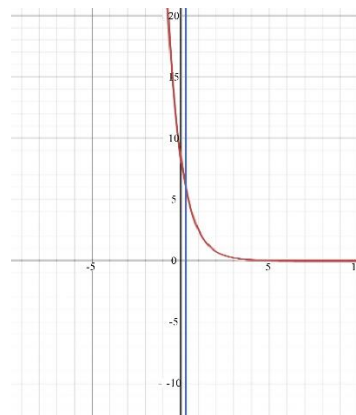


Fig. 4 Function: x=14, p=6

According to CR, the natural boundary is  $\cot(x)$ :

$$\sum a_n(Z-z_0)^n \lim_{b \rightarrow 0} R = \frac{a_{n+1}}{a_n} = \frac{1}{n}$$

Given the radius of convergence of the Cauchy Reimann region, we can use the Taylor Polynomial Approximation to measure the closeness of the zeros of the zeta location. Finding the Taylor polynomial approximation at a specific location:  $P(x)=1+(x-L)\dots$

**7.2. Taylor Approximation Polynomial**

Hence for the number 14:

$$p(x)=1+(.13549-1.2074420274)=-.0719520274$$

$$1+(.13549-1.2074420274)=-.0719520274$$

The slope  $p(x)+1+x=L$  predicts the number of prime up to  $n$ , thus the number of zeros. Hence:  $p(x)$  is a good estimation for the slope. The working assumption is the zeta location and the  $x$  value counting the number of zeros is close to each other. To delineate this reality, we need to change bases to the  $e$  function and proceed accordingly.

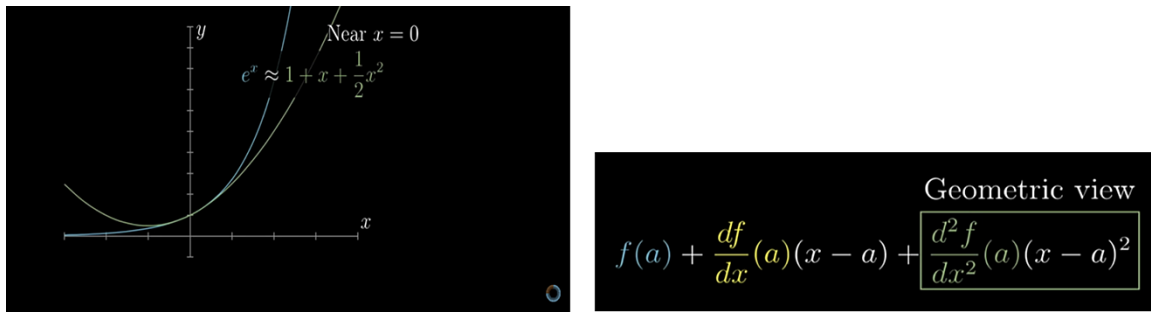


Fig. 5 The Point of Tangency

$e^z - e^{p(x)} = .0794939535$  is the closest point according to the original assumption (Figure 5).

Using Euler’s formula, we can show the region of the dead-end region. Below the  $x$  axis is not our concern. The behavior of Figure 5 mimics the behavior of Figure 4 and the actual example.

$$= \cos(x) + i\sin(x)$$

$$\text{Thus: } \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

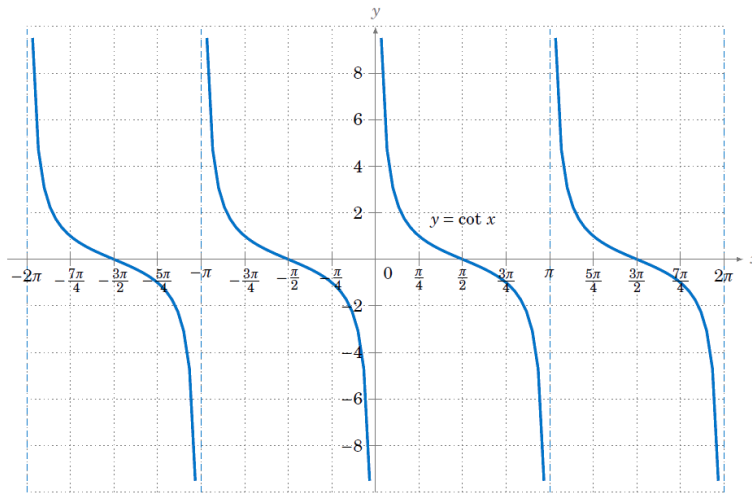


Fig. 6 Cot Example

Reiterating what was stated earlier, Figure 6 displays the same nature as Figure 4, asymptotically. This behaves like a dead-end street controlled by the radius of convergence. There exists a saddle point where the final prime lies. The 2<sup>nd</sup> property for large numbers greater than or equal to 13 digits, the radius of convergence remains at 1.

**Table 1 of convergence from π to e and Location of L**

large x→	324543343	0.272999998→	→limit Fibo	pt	↓e base,curve L
		change of base from π→e↓ 21.7544696263 3.0798192365 && Knapsack vol saddle linear 17506978.0540345000 pts↓ for conversion result↑ .324543343 to e ref f33 0.273  17329647.8606202000 17503813.7171292000 17506978.0540345000 17857414.2097939000 Taylor polynomial slope on the following numbers -1.135700 1+((0.9543)-3.09) -1.125700 1+((0.9543)-3.08) -1.1255191+((0.9543)-3.07) -1.1057001+((0.9543)-3.06) 0.3211972046 0.3244839515 0.3309791158 0.3244839515 saddle point transfer to table adjusted by the length of L (3.0798192365)			
	324543343	17405074.52	17506978.05		17506978.71 140108732.8

Now, going back to the choice of using the e-curve beyond the point of its natural properties, one of the nice qualities of the zeta zeros is that vertically the zeros attract each other, whereas horizontally they repel each other. A significant gain when counting zeros. Needless to say, we are interested in the area below the slope where the final high of the curve is under study at L. This ability of attraction simplified our work significantly. The zeta zeros are well behaved and dense as we move to larger values of x for consideration. This last piece of information is due to Terence Tao in his YouTube video lecture on vaporization (ref. "Terence Tao: Vaporization and freezing the Reimann zeta function").

**7.3. Lagrange Error Bound of Taylor Polynomial**

Hence for the number 14:

$$T(a)=f(a)+f'(a)(x-a)+f''(x - a)^2/2$$

$$\text{ERROR: } \leq M \frac{(c-a)^{n+1}}{n+1!}$$

- C is the value we substitute in
- a is where the poly is centered in.
- n is the higher derivative used for poly
- M is the max pf abs value of  $F^{n+1}(x)$  on the interval with a and c as endpoints, or a convenient chosen point.

For example:

$$e^x = 1 + x + \frac{x^2}{2} \dots$$

Finding the Taylor Polynomial Approximation at a specific location:

$P(x)=1+(x-L)\dots$

$1+(.13549-1.2074420274)=-.0719520274$  the slope.

Testing the error bound:  $=r.0025885471$  error

#### 7.4. The Error Analysis

We are familiar with the concept of error analysis in a procedural method. Here we identify two issues that could surface. We will focus closely to rounding up or down errors in each extension field. The error may lie in not carrying all decimal digits required, or in certain cases too many digits in consideration with others. This may render the result less accurate. This scissor relationship is common. In this paper, we relied on the closeness of the boundary of the singularities in the Riemann Cauchy dead-end sector of convergence as it is related to distance. However, an old technique used in the past called Alpha Beta Pruning, commonly known as backtracking, can be used to resolve this issue. It is a well-known algorithm. We then used continuous analysis to extrapolate further. However, more needs to be done in this arena.

## 8. Conclusion

We begin our work by assuming that the zeta location is the closest distribution where all the prime numbers are stable. This means the slope of the Taylor Approximation Polynomial is the highest of the graph. It is located to the right of the strip of the zeta zero location. The point reveals the area of the prime number, the Asymptotic estimate value for the number of prime up to a given value of x. It is conceivable to use the base e formula center close to the zeta zero location to estimate the number of zeros that pass through our periodic domain. The graph which compares the two, estimates one linear vs one exponential, which leads to the same knapsack volume.

Between data and result:

Linear  $\pi_k.. n$  Vs  $e^L$  where L or 1-L is the neighborhood of y

A complete excel table is available at the following location on OneDrive: <https://bit.ly/3F2wap9>

## References

- [1] Alan Ross Anderson, Nuel D. Belnap, and J. Michael Dunn, *Entailment, Vol II: The Logic of Relevance and Necessity*, Princeton University Press, Reprint edition, 2017. [[Google Scholar](#)] [[Publisher Link](#)].
- [2] John C. Amazigo, and Lester A. Rubinfeld, *Advanced Calculus And Its Applications to the Engineering And Physical Sciences*, Wiley, 1<sup>st</sup> Edition, 1980. [[Google Scholar](#)] [[Publisher Link](#)]
- [3] Thomas Becker, and Volker Weispfenning, *A Computational Approach to Commutative Algebra*, Graduate Texts in Mathematics (GTM, volume 141), Springer, 1993. [[Google Scholar](#)] [[Publisher Link](#)]
- [4] Samuel R. Buss, *Bounded Arithmetic*, University of California, Berkley, 1985. [[Google Scholar](#)]
- [5] Folkmar Bornemann, "PRIMES Is in P: Breakthrough for "Everyman"," *Notices of the American Mathematical Society*, vol. 50, no. 5, pp. 545-552, 2003. [[Google Scholar](#)]
- [6] Harvey Cohn, *Advanced Number Theory*, Dover Publications, 1980.
- [7] John Derbyshire, *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics*, Joseph Henry Press, 2003. [[Google Scholar](#)]
- [8] A. Evyatar, and P. Rosenbloom, *Motivated Mathematics*, Cambridge University Press, 1981. [[Publisher Link](#)]
- [9] Donald E. Knuth, *The Art of Computer Programming, Vol 1: Fundamental Algorithms*, 2<sup>nd</sup> Edition, Addison-Wesley, 1973.
- [10] Jean Louis Lassez, and Gordon Plotkin, *Computational Logic Essays in Honor of Alan Robinson*, The MIT Press, 1991. [[Google Scholar](#)] [[Publisher Link](#)]
- [11] Harry R. Lewis, and Christos H. Papadimitriou, *Elements of the Theory of Computation*, Prentice-Hall, 2<sup>nd</sup> edition, 1997. [[Google Scholar](#)]
- [12] Melvin J. Maron, and Robert J. Lopez, *Numerical Analysis: A Practical Approach*, International Thomson Publishing, 1991.
- [13] Piergiorgio Odifreddi, *Logic and Computer Science (Apic Studies in Data Processing)*, Academic Press, 1<sup>st</sup> edition, 1990.

- [14] Charles Stanley Ogilvy, and John Timothy Anderson, *Excursions in Number Theory*, Oxford University Press, 1988. [[Google Scholar](#)]
- [15] Oystein Ore, *Number Theory and Its History*, Dover Publications, 1988. [[Google Scholar](#)]
- [16] George Pólya, and John H. Conway, *How to Solve It: A New Aspect of Mathematical Method*, Princeton University Press, Reprint Edition, 2014. [[Publisher Link](#)]
- [17] Gerald E. Sacks, *Higher Recursion Theory, Perspective in Logic, vol. 2*, Association for Symbolic Logic, 1990.
- [18] Stoica Codruta Simona, *Essays on Mathematics and Statistics, Volume 5*, ATINER BOOKS, 2015.
- [19] William M. Trench, *Advanced Calculus*, Harper & Row, 1978.
- [20] Raymond Turner, *Truth and Modality for Knowledge Representation (Artificial Intelligence)*, The MIT Press, 1991. [[Google Scholar](#)]
- [21] Lincoln A. Wallen, *Automated Deduction in Non-Classical Logic*, The MIT Press, 1989. [[Publisher Link](#)]
- [22] Albert Wilansky, *Topology for Analysis*, Dover Publications, Illustrated edition, 2008. [[Google Scholar](#)]
- [23] Wilson M. Zaring, *An Introduction to Analysis*, The Macmillan Company, 1967.
- [24] Dennis G. Zill, *A First Course in Differential Equations with Modeling Applications*, 10<sup>th</sup> edition, Cengage Learning, 2012. [[Google Scholar](#)]