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*Original Article*

# Optimal Policy for an Inventory Model using Pentagonal Fuzzy Number

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***Abstract** - This article explores an inventory system in fuzzy scenario with two models. In this research, ordering cost and holding cost are considered as pentagonal fuzzy numbers. Initially, the parameters are considered as crisp parameters. Secondly, the parameters are uncertain and are caned as fuzzy parameters. The total inventory cost is defuzzied using the graded mean integration formula, and the optimal order quantity is estimated using the Kuhn–tucker method. To find the optimal order quantity and minimum total inventory cost, an algorithm is designed. Numerical examples are used to compared a fuzzy inventory model with the traditional crisp inventory model. Finally, a graphical depiction of the proposed model is shown. The result demonstrates that the advantages of the application of fuzzy model in real-life environment.*

***Keyword** - Defuzzification, Fuzzy inventory model, Graded mean integration, Kuhn-Tucker condition, Pentagonal fuzzy numbers.*

## 1. Introduction

Supply chain operation has the power to boost client service, reduce operating costs and ameliorate the fiscal standing of a company. Multi-echelon reservoir system has formed by an item in which it moves through further than one stage before reaching the final client. Clark and Scarf (1960) were the first to study the two- level echelon model. Integrated inventory model for the buyer-vendor coordination was proposed by Goyal and Gupta (1989). Recent research has seen the development of numerous inventory models that take lead time into consideration as a determinant variable. According to [2, 3], the order quantity, reorder point, lead time and setup cost were analyzed as decision variables.

Inventory managers must apply some flexibility when choosing on the sizes of the order quantities in EOQ-based inventory models in order to reduce the expense of uncertainty. The fuzzy inventory models suggested by Guiffrida demonstrate that by using fuzzy set theory to solve inventory problems, as compared to traditional probability theory, generates more accurate results. Kohila Gowri, Thamil selvi and Kavitha (2022) propounded a comparison of combined filtering recommendation grounded on cloud model and trapezoidal fuzzy number. Syil Viya Rivika, Mashadi and Sri Gemawati (2023) discussed about triangular fuzzy matrix inverse. Sachin Kumar Rana, Amit Kumar and Rahul Kumar (2022) investigated an inventory model that deals for deterioration items with a fixed expiry date with price and trade credit.

In inventory modeling, the closest possible approach in reality is the fuzzy set theory. Taha (1997) discussed the kuhn-tucker condition. Fuzzy arithmetic operations and applications were presented in [5, 16]. Graded mean integration representation of the generalized fuzzy number was proposed by Chen and Hsieh (1999). By employing inventory parameters as pentagonal fuzzy numbers, Harish Nagar and Priyanka Surana devised a fuzzy inventory model for deteriorating commodities with fluctuating demand (2015). Using a trapezoidal fuzzy number, Dutta and Pavan Kumar (2012) developed a fuzzy inventory model without shortage. Several researchers [9,13,14, 15,18,19,20,22,24] addressed EOQ inventory problem in fuzzy random environment under various assumptions. Apurva Rawat (2011) developed an inventory model in fuzzy environments without shortage by employing triangular fuzzy number. Amit Nalvade, Ashok Mhaske, Sagar Waghmare and Smt. Shilpa Todmal developed a fuzzy approach to solve fuzzy game theory problem by employing two types of fuzzy numbers such as pentagonal and hexagonal.

In this article, we address a fuzzy environment for an optimal inventory model. For crisp-order quantities, we initially start with the fuzzy inventory model. To determine the total inventory cost in a fuzzy sense, a fuzzy inventory model is then proposed for a fuzzy order quantity. Relevant optimal order quantity is then determined. The rest of the paper is organized as follows. In section 2 introduces the notations and assumptions required to state the problem. In section 3, we mathematically formulate the problem. In section 4, the methodology of Graded mean integration method, the basic concepts of fuzzy sets, fuzzy numbers, pentagonal fuzzy number and the Kuhn-Tucker condition is discussed. In section 5 describes the fuzzy inventory models. The numerical example along with the graphical representation is done to illustrate crisp and fuzzy sense in section 6. Finally, certain important managerial insights are also surmise from the sensitivity analysis in section 7. A comparative study of the proposed model is described in section 8. We conclude with final remarks in section 9.

## 2. Notations and Assumptions

In this paper, the proposed model is developed on the basis of the following notations and assumptions.

### *2.1. Notations*

$TIC$: Total inventory cost.
$h$: Inventory holding cost per unit time.
$Q$: Order quantity per cycle.
$K$: Ordering cost per order.
$D$: Demand quantity.
$T$: Length of the plan.
$Q^*$: Optimal order quantity.
$\widetilde{TIC}$: Fuzzy total inventory cost.
$\tilde{h}$ : Fuzzy holding cost per item per unit time.
$\tilde{K}$: Fuzzy Ordering cost per order.
$\tilde{Q}^*$: Fuzzy optimal order quantity.

### *2.2 Assumptions*

- Total demand is considered as a constant.
- No Shortages.
- Time plan is constant.
- To fuzzify ordering cost, holding cost, and replenishment processing cost.

## 3. Mathematical Model

The total inventory cost, which is composed of buyer and vendor ordering cost and inventory carrying cost is given by

$$TIC = K\frac{D}{Q} + h\frac{Q}{2} \tag{1}$$

We wish to determine the value of $Q$which minimizes the TIC of the Eq. (1). If the optimal value $Q^*$ satisfy $0 < Q^* < \infty$, then $Q^*$ must satisfy the equation

$$\frac{\partial TIC}{\partial Q} = 0 = \frac{-KD}{Q^2} + \frac{h}{2} \tag{2}$$

Solving the equation (2) for$Q$, we obtain $Q^* = Q = \sqrt{\frac{2DK}{h}}$. (3)

## 4. Methodology

The optimal order quantity with a fuzzy inventory model is determined in this study using the graded mean integration approach and the function principle. The model is solved using the Kuhn-Tucker technique when the quantities are fuzzy numbers.

### *4.1. Graded mean Integration Representation Method*

Generalized fuzzy number [6] is the fuzzy subset of the real line$R$, whose membership function satisfies the following conditions.

i. $\mu_{\tilde{A}}(y)$ is a continuous mapping from R to the closed interval [0,1],
ii. $\mu_{\tilde{A}}(y) = 0, -\infty \leq y \leq u_1$,
iii. $\mu_{\tilde{A}}(y) = L(y)$ is strictly increasing on $[u_1, u_2]$.
iv. $\mu_{\tilde{A}}(y) = v_A,\ u_2 \leq y \leq u_3$,
v. $\mu_{\tilde{A}}(y) = R(y)$ is strictly decreasing on $[u_3, u_4]$,
vi. $\mu_{\tilde{A}}(y) = 0, u_4 \leq y \leq \infty$ , where $u_1, u_2, u_3$ and $u_4$ are real numbers and $0 < v_A \leq 1$.

This type of generalized fuzzy number is also denoted as $\tilde{A} = (u_1, u_2, u_3, u_4; v_A)_{LR}$. . When $v_A = 1$, it can be simplified as $\tilde{A} = (u_1, u_2, u_3, u_4)_{LR}$.

Defuzzification of $\widetilde{D}$ can be found by graded mean integration representation. If $\widetilde{D}$ is a pentagonal fuzzy number $(a, b, c, d, e)$ then the graded mean integration representation formula is given by

$$G(\widetilde{D},\widetilde{0}) = \int_0^1 d([A_L(\alpha), A_R(\alpha)], \widetilde{0})\, d\alpha = \frac{1}{12}(a + 3b + 4c + 3d + e). \tag{4}$$

***4.2. Pentagonal Fuzzy Number***

A pentagonal fuzzy number $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}} = \begin{cases} L_1(t) = \dfrac{t-a}{b-a}, a \le t \le b, \\ L_2(t) = \dfrac{t-b}{c-b}, b \le t \le c, \\ 1, t = c, \\ R_1(t) = \dfrac{d-t}{d-c}, c \le t \le d, \\ R_2(t) = \dfrac{e-t}{e-d}, d \le t \le e, \\ 0, otherwise. \end{cases}$$

The $\alpha$ – cut of a pentagonal fuzzy number $\tilde{A}=(a, b, c, d, e)$ , $0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$, where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = d + (d-c)\alpha$ are left and right end points of $A_\alpha$. Here, we have $A_{(L_1)}(\alpha) = a + (b-a)\alpha = {L_1}^{(-1)}(\alpha)$ and $A_{(L_2)}(\alpha) = b + (c-b)\alpha = {L_2}^{(-1)}(\alpha)$, and

$A_{R_1}(\alpha) = d - (d-c)\alpha = R_1^{-1}(\alpha)$ and $A_{R_2}(\alpha) = e - (e-d)\alpha = R_2^{-1}(\alpha)$.

The graphical representation of the $\alpha$ – cut of pentagonal fuzzy number is shown in the Figure.1.

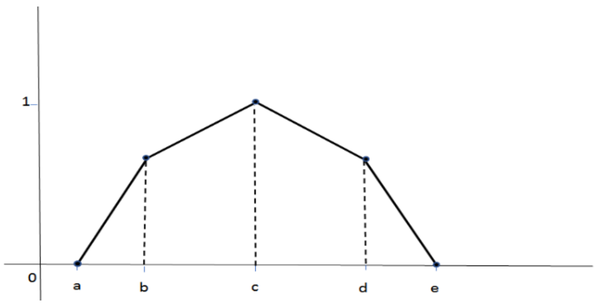


**Fig. 1 $\alpha$ – cut of pentagonal fuzzy number**

***4.3. Arithmetic Operations on Pentagonal Fuzzy Numbers under Function Principle***

For the pentagonal fuzzy numbers, some fuzzy arithmetic operations are expressed as in [5].

Let $\tilde{L} = (l_1, l_2, l_3, l_4, l_5)$ and $\widetilde{M} = (m_1, m_2, m_3, m_4, m_5)$ be two pentagonal fuzzy numbers, then

1. The addition of $\tilde{L} = (l_1, l_2, l_3, l_4, l_5)$ and $\widetilde{M} = (m_1, m_2, m_3, m_4, m_5)$ is given by
$$\tilde{L} \oplus \widetilde{M} = (l_1 + m_1, l_2 + m_2, l_3 + m_3, l_4 + m_4, l_5 + m_5),$$
where $l_1, l_2, l_3, l_4, l_5$ and $m_1, m_2, m_3, m_4, m_5$ are any real numbers.

2. The multiplication of $\tilde{L} = (l_1, l_2, l_3, l_4, l_5)$ and $\widetilde{M} = (m_1, m_2, m_3, m_4, m_5)$ is given by
$$\tilde{L} \otimes \widetilde{M} = (l_1 m_1, l_2 m_2, l_3 m_3, l_4 m_4, l_5 m_5),$$
where $l_1, l_2, l_3, l_4, l_5$ and $m_1, m_2, m_3, m_4, m_5$ are any non-zero positive real numbers.

3. $-\widetilde{M} = (-m_5, -m_4, -m_3, -m_2, -m_1)$ then the subtraction of $\widetilde{M}$ from $\tilde{L}$ is
$$\tilde{L} \ominus \widetilde{M} = (l_1 - m_5, l_2 - m_4, l_3 - m_3, l_4 - m_2, l_5 - m_1),$$
where $l_1, l_2, l_3, l_4, l_5$ and $m_1, m_2, m_3, m_4, m_5$ are any real numbers.

4. $\frac{1}{\widetilde{M}} = \widetilde{M}^{-1} = \left(\frac{1}{m_5}, \frac{1}{m_4}, \frac{1}{m_3}, \frac{1}{m_2}, \frac{1}{m_1}\right)$, where $m_1, m_2, m_3, m_4, m_5$ are all non-zero positive real numbers, then division of $\tilde{L}$ and $\widetilde{M}$ is
$$\tilde{L} \oslash \widetilde{M} = \left(\frac{l_1}{m_5}, \frac{l_2}{m_4}, \frac{l_3}{m_3}, \frac{l_4}{m_2}, \frac{l_5}{m_1}\right).$$

5. For any real number t,

$$t \otimes \tilde{L} = \begin{cases}(tl_1, tl_2, tl_3, tl_4, tl_5), t \geq 0 \\ (tl_5, tl_4, tl_3, tl_2, tl_1), t < 0.\end{cases}$$

### *4.4. Kuhn-Tucker Condition*

Consider the problem stated as in [25]. Minimize $z = p(t)$, subject to $h_j(t) \geq 0$, $j = 1,2,\ldots,m$. If any of the nonnegative constraints $t \geq 0$, are included in the m constraints. Through the use of nonnegative surplus variables, the inequality restrictions can be transformed into equations. To the jth constraint $h_j(t) \geq 0$, the surplus quantity $S_j^2$ is added. Let $\psi = (\psi_1, \psi_2, \ldots, \psi_m)$, $h(t) = (h_1(t), h_2(t), \ldots, h_m(t))$ and $S^2 = (S_1^2, S_2^2, \ldots, S_m^2)$. Hence, the Lagrangian equation is $N(t, S, y) = p(t) - y(h(t) - S^2)$.

Then, $h_j(t) \geq 0$.

By taking the first order derivative of N with respect to t, S, and $\psi$ we get
$\frac{\partial N}{\partial t} = \nabla f(t) - \psi \nabla h(t) = 0$, $\frac{\partial N}{\partial S_j} = 2\psi_j S_j = 0, j = 1,2,\ldots,m$, and $\frac{\partial N}{\partial \psi_j} = -h_j(t) + S_j^2 = 0, j = 1,2,\ldots,m$.
It shows that $y_j h_j(t) = 0, j = 1,2,\ldots,m$.

The Kuhn-Tucker conditions must be the fixed point of t and $\psi$. Then, the minimization problem can be summarized as:

$$\psi \leq 0,$$
$$\nabla f(t) - \psi \nabla h(t) = 0,$$
$y_j h_j(t) = 0, j = 1,2,\ldots,m,$
$$h_j(t) \geq 0.$$

## 5. Fuzzy Inventory Models

### *5.1. Fuzzy inventory model for crisp order quantity*

In this model, the fuzzy total inventory cost $\tilde{TIC}$ for buyer and vendor including the cost of ordering, and inventory carrying cost is given by $\tilde{TIC} = \tilde{K}\frac{D}{Q} + \tilde{h}\frac{Q}{2}$,
where Fuzzy Ordering cost $= (\tilde{K} * \tilde{D})/Q$.
Fuzzy total carrying cost $= (\tilde{h} * Q)/2$.
Here, the variables K and $h$ are treated as a fuzzy parameter.

$$\tilde{TIC} = [((\tilde{K} \otimes D) \oslash Q) \oplus ((\tilde{h} \otimes Q) \oslash 2)], \quad (5)$$

where $\oplus, \otimes, \oslash$ and $\ominus$ are the fuzzy arithmetical operators under Function Principle. Suppose, $\tilde{K} = (K_1, K_2, K_3, K_4, K_5)$, and $\tilde{h} = (h_1, h_2, h_3, h_4, h_5)$ are non-negative pentagonal fuzzy numbers. The fuzzy total inventory cost is given by

$$\tilde{TIC} = \left\{\left(\frac{K_1 D}{Q} + \frac{h_1 Q}{2}\right), \left(\frac{K_2 D}{Q} + \frac{h_2 Q}{2}\right), \left(\frac{K_3 D}{Q} + \frac{h_3 Q}{2}\right), \left(\frac{K_4 D}{Q} + \frac{h_4 Q}{2}\right), \left(\frac{K_5 D}{Q} + \frac{h_5 Q}{2}\right)\right\} \quad (6)$$

We defuzzify the fuzzy total inventory cost of formula (6) by using graded mean integration representation method. The representation of the fuzzy total inventory cost $\tilde{TIC}$ by graded mean integration representation method is given by

$$S(\tilde{TIC}) = \frac{1}{12}\left\{\left(\frac{K_1 D}{Q} + \frac{h_1 Q}{2}\right) + 3\left(\frac{K_2 D}{Q} + \frac{h_2 Q}{2}\right) + 4\left(\frac{K_3 D}{Q} + \frac{h_3 Q}{2}\right) + 3\left(\frac{K_4 D}{Q} + \frac{h_4 Q}{2}\right) + \left(\frac{K_5 D}{Q} + \frac{h_5 Q}{2}\right)\right\} \quad (7)$$

By taking first order partial derivative of the Eq. (7) with respect to $Q$ and equating to zero, the fuzzy order quantity $Q^*$ is obtained. Then the result is given by

$$\frac{1}{12}\left\{\frac{-D}{Q^2}(K_1 + 3K_2 + 4K_3 + 3K_4 + K_5) + \frac{1}{2}(h_1 + 3h_2 + 4h_3 + 3h_4 + h_5)\right\} = 0$$

By Simplification, the optimal order quantity $Q^*$ is

$$Q = Q^* = \sqrt{\frac{2D(K_1 + 3K_2 + 4K_3 + 3K_4 + K_5)}{h_1 + 3h_2 + 4h_3 + 3h_4 + h_5}} \quad (8)$$

### *5.2. Fuzzy inventory model for Fuzzy Order Quantity*

Suppose the fuzzy order quantity $\tilde{Q}$ be a pentagonal fuzzy number $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4, Q_5)$ with$0 \leq Q_1 \leq Q_2 \leq Q_3 \leq Q_4 \leq Q_5$. The fuzzy total cost is given by

$$\widetilde{TIC} = \left\{\left(\frac{K_1 D}{Q_5} + \frac{h_1 Q_1}{2}\right), \left(\frac{K_2 D}{Q_4} + \frac{h_2 Q_2}{2}\right), \left(\frac{K_3 D}{Q_3} + \frac{h_3 Q_3}{2}\right), \left(\frac{K_4 D}{Q_2} + \frac{h_4 Q_4}{2}\right), \left(\frac{K_5 D}{Q_1} + \frac{h_5 Q_5}{2}\right)\right\} \tag{9}$$

The graded mean integration representation of $\widetilde{TIC}$ is given by

$$S(\widetilde{TIC}) = \frac{1}{12}\left\{\left(\frac{K_1 D}{Q_5} + \frac{h_1 Q_1}{2}\right) + 3\left(\frac{K_2 D}{Q_4} + \frac{h_2 Q_2}{2}\right) + 4\left(\frac{K_3 D}{Q_3} + \frac{h_3 Q_3}{2}\right) + 3\left(\frac{K_4 D}{Q_2} + \frac{h_4 Q_4}{2}\right) + \left(\frac{K_5 D}{Q_1} + \frac{h_5 Q_5}{2}\right)\right\} \tag{10}$$

with $0 \leq Q_1 \leq Q_2 \leq Q_3 \leq Q_4 \leq Q_5$. The minimization of $S(\widetilde{TIC})$ is obtained by differentiating the Eq. (10) with respect to$Q_1, Q_2, Q_3, Q_4, Q_5$ . Thereby, letting all the partial derivatives equals to zero and solving for$Q_1, Q_2, Q_3, Q_4, Q_5$. We get,

$Q_1 = \sqrt{\frac{2K_5 D}{h_1}}$, $Q_2 = \sqrt{\frac{2(3K_4 D)}{3h_2}}$, $Q_3 = \sqrt{\frac{2(4K_3 D)}{4h_3}}$, $Q_4 = \sqrt{\frac{2(3K_2 D)}{3h_4}}$ and $Q_5 = \sqrt{\frac{2(K_1 D)}{h_5}}$.

Kuhn-Tucker condition is applied for the following linear programming problem to get the optimal order quantity$\tilde{Q}^*$.

Minimize $S(\widetilde{TIC})$

Subject to the constraints

$$Q_2 - Q_1 \geq 0$$
$$Q_3 - Q_2 \geq 0$$
$$Q_4 - Q_3 \geq 0$$
$$Q_5 - Q_4 \geq 0$$
$$Q_1 \geq 0$$

We get,

$$KT(Q_1, Q_2, Q_3, Q_4, Q_5, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) = S(\widetilde{TIC}(Q, K, h)) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2) - \lambda_3(Q_4 - Q_3) - \lambda_4(Q_5 - Q_4) - \lambda_5 Q_1 \tag{11}$$

The Kuhn-Tucker conditions are given as follows,

$$\lambda \leq 0,$$

$\nabla f(S(\widetilde{TIC})) - \lambda_i \nabla h(\tilde{Q}) = 0,$

$\lambda_i h_i(Q) = 0, i = 1, 2, \ldots, m,$

$h_i(Q) \geq 0.$

These conditions simplify to the following,

$$\lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, \lambda_4 \leq 0, \text{ and } \lambda_5 \leq 0, \tag{12.1}$$

$$\left[\frac{h_1}{2} - \frac{K_5 D}{Q_1^2}\right] + \lambda_1 - \lambda_5 = 0, \tag{12.2}$$

$$\left[\frac{h_2}{2} - \frac{K_4 D}{Q_2^2}\right] - \lambda_1 + \lambda_2 = 0, \tag{12.3}$$

$$\left[\frac{h_3}{2} - \frac{K_3 D}{Q_3^2}\right] - \lambda_2 + \lambda_3 = 0, \tag{12.4}$$

$$\left[\frac{h_4}{2} - \frac{K_2 D}{Q_4^2}\right] - \lambda_3 + \lambda_4 = 0, \tag{12.5}$$

$$\left[\frac{h_5}{2} - \frac{K_1 D}{Q_5^2}\right] - \lambda_4 = 0, \tag{12.6}$$

$$\lambda_1(Q_2 - Q_1) = 0, \tag{12.7}$$

$$\lambda_2(Q_3 - Q_2) = 0, \tag{12.8}$$

$$\lambda_3(Q_4 - Q_3) = 0, \tag{12.9}$$

$$\lambda_4(Q_5 - Q_4) = 0, \tag{12.10}$$

$$\lambda_5(Q_1) = 0, \tag{12.11}$$

$Q_2 - Q_1 \geq 0$, $Q_3 - Q_2 \geq 0$, $Q_4 - Q_3 \geq 0$, $Q_5 - Q_4 \geq 0$, and $Q_1 \geq 0$.
Since $Q_1 > 0$ and $\lambda_5(Q_1) = 0$, then $\lambda_5 = 0$. If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$, then $Q_5 < Q_4 < Q_3 < Q_2 < Q_1$ and it does not satisfy the constraints $0 \leq Q_1 \leq Q_2 \leq Q_3 \leq Q_4 \leq Q_5$. Therefore $Q_2 = Q_1$, $Q_3 = Q_2$, $Q_4 = Q_3$, and $Q_5 = Q_4$. That is $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = \tilde{Q}^*$. Then the optimal fuzzy order quantity is defined as follows. $\tilde{Q}^* = (Q^*, Q^*, Q^*, Q^*, Q^*)$. Hence, we get the optimal order quantity $\tilde{Q}^*$ from the Eqs. (12.2 - 12.6).

$$\tilde{Q}^* = \sqrt{\frac{2D(K_1+3K_2+4K_3+3K_4+K_5)}{h_1+3h_2+4h_3+3h_4+h_5}} \tag{13}$$

It shows that the Eq. (13) becomes Eq. (3). That is, if $K_1 = K_2 = K_3 = K_4 = K_5 = K$, and $h_1 = h_2 = h_3 = h_4 = h_5 = h$, then Eq. (13) can be revised as $Q = \sqrt{2DK/h}$.

***5.3. Algorithm for Finding Fuzzy Total Cost and Fuzzy Optimal Order Quantity***

Step 1: Calculate total inventory cost for the crisp model for the given crisp value of $K$ and $h$.

Step 2: Determine the fuzzy total inventory cost using fuzzy arithmetic operations, where the ordering cost and holding cost are regarded as a pentagonal fuzzy number.

Step 3: Use graded mean integration representation method to defuzzify the total inventory cost $\tilde{TIC}$, in order to find the fuzzy order quantity $Q^*$. It can be obtained by putting the first derivative of the defuzzified fuzzy total inventory cost $S(\tilde{TIC})$ to zero.

Step 4: Use Kuhn-Tucker condition to find the fuzzy optimal order quantity $\tilde{Q}^*$.

Step 5: To check whether the fuzzy order quantity obtained from the Kuhn-Tucker condition is same as the crisp order quantity and compares the total inventory cost and savings obtained by both the crisp and fuzzy models.

## 6. Numerical Example

This section presents numerical examples that illustrate the proposed algorithm's use in solving the above problem. The solutions to these examples are obtained with the computer Matlab software. The following Examples 1and 2 are the same as in Apurva Rawat (2011).

**Example 1:** To illustrate the solution method developed in the crisp model, let us consider the system with the data as in Apurva Rawat (2011), $D$ =500 unit, $K$ =Rs.20 per unit, $h$ = Rs.12 per unit and $T$ =6 days. Using Eqs. (1) and (3), we get the minimum total inventory cost $TIC$ = Rs.1200 and optimal order quantity $Q^*$ = 16.67 units. Here $K$ and $h$ are transferred into the fuzzy parameters. To illustrate the solution method developed in the fuzzy model, let us consider the system with the data $K$ = (10, 12, 17, 30, 36) and $\tilde{h}$ = (7, 9, 11, 16, 18). Using Eqs. (10) and (13) we get the fuzzy total cost $S(\tilde{TIC})$ = 1129 and the optimal order quantity $\tilde{Q}^*$ = 17.

**Example 2:** The company uses 24000units of raw material costing 1.25 rupees per unit. Placing each order costs 2.5 rupees and carrying costs are 5.4% per year. Find the EOQ as well as total inventory in both crisp and fuzzy sense.

**Solution**: Here,$D$ =24000 units, $K$ =Rs.22.5 per unit, $h$ = Rs.1.25 x 5.4% = 0.675 per unit and $T$ = 1 day. Using Eqs. (1) and (3), we get the minimum total inventory cost $TIC$ = Rs.854 and optimal order quantity $Q^*$ = 1265 units. Here $K$ and $h$ are transferred into the fuzzy parameters. To illustrate the solution developed in the fuzzy model, let us consider the system with the data $K$ = (12.5, 14.5, 19.5, 32.5, 38.5) and $h$ = (0.175, 0.375, 0.575, 1.075, 1.275). Using Eqs. (10) and (13) we get the fuzzy total cost $S(\tilde{TIC})$ = 772 and the optimal order quantity $\tilde{Q}^*$ = 1265.

The total cost is regarded effectively minimized for the fuzzy model analogized to the crisp model. A graphical representation of optimal order quantity and total cost versus demand $D$ is likened for both the crisp and fuzzy models as shown in Figures 2 and 3.

## 7. Sensitivity Analysis

Sensitivity analysis is ensured by changing the value of demand as $D$ = (450, 475, 500, 525, 550), $D$ = (23250, 23500, 23750, 24000, 24250, 24500, 24750, 25000) and analyze its effects on the optimal solution for Example 1 and Example 2. The results of sensitiveness analysis are presented in Tables 1 and 2. Predicated on the sensitiveness analysis, we acquire the following executive insights. In both the cases, we observe that total cost increases as demand increases. This can be observed from the Figure 4 and 5.

**Table 1. Sensitivity analysis for Example 1**

| $D$ | Crisp model | | Fuzzy model | | Total inventory cost |
|---|---|---|---|---|---|
| | $Q^*$ | $TIC$ | $\tilde{Q}^*$ | $T\tilde{I}C$ | Savings (%) |
| 450 | 15.81 | 1138 | 15.81 | 1071 | 5.9 |
| 475 | 16.24 | 1170 | 16.24 | 1100 | 5.9 |
| 500 | 16.67 | 1200 | 16.67 | 1129 | 5.9 |
| 525 | 17.08 | 1230 | 17.08 | 1157 | 5.9 |
| 550 | 17.48 | 1259 | 17.48 | 1184 | 5.9 |

**Table 2. Sensitivity analysis for Example 2**

| $D$ | Crisp | | Fuzzy | | Total inventory cost |
|---|---|---|---|---|---|
| | $Q^*$ | $TIC$ | $\tilde{Q}^*$ | $T\tilde{I}C$ | Savings (%) |
| 23250 | 1245 | 840.37 | 1245 | 760.11 | 9.6 |
| 23500 | 1252 | 844.87 | 1252 | 764.19 | 9.5 |
| 23750 | 1258 | 849.36 | 1258 | 768.24 | 9.6 |
| 24000 | 1265 | 853.82 | 1265 | 772.28 | 9.6 |
| 24250 | 1272 | 858.25 | 1272 | 776.29 | 9.5 |
| 24500 | 1278 | 862.66 | 1278 | 780.28 | 9.5 |
| 24750 | 1285 | 867.05 | 1285 | 784.25 | 9.5 |
| 25000 | 1291 | 871.42 | 1291 | 788.20 | 9.5 |

**Table 3. Summary of the comparisons**

| Demand | Holding cost | Ordering cost | Length of the plan | Proposed fuzzy inventory model with crisp order quantity | | Proposed fuzzy inventory model with fuzzy order quantity | | Apurva Rawat (2011) model | Savings (%) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $Q^*$ | $TIC$ | $\tilde{Q}^*$ | $T\tilde{I}C$ | $T\tilde{I}C$ | Total inventory cost |
| 500 | 12 | 20 | 6 | 16.67 | 1200 | 16.67 | 1129 | 1200 | 5.9 |
| 24000 | 22.5 | 0.675 | 1 | 1265 | 854 | 1265 | 772 | 917 | 15.8 |

## 8. Comparative Study

In Table 3, the suggested model is compared with the optimal order quantity and integrated total cost. In comparison to the model with crisp optimal order quantity and total cost, it has been found that the model with fuzzy optimal order quantity and total cost has higher utilization. The preceding model is contrasted with this fuzzy inventory model. In Apurva Rawat (2011), the authors used the triangular fuzzy number and signed distance approach for defuzzification and observed that the optimal order quantity increases or decreases with a tiny amount. This leads to a productive outcome for the suggested fuzzy model.

Thus, the company may manage ambiguous inventory cost factors with the aid of our fuzzy integrated inventory model. From Examples 1 and 2, it is noted that uncertain cost parameters result in savings of 5.9% and 15.8% of the total cost, respectively. Additionally, it is shown that the solutions for the fuzzy situation differ significantly from those for the crisp example in terms of optimal order quantity and minimal total cost.

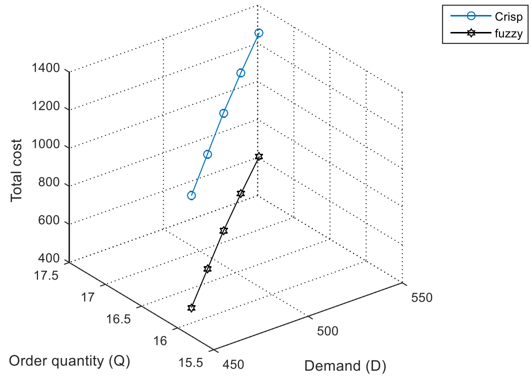


**Fig.2 Graphical representation of Example 1**

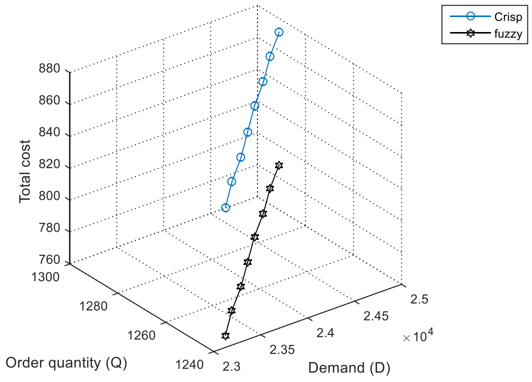


**Fig. 3 Graphical representation of Example 2**

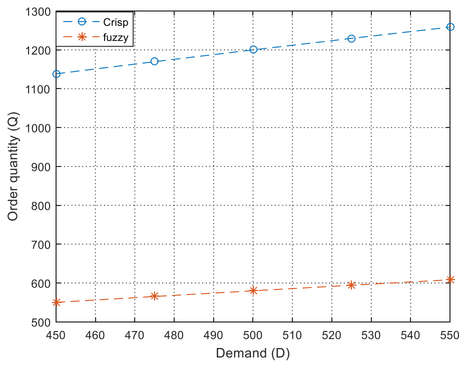


**Fig. 4 Graphical representation of optimal order quantity versus demand for Example 1**

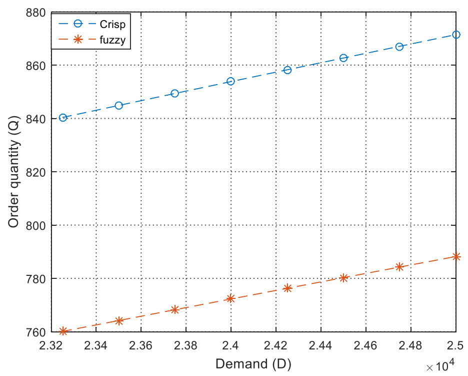


**Fig. 5 Graphical representation of optimal order quantity versus demand for Example 2**

## 9. Conclusion

This study develops a mathematical model for an optimal inventory model in both crisp and fuzzy contexts. In a fuzzy environment, it is assumed that all related inventory parameters are pentagonal fuzzy numbers. The optimal integrated total cost is assessed for defuzzification using the graded mean integration method. The optimal order quantity is established using the Kuhn - Tucker method. The impacts of fuzzy parameters on the optimal order quantity and minimum integrated total cost of the suggested model are examined using a computational algorithm. Graphical representation of numerical examples shows that by using the proposed fuzzy model, one can obtain a significant amount of savings. An effective outcome for the proposed model is obtained by contrasting it with the previous models. The standard crisp and fuzzy models were compared, and it was found that the fuzzy model performed better.

Constraints on ordering, inventory and other factors can be covered in upcoming studies on this issue. Moreover, different multi-echelon supply chain models can be considered in crisp sense, fuzzy sense or both.

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