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Original Article

Analytical Study of Priority Biserial Queue System

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Abstract - *This paper is a study of biserial servers connected in series with the common intermediate server. Queue discipline before the entry-level servers in the system is considered pre-emptive priority discipline. The arrival rate is supposed to follow Poisson distribution, and the service pattern is exponentially distributed. A steady-state analysis of the model is done by using various statistical tools. The methodology used to obtain the Probability distribution function is G.F and P.G.F. The present model helps reduce congestion and enhance the optimum utilization of servers in such types of real-world problems. A numerical illustration is given to validate the study.*

Keywords - *Biserial, Numerical illustration, Priority, Poisson Law, Variance.*

1. Introduction

Waiting line theory is a part of daily- life. A Danish Mathematician, A.K. Erlang, first introduced the concept of waiting line theory in the 20^{th} century by designing a model on Telephone networks. After that, many researchers and mathematicians contribute their work in the field of queueing theory and priority queues. Stephan [1] discussed two queues under pre-emptive priority with Poisson arrivals and service rates. A preemptive priority queue with a general bulk service rule was studied by Sivasamy R [2]. Singh T.P et al. [4-5] present a stochastic analysis of bi- tandem and semi-bi-serial queue network model with a feedback facility. Sharma S and Gupta Deepak [6] made an analysis of biserial queues with the centrally connected server. Agrawal S.K. and Singh B.K. [7-8] analyzed various queue characteristics of a complex queuing model having three servers connected in tri-cum biserial way. Singh H. [9] discussed the practical situation in hospitals to justify the queuing network model with parallel servers linked in series. After that, Gupta D. & Gupta R. [10] explored the such type of model with batch arrival. Recently, Saini V and Gupta Deepak [11] extended this work by analyzing a complex feedback queue model with the condition of revisiting at most one time by a customer at any of the servers with changed moving probabilities. Selvakumaria K. and Revathi S. [12] made an effort to discuss non-preemptive priority queues in a fuzzy environment with unequal service rates. A hysteresis policy was used by Alexander D. et al. [13] for server reservation in a multi-server queuing model to neglect the effect of interruption of service of low-priority customers. Seokjun L. et al. [14] were invented a flexible priority scheme to enhance the protocol of scheduling servers in many real-world situations by using the Markov chain process, including the problem of a cognitive radio network with channel leasing.

In the present paper, we further expand a model by an Analytical Study of a Priority Biserial Queue System consisting of bi-serial servers connected centrally to a common server. In the study, low and high-priority customers' arrival at entry-level biserial subsystems is assumed because most of the time, we see importance is given to one other than others in our daily- life. Queue behavior is analyzed by using the steady-state solution of the proposed model.

2. Model description

In the proposed model, there are three subsystems C_1, C_2 and C_3. The subsystems C_1 and C_2 have biserial service channels C_{11} & C_{12} and C_{21} & C_{22}, respectively. The subsystems C_1 and C_2 are linked to a common subsystem C_3 in series. At first, the customer of Low and high priority with arrival rates λ_{1L}, λ_{1H} & λ_{2L}, λ_{2H} will arrive at service channels C_{11} & C_{12}. After being served at C_{11}, the customer will either move to service channel C_{12} with transition probabilities α_{12} or C_3 with moving probabilities α_{13} such that $\alpha_{12} + \alpha_{13} = 1$. From server C_{12}, the customer either visits C_{11} with moving probabilities α_{21} or direct move to C_3 with probabilities α_{23} with condition $\alpha_{21} + \alpha_{23} =1$.

After availing of the service of service channel C_3 where the service rate is the same for all customers, the customer may either go C_{21} or C_{22} with transition probabilities α_{34} & α_{35}, $\alpha_{34}+ \alpha_{35} =1$ for receiving the service of the next phase. Moreover, from server C_{21}, the customer either visit C_{22} with α_{45} or leave the system with leaving probability α_4, where $\alpha_{45}+ \alpha_4=1$. In the same way, those who arrive at C_{22} to avail of service either visit C_{21} with moving probabilities α_{54} or exit the system with leaving probability α_5 such that $\alpha_{54} + \alpha_5 = 1$ after successful completion of the service.

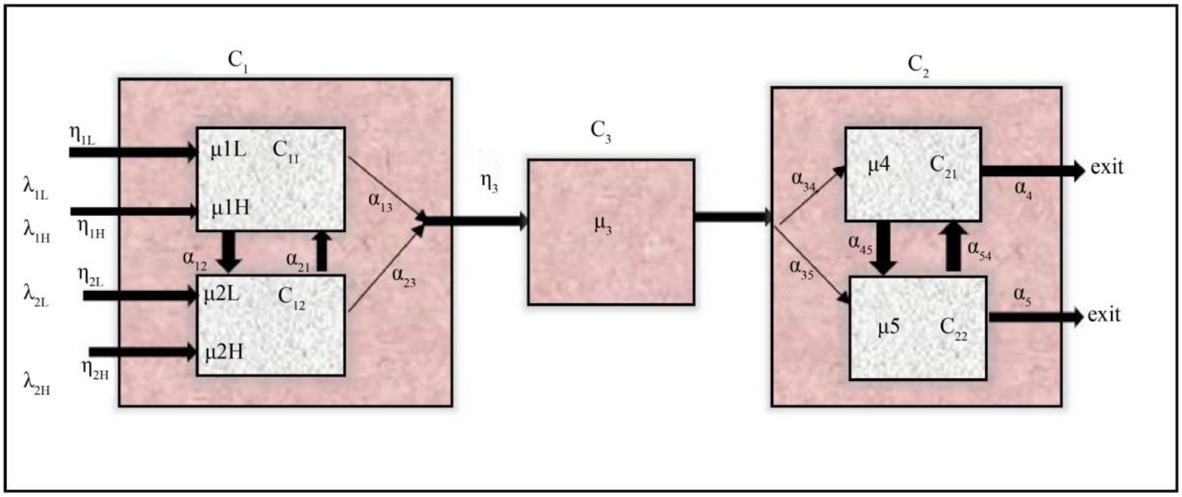

Fig. 1 Proposed Model

Table 1. Notations

Servers	C11	C12	C21	C22	C3
No. of customers	Ƞ1L Ƞ1H	Ƞ2L Ƞ2H	Ƞ4	Ƞ5	Ƞ3
Service Rate	µ1L µ1H	µ2L µ2H	µ3	µ4	µ5
Probabilities	$C11 \to C12$ α12 $C11 \to C3$ α13	$C12 \to C11$ α21 $C12 \to C3$ α23	$C3 \to C21$ α34 $C3 \to C22$ α35	$C22 \to C21$ α54 $C21 \to C22$ α45	$C21 \to$ exit α4 $C22 \to$ exit α5

3. Mathematical Description of The Model

Define Probability function $P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5}(t)$ and $\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5$ number of customers in queues Q_{1L}, Q_{1H}, Q_{2L}, Q_{2H}, Q_3, Q_4, Q_5 in front of servers S_{11}, S_{12}, S_3, S_{21}, S_{22} respectively, where $\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5 \geq 0$.

In Steady-State, the Differential Difference equation is defined as

$$(\lambda_{1L}+\lambda_{1H}+\lambda_{2L}+\lambda_{2H}+\mu_{1H}+\mu_{2H}+\mu_3+\mu_4+\mu_5)P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5} = \lambda_{1L}P_{\eta_{1L-1},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5}+\lambda_{1H}P_{\eta_{1L},\eta_{1H-1},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5}+\lambda_{2L}P_{\eta_{1L},\eta_{1H},\eta_{2L-1},\eta_{2H},\eta_3,\eta_4,\eta_5}+\lambda_{2H}P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H-1},\eta_3,\eta_4,\eta_5}+\mu_{1H}\alpha_{12}P_{\eta_{1L},\eta_{1H+1},\eta_{2L},\eta_{2H-1},\eta_3,\eta_4,\eta_5}+\mu_{1H}\alpha_{13}P_{\eta_{1L},\eta_{1H+1},\eta_{2L},\eta_{2H},\eta_{3-1},\eta_4,\eta_5}+\mu_{2H}\alpha_{21}P_{\eta_{1L},\eta_{1H-1},\eta_{2L},\eta_{2H+1},\eta_3,\eta_4,\eta_5}+\mu_{2H}\alpha_{23}P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H+1},\eta_{3-1},\eta_4,\eta_5}+\mu_3\alpha_{34}P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_{3+1},\eta_{4-1},\eta_5}+\mu_3\alpha_{35}P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_{3+1},\eta_4,\eta_{5-1}}+\mu_4\alpha_{45}P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_{4+1},\eta_{5-1}}+\mu_4\alpha_4P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_{4+1},\eta_5}+\mu_5\alpha_{54}P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_{4-1},\eta_{5+1}}+\mu_5\alpha_5P_{\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_{5+1}}$$

$$\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5 > 0 \qquad (A_1)$$

Taking all possible combinations of $\eta_{1L},\eta_{1H},\eta_{2L},\eta_{2H},\eta_3,\eta_4,\eta_5$, 128 more steady state equations obtained.

To solve the steady state equations (A_1) to (A_{128}), introduce the generating function as,

$$H(R_1',R_2',R_3',R_4',R_5',R_6',R_7') = \sum_{\eta1L=0}^{\infty}\sum_{\eta1H=0}^{\infty}\sum_{\eta2L=0}^{\infty}\sum_{\eta2H=0}^{\infty}\sum_{\eta3=0}^{\infty}\sum_{\eta4=0}^{\infty}\sum_{\eta5=0}^{\infty} P_{\eta1L,\eta1H,\eta2L,\eta2H\,\eta3,\eta4,\eta5}\, {R_1'}^{\eta1L}{R_2'}^{\eta1H}{R_3'}^{\eta2L}{R_4'}^{\eta2H}{R_5'}^{\eta3}{R_6'}^{\eta4}{R_7'}^{\eta5}$$

Where, $|R_1'|=1$, $|R_2'|=1$, $|R_3'|=1$, $|R_4'|=1$, $|R_5'|=1$, $|R_6'|=1$, $|R_7'|=1$, also partial generating functions are

$$H_{\eta1H,\eta2L,\eta2H\,\eta3,\eta4,\eta5}(R_1') = \sum_{\eta1L=0}^{\infty} P_{\eta1L,\eta1H,\eta2L,\eta2H\,\eta3,\eta4,\eta5}{R_1'}^{\eta1L} \tag{1}$$

$$H_{\eta2L,\eta2H\,\eta3,\eta4,\eta5}(R_1',R_2') = \sum_{\eta1H=0}^{\infty} H_{\eta1H,\eta2L,\eta2H\,\eta3,\eta4,\eta5}(R_1'){R_2'}^{\eta1H} \tag{2}$$

$$H_{\eta2H\,\eta3,\eta4,\eta5}(R_1',R_2',R_3') = \sum_{\eta2L=0}^{\infty} H_{\eta2L,\eta2H\,\eta3,\eta4,\eta5}(R_1',R_2'){R_3'}^{\eta2L} \tag{3}$$

$$H_{\eta3,\eta4,\eta5}(R_1',R_2',R_3',R_4') = \sum_{\eta2H=0}^{\infty} H_{\eta2H\,\eta3,\eta4,\eta5}(R_1',R_2',R_3'){R_4'}^{\eta2H} \tag{4}$$

$$H_{\eta4,\eta5}(R_1',R_2',R_3',R_4',R_5') = \sum_{\eta3=0}^{\infty} H_{\eta3,\eta4,\eta5}(R_1',R_2',R_3',R_4'){R_5'}^{\eta3} \tag{5}$$

$$H_{\eta5}(R_1',R_2',R_3',R_4',R_5',R_6') = \sum_{\eta4=0}^{\infty} H_{\eta4,\eta5}(R_1',R_2',R_3',R_4',R_5'){R_6'}^{\eta4} \tag{6}$$

$$H(R_1',R_2',R_3',R_4',R_5',R_6',R_7') = \sum_{\eta5=0}^{\infty} H_{\eta5}(R_1',R_2',R_3',R_4',R_5',R_6'){R_7'}^{\eta5} \tag{7}$$

By using equations (1) to (7) and solving steady-state equations, then we get the Probability Distribution function as,

$$H(R_1',R_2',R_3',R_4',R_5',R_6',R_7') = \frac{\begin{array}{c} G_1\left[\mu_{1H}\left(1-\frac{\alpha_{12}R_4'}{R_2'}-\frac{\alpha_{13}R_5'}{R_2'}\right)-\mu_{1L}\left(1-\frac{\alpha_{12}R_3'}{R_1'}-\frac{\alpha_{13}R_5'}{R_1'}\right)\right]+\mu_3\left(1-\frac{\alpha_{34}R_6'}{R_5'}-\frac{\alpha_{35}R_7'}{R_5'}\right)G_3+ \\ G_2\left[\mu_{2H}\left(1-\frac{\alpha_{21}R_2'}{R_4'}-\frac{\alpha_{23}R_5'}{R_4'}\right)-\mu_{2L}\left(1-\frac{\alpha_{21}R_1'}{R_3'}-\frac{\alpha_{23}R_5'}{R_3'}\right)\right]+\mu_4\left(1-\frac{\alpha_{45}R_7'}{R_6'}-\frac{\alpha_4}{R_6'}\right)G_4+\mu_5\left(1-\frac{\alpha_{54}R_6'}{R_7'}-\frac{\alpha_5}{R_7'}\right)G_5 \\ +\mu_{1L}\left(1-\frac{\alpha_{12}R_3'}{R_1'}-\frac{\alpha_{13}R_5'}{R_1'}\right)G_7+\mu_{2L}\left(1-\frac{\alpha_{21}R_1'}{R_3'}-\frac{\alpha_{23}R_5'}{R_3'}\right)G_6 \end{array}}{\begin{array}{c} \lambda_{1L}(1-R_1')+\lambda_{1H}(1-R_2')+\lambda_{2L}(1-R_3')+\lambda_{2H}(1-R_4')+\mu_{1H}\left(1-\frac{\alpha_{12}R_4'}{R_2'}-\frac{\alpha_{13}R_5'}{R_2'}\right)+ \\ \mu_3\left(1-\frac{\alpha_{34}R_6'}{R_5'}-\frac{\alpha_{35}R_7'}{R_5'}\right)+\mu_{2H}\left(1-\frac{\alpha_{21}R_2'}{R_4'}-\frac{\alpha_{23}R_5'}{R_4'}\right)+\mu_4\left(1-\frac{\alpha_{45}R_7'}{R_6'}-\frac{\alpha_4}{R_6'}\right)+\mu_5\left(1-\frac{\alpha_{54}R_6'}{R_7'}-\frac{\alpha_5}{R_7'}\right) \end{array}} \tag{8}$$

Here for convenience, we denote

$G_1=H_0(R_1',R_3',R_4',R_5',R_6',R_7')$, $G_2=H_0(R_1',R_2',R_3',R_5',R_6',R_7')$, $G_3=H_0(R_1',R_2',R_3',R_4',R_6',R_7')$,
$G_4=H_0(R_1',R_2',R_3',R_4',R_5',R_7')$, $G_5=H_0(R_1',R_2',R_3',R_4',R_5',R_6')$, $G_6=H_{0,0}(R_1',R_2',R_5',R_6',R_7')$,
$G_7=H_{0,0}(R_3',R_4',R_5',R_6',R_7')$

At $|R_1'|=|R_2'|=|R_3'|=|R_4'|=|R_5'|=|R_6'|=|R_7'|=1$and $H(R_1',R_2',R_3',R_4',R_5',R_6',R_7')=1$, the equation (5) reduces to indeterminate form. Therefore, applying the L'Hospital rule on (5) and differentiating it w.r.t to one -by- one variable, we get the results

$$-\lambda_{1L} = -\mu_{1L}G_1 + \mu_{1L}G_7 + \mu_{2L}\alpha_{21}G_2 - \mu_{2L}\alpha_{21}G_6 \tag{9}$$

$$-\lambda_{1H} + \mu_{1H} - \mu_{2H}\alpha_{21} = \mu_{1H}G_1 - \mu_{2H}\alpha_{21}G_2 \tag{10}$$

$$-\lambda_{2L} = \mu_{1L}\alpha_{12}G_1 - \mu_{1L}\alpha_{12}G_7 - \mu_{2L}G_2 + \mu_{2L}G_6$$
(11)

$$-\lambda_{2H} - \mu_{1H}\alpha_{12} + \mu_{2H} = -\mu_{1H}\alpha_{12}G_1 + \mu_{2H}G_2 \tag{12}$$

$$-\mu_3\alpha_{34} + \mu_4 - \mu_5\alpha_{54} = -\mu_3\alpha_{34}G_3 + \mu_4G_4 - \mu_5\alpha_{54}G_5 \tag{13}$$

$$-\mu_3\alpha_{35} + \mu_5 - \mu_4\alpha_{45} = -\mu_3\alpha_{35}G_3 + \mu_5G_5 - \mu_4\alpha_{45}G_4 \quad (14)$$

$$-\mu_{1H}\alpha_{13} - \mu_{2H}\alpha_{23} + \mu_3 = \mu_3G_3 - \mu_{1L}\alpha_{13}G_7 - \mu_{2L}\alpha_{23}G_6 - \mu_{1H}\alpha_{13}G_1 + \mu_{1L}\alpha_{13}G_1$$
$$-\mu_{2H}\alpha_{23}G_2 + \mu_{2L}\alpha_{23}G_2 \quad (15)$$

Solve equations (9) to (15), we get

$$G_1 = 1 - \frac{\lambda_{1H}+\lambda_{2H}\alpha_{21}}{\mu_{1H}(1-\alpha_{12}\alpha_{21})} \quad (16)$$

$$G_2 = 1 - \frac{\lambda_{2H}+\lambda_{1H}\alpha_{12}}{\mu_{2H}(1-\alpha_{12}\alpha_{21})} \quad (17)$$

$$G_3 = 1 - \frac{\alpha_{13}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{23}[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_3\,(1-\alpha_{12}\alpha_{21})} \quad (18)$$

$$G_4 = 1 - (\alpha_{34} + \alpha_{35}\alpha_{54})[\frac{\alpha_{13}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{23}[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_4(1-\alpha_{12}\alpha_{21})(1-\alpha_{45}\alpha_{54})}] \quad (19)$$

$$G_5 = 1 - \alpha_{35}\,[\frac{\alpha_{13}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{23}[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_5(1-\alpha_{12}\alpha_{21})(1-\alpha_{45}\alpha_{54})}] \quad (20)$$

$$G_6 = 1 - \frac{\mu_{2L}(\lambda_{2H}+\lambda_{1H}\alpha_{12})+\mu_{2H}(\lambda_{2L}+\lambda_{1L}\alpha_{12})}{\mu_{2L}\mu_{2H}(1-\alpha_{12}\alpha_{21})} \quad (21)$$

$$G_7 = 1 - \frac{\mu_{1L}(\lambda_{1H}+\lambda_{2H}\alpha_{21})+\mu_{1H}(\lambda_{1L}+\lambda_{2L}\alpha_{21})}{\mu_{1L}\mu_{1H}(1-\alpha_{12}\alpha_{21})} \quad (22)$$

In steady-state, the solution of the model is,

$$P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H\,\eta 3,\eta 4,\eta 5} = (1-G_1)^{\eta 1H}(1-G_2)^{\eta 2H}(1-G_3)^{\eta 3}(1-G_4)^{\eta 4}(1-G_5)^{\eta 5}(1-G_6)^{\eta 1L}$$
$$(1-G_7)^{\eta 2L}G_1G_2G_3G_4G_5G_6G_7$$
$$=\gamma_1^{\eta 1H}\gamma_2^{\eta 2H}\gamma_3^{\eta 3}\gamma_4^{\eta 4}\gamma_5^{\eta 5}\gamma_6^{\eta 1L}\gamma_7^{\eta 2L}(1-\gamma_1)(1-\gamma_2)(1-\gamma_3)$$
$$(1-\gamma_4)(1-\gamma_5)(1-\gamma_6)(1-\gamma_7)$$

And

$$\gamma_1 = \frac{\lambda_{1H}+\lambda_{2H}\alpha_{21}}{\mu_{1H}(1-\alpha_{12}\alpha_{21})} \quad (23)$$

$$\gamma_2 = \frac{\lambda_{2H}+\lambda_{1H}\alpha_{12}}{\mu_{2H}(1-\alpha_{12}\alpha_{21})} \quad (24)$$

$$\gamma_3 = \frac{\alpha_{13}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{23}[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_3\,(1-\alpha_{12}\alpha_{21})} \quad (25)$$

$$\gamma_4 = (\alpha_{34} + \alpha_{35}\alpha_{54})[\frac{\alpha_{13}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{23}[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_4(1-\alpha_{12}\alpha_{21})(1-\alpha_{45}\alpha_{54})}] \quad (26)$$

$$\gamma_5 = \alpha_{35}\,[\frac{\alpha_{13}\,[(\lambda_{1H}+\lambda_{2H}\alpha_{21})+(\lambda_{1L}+\lambda_{2L}\alpha_{21})]+\alpha_{23}[(\lambda_{2H}+\lambda_{1H}\alpha_{12})+(\lambda_{2L}+\lambda_{1L}\alpha_{12})]}{\mu_5(1-\alpha_{12}\alpha_{21})(1-\alpha_{45}\alpha_{54})}] \quad (27)$$

$$\gamma_6 = \frac{\mu_{2L}(\lambda_{2H}+\lambda_{1H}\alpha_{12})+\mu_{2H}(\lambda_{2L}+\lambda_{1L}\alpha_{12})}{\mu_{2L}\mu_{2H}(1-\alpha_{12}\alpha_{21})} \quad (28)$$

$$\gamma_7 = \frac{\mu_{1L}(\lambda_{1H}+\lambda_{2H}\alpha_{21})+\mu_{1H}(\lambda_{1L}+\lambda_{2L}\alpha_{21})}{\mu_{1L}\mu_{1H}(1-\alpha_{12}\alpha_{21})} \quad (29)$$

The solution of the model exists if $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \leq 1$

4. Queuing Model Characteristics

a) Expected Queue Length of the entire system

$$L = L_{q1L} + L_{q1H} + L_{q2L} + L_{q2H} + L_{q3} + L_{q4} + L_{q5}$$

Where $L_{q1L} = \frac{\gamma_7}{(1-\gamma_7)}$, $L_{q1H} = \frac{\gamma_1}{(1-\gamma_1)}$, $L_{q2L} = \frac{\gamma_6}{(1-\gamma_6)}$, $L_{q2H} = \frac{\gamma_2}{(1-\gamma_2)}$,

$L_{q3} = \frac{\gamma_3}{(1-\gamma_3)}$, $L_{q4} = \frac{\gamma_4}{(1-\gamma_4)}$, $L_{q5} = \frac{\gamma_5}{(1-\gamma_5)}$

b) *Variance in queue length*

$$Var = V_{n1L} + V_{n1H} + V_{n2L} + V_{n2H} + V_3 + V_4 + V_5$$

Where $V_{n1L} = \frac{\gamma_7}{(1-\gamma_7)^2}$, $V_{n1H} = \frac{\gamma_1}{(1-\gamma_1)^2}$, $V_{n2L} = \frac{\gamma_6}{(1-\gamma_6)^2}$, $V_{n2H} = \frac{\gamma_2}{(1-\gamma_2)^2}$,

$V_3 = \frac{\gamma_3}{(1-\gamma_3)^2}$, $V_4 = \frac{\gamma_4}{(1-\gamma_4)^2}$, $V_5 = \frac{\gamma_5}{(1-\gamma_5)^2}$

c) *Expected time spent by the customer in the system*

$$E = \frac{L}{\lambda}, \quad \lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H}$$

5. Behavior Analysis

Table 1. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. the different Arrival rates of High Priority Customers at S_{11}

$\lambda_{1H} = 8, \lambda_{1L} = 5, \lambda_{2L} = 4, \lambda_{2H} = 6, \mu_{1L} = 25, \mu_{1H} = 40, \mu_{2L} = 20, \mu_{2H} = 30, \mu_3 = 38, \mu_4 = 55, \mu_5 = 46, \alpha_{12} = .4, \alpha_{21} = .7, \alpha_{13} = .6, \alpha_{23} = .3, \alpha_{35} = .5, \alpha_{34} = .5, \alpha_{45} = .6, \alpha_4 = .4, \alpha_{54} = .8, \alpha_5 = .2, \eta 1L = 2, \eta 1H = 4, \eta 2L = 3, \eta 2H = 6, \eta 3 = 15, \eta 4 = 8, \eta 5 = 7$										
λ_{1H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	**Var**	**L**	**E(W)**
4	.2847	.3518	.5000	.5979	.3971	.7685	.7180	31.5937	9.9548	.5239
5	.3194	.3703	.5263	.6293	.4180	.7870	.7527	39.4799	11.3263	.5663
6	.3541	.3888	.5526	.6608	.4389	.8055	.7875	50.5629	13.0001	.6190
7	.3888	.4074	.5789	.6923	.4598	.8240	.8222	67.0482	15.1118	.6869
8	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
9	.4583	.4444	.6315	.7552	.5016	.8611	.8916	143.3373	21.8894	.9120
10	.4930	.4629	.6578	.7867	.5225	.8796	.9263	1805.2205	28.4388	1.1375
11	.5277	.4814	.6842	.8181	.5434	.8981	.9611	6532.9465	43.4937	1.6728
12	.5625	.5000	.7105	.8496	.5643	.9166	.9958	587147.1066	2512.1923	93.0441

Table 2. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. the different Arrival rates of High Priority Customers at S_{12}

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55, \mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8, \alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$										
λ_{2H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	**Var**	**L**	**E(W)**
2	.3263	.2407	.5000	.5979	.3971	.6574	.7597	26.6998	9.0295	.4752
3	.3506	.2870	.5263	.6293	.4180	.7037	.7840	34.4083	10.4769	.5238
4	.3750	.3333	.5526	.6608	.4389	.7500	.8083	45.6367	12.2849	.5849
5	.3993	.3796	.5789	.6923	.4598	.7962	.8326	63.2218	14.6388	.6654
6	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
7	.4479	.4722	.6315	.7552	.5016	.8888	.8812	157.2032	22.9368	.9557
8	.4729	.5185	.6578	.7867	.5225	.9351	.9055	3273.01943	32.7015	1.3080
9	.4965	.5648	.6842	.8181	.5434	.9814	.9298	30801.4606	76.4542	2.9405

Table 3. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. the different Arrival rates of Low Priority Customers at S_{11}

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55, \mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8, \alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$										
λ_{1L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	**Var**	**L**	**E(W)**
2	.4236	.4259	.5263	.6293	.4180	.7592	.6902	31.0409	10.3871	.5193
2.5	.4236	.4259	.5394	.6451	.4285	.7731	.7180	35.6318	11.1172	.5449
3	.4236	.4259	.5526	.6608	.4389	.7870	.7458	41.3876	12.0747	.5749
3.5	.4236	.4259	.5657	.6765	.4494	.8009	.7736	48.8526	13.1308	.6189
4	.4236	.4259	.5789	.6923	.4598	.8148	.8013	58.8908	14.3904	.6541
4.5	.4236	.4259	.5921	.7080	.4703	.8287	.8291	72.8890	15.9369	.7083
5	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
5.5	.4236	.4259	.6184	.7395	.4918	.8564	.8847	128.4180	20.5506	.8744
6	.4236	.4259	.6315	.7552	.5016	.8703	.9125	1274.3102	24.4275	1.0178
6.5	.4236	.4259	.6447	.7709	.5121	.8842	.9402	2777.3064	31.0988	1.2693
7	.4236	.4259	.6578	.7867	.5225	.8981	.9680	9794.9998	47.3504	1.8940

Table 4. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. the different Arrival rates of Low Priority Customers at S_{12}

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55, \mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8, \alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$										
λ_{2L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	**Var**	**L**	**E(W)**
2	.4236	.4259	.5526	.6608	.4389	.7037	.7791	34.7735	11.3474	.5403
2.5	.4236	.4259	.5657	.6765	.4494	.7384	.7986	44.0324	12.4787	.5804
3	.4236	.4259	.5789	.6923	.4598	.7731	.8180	54.4825	13.6578	.6208
3.5	.4236	.4259	.5921	.7080	.4703	.8078	.8375	69.7293	15.6011	.6933
4	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
4.5	.4236	.4259	.6184	.7395	.4912	.8773	.8763	135.7576	21.1486	.8999
5	.4236	.4259	.6315	.7552	.5016	.9120	.8958	1289.2088	26.2643	1.0943
5.5	.4236	.4259	.6447	.7709	.5121	.9467	.9152	4694.6249	36.3082	1.4819
6	.4236	.4259	.6578	.7867	.5225	.9814	.9347	31117.9924	75.5684	3.0227

Table 5. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate of High Priority Customers at S_{11}

$\lambda_{1H} = 8, \lambda_{1L} = 5, \lambda_{2L} = 4, \lambda_{2H} = 6, \mu_{1L} = 25, \mu_{1H} = 40, \mu_{2L} = 20, \mu_{2H} = 30, \mu_3 = 38, \mu_4 = 55, \mu_5 = 46, \alpha_{12} = .4, \alpha_{21} = .7, \alpha_{13} = .6, \alpha_{23} = .3, \alpha_{35} = .5, \alpha_{34} = .5, \alpha_{45} = .6, \alpha_4 = .4, \alpha_{54} = .8, \alpha_5 = .2, \eta 1L = 2, \eta 1H = 4, \eta 2L = 3, \eta 2H = 6, \eta 3 = 15, \eta 4 = 8, \eta 5 = 7$

μ_{1H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
30	.5648	.4259	.6052	.7237	.4807	.8425	.9981	2935641.785	5557.4718	241.6295
35	.4841	.4259	.6052	.7237	.4807	.8425	.9174	1401.5033	23.2323	1.0101
40	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
45	.3765	.4259	.6052	.7237	.4807	.8425	.8098	73.9674	16.0375	.6972
50	.3388	.4259	.6052	.7237	.4807	.8425	.7722	66.2490	15.0775	.6555
55	.3080	.4259	.6052	.7237	.4807	.8425	.7414	62.3087	14.4869	.6298
60	.2824	.4259	.6052	.7237	.4807	.8425	.7157	59.9725	14.0855	.6124
65	.2606	.4259	.6052	.7237	.4807	.8425	.6940	58.4578	13.7949	.5997
70	.2420	.4259	.6052	.7237	.4807	.8425	.6753	57.4009	13.5735	.5901

Table 6. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate of High Priority Customers at S_{12}

$\lambda_{1H} = 8, \lambda_{1L} = 5, \lambda_{2L} = 4, \lambda_{2H} = 6, \mu_{1L} = 25, \mu_{1H} = 40, \mu_{2L} = 20, \mu_{2H} = 30, \mu_3 = 38, \mu_4 = 55, \mu_5 = 46, \alpha_{12} = .4, \alpha_{21} = .7, \alpha_{13} = .6, \alpha_{23} = .3, \alpha_{35} = .5, \alpha_{34} = .5, \alpha_{45} = .6, \alpha_4 = .4, \alpha_{54} = .8, \alpha_5 = .2, \eta 1L = 2, \eta 1H = 4, \eta 2L = 3, \eta 2H = 6, \eta 3 = 15, \eta 4 = 8, \eta 5 = 7$

μ_{2H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
25	.4236	.5111	.6052	.7237	.4807	.9277	.8569	1844.6215	25.7012	1.1174
30	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
35	.4236	.3650	.6052	.7237	.4807	.7817	.8569	75.7723	15.9638	.6940
40	.4236	.3194	.6052	.7237	.4807	.7361	.8569	69.7104	15.0662	.6550
45	.4236	.2839	.6052	.7237	.4807	.7006	.8569	66.8175	14.5438	.6323
50	.4236	.2555	.6052	.7237	.4807	.6722	.8569	65.1646	14.2010	.6174
55	.4236	.2323	.6052	.7237	.4807	.6489	.8569	64.1058	13.9579	.6068
60	.4236	.2129	.6052	.7237	.4807	.6296	.8569	63.3806	13.7773	.5990
65	.4236	.1965	.6052	.7237	.4807	.6132	.8569	62.8507	13.6368	.5929

Table 7. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate of Low Priority Customers at S_{11}

$\lambda_{1H} = 8, \lambda_{1L} = 5, \lambda_{2L} = 4, \lambda_{2H} = 6, \mu_{1L} = 25, \mu_{1H} = 40, \mu_{2L} = 20, \mu_{2H} = 30, \mu_3 = 38, \mu_4 = 55, \mu_5 = 46, \alpha_{12} = .4, \alpha_{21} = .7, \alpha_{13} = .6, \alpha_{23} = .3, \alpha_{35} = .5, \alpha_{34} = .5, \alpha_{45} = .6, \alpha_4 = .4, \alpha_{54} = .8, \alpha_5 = .2, \eta 1L = 2, \eta 1H = 4, \eta 2L = 3, \eta 2H = 6, \eta 3 = 15, \eta 4 = 8, \eta 5 = 7$

μ_{1L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
20	.4236	.4259	.6052	.7237	.4807	.8425	.9652	8095.1749	39.7243	1.7271
25	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
30	.4236	.4259	.6052	.7237	.4807	.8425	.7847	68.7897	15.5551	.6763
35	.4236	.4259	.6052	.7237	.4807	.8425	.7331	62.1379	14.6565	.6372
40	.4236	.4259	.6052	.7237	.4807	.8425	.6944	59.2842	14.1817	.6165
45	.4236	.4259	.6052	.7237	.4807	.8425	.6643	57.7412	13.8882	.6038
50	.4236	.4259	.6052	.7237	.4807	.8425	.6402	56.7890	13.6886	.5951
55	.4236	.4259	.6052	.7237	.4807	.8425	.6205	56.1536	13.5442	.5888
60	.4236	.4259	.6052	.7237	.4807	.8425	.6041	55.6991	13.4350	.5841

Table 8. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate of Low Priority Customers at S_{12}

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55,$
$\mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8,$
$\alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$

μ_{2L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
15	.4236	.4259	.6052	.7237	.4807	.9814	.8569	28924.4430	65.5971	2.8520
20	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
25	.4236	.4259	.6052	.7237	.4807	.7592	.8569	72.8494	15.7026	.6827
30	.4236	.4259	.6052	.7237	.4807	.7037	.8569	67.7611	14.9242	.6488
35	.4236	.4259	.6052	.7237	.4807	.6640	.8569	65.6237	14.5252	.6315
40	.4236	.4259	.6052	.7237	.4807	.6342	.8569	64.4806	14.2827	.6209
45	.4236	.4259	.6052	.7237	.4807	.6111	.8569	63.7788	14.1202	.6139
50	.4236	.4259	.6052	.7237	.4807	.5925	.8569	63.3086	14.0028	.6088
55	.4236	.4259	.6052	.7237	.4807	.5774	.8569	62.9719	13.9151	.6050

Table 9. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate at S_3

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55,$
$\mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8,$
$\alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$

μ_3	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
30	.4236	.4259	.7666	.7237	.4807	.8425	.8569	104.0540	19.6536	.8545
34	.4236	.4259	.6764	.7237	.4807	.8425	.8569	96.4286	18.4586	.8025
38	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
42	.4236	.4259	.5476	.7237	.4807	.8425	.8569	92.6385	17.5785	.7642
46	.4236	.4259	.5000	.7237	.4807	.8425	.8569	91.9621	17.3678	.7551
50	.4236	.4259	.4600	.7237	.4807	.8425	.8569	91.5396	17.2196	.7486
54	.4236	.4259	.4259	.7237	.4807	.8425	.8569	91.2546	17.1097	.7439
58	.4236	.4259	.3965	.7237	.4807	.8425	.8569	91.0510	17.0249	.7402
62	.4236	.4259	.3709	.7237	.4807	.8425	.8569	90.8996	16.9574	.7372

Table 10. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate at S_{21}

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55,$
$\mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8,$
$\alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$

μ_4	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
45	.4236	.4259	.6052	.8846	.4807	.8425	.8569	150.8604	22.9530	.9979
50	.4236	.4259	.6052	.7961	.4807	.8425	.8569	103.5323	19.1871	.8342
55	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
60	.4236	.4259	.6052	.6634	.4807	.8425	.8569	90.2096	17.2523	.7501
65	.4236	.4259	.6052	.6124	.4807	.8425	.8569	88.4264	16.8612	.7330
70	.4236	.4259	.6052	.5686	.4807	.8425	.8569	87.4061	16.5992	.7217
75	.4236	.4259	.6052	.5307	.4807	.8425	.8569	86.7603	16.4112	.7135
80	.4236	.4259	.6052	.4975	.4807	.8425	.8569	86.3202	16.2711	.7074
85	.4236	.4259	.6052	.4683	.4807	.8425	.8569	86.0063	16.1618	.7026

Table 11. Utilization of Server, Variance of the queue, Mean Queue length, Average Waiting time for customer w. r. t. different Service rate at S_{22}

$\lambda_{1H}=8, \lambda_{1L}=5, \lambda_{2L}=4, \lambda_{2H}=6, \mu_{1L}=25, \mu_{1H}=40, \mu_{2L}=20, \mu_{2H}=30, \mu_3=38, \mu_4=55, \mu_5=46, \alpha_{12}=.4, \alpha_{21}=.7, \alpha_{13}=.6, \alpha_{23}=.3, \alpha_{35}=.5, \alpha_{34}=.5, \alpha_{45}=.6, \alpha_4=.4, \alpha_{54}=.8, \alpha_5=.2, \eta 1L=2, \eta 1H=4, \eta 2L=3, \eta 2H=6, \eta 3=15, \eta 4=8, \eta 5=7$										
μ_5	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	Var	L	E(W)
28	.4236	.4259	.6052	.7237	.7898	.8425	.8569	109.9727	20.7344	.9014
34	.4236	.4259	.6052	.7237	.6504	.8425	.8569	97.3902	18.8362	.8189
40	.4236	.4259	.6052	.7237	.5528	.8425	.8569	94.8288	18.2117	.7918
46	.4236	.4259	.6052	.7237	.4807	.8425	.8569	93.8465	17.9011	.7783
52	.4236	.4259	.6052	.7237	.4252	.8425	.8569	93.3512	17.7151	.7702
58	.4236	.4259	.6052	.7237	.3812	.8425	.8569	93.0595	17.5914	.7648
64	.4236	.4259	.6052	.7237	.3455	.8425	.8569	92.8701	17.5032	.7610
70	.4236	.4259	.6052	.7237	.3159	.8425	.8569	92.7386	17.4371	.7581
76	.4236	.4259	.6052	.7237	.2909	.8425	.8569	92.6422	17.3855	.7558

6. Results

In the present study, two servers C_1 and C_2, are in series, and both comprise two biserial subsystems connected to a common server C_3. A detailed model description has been done with pictorial representation in section 3. In section 4, the mathematical modelling of the presented model is done, and derive governing equations which have been used to find out various queue characteristics. From Table1and Table 2, it is clear that while changing the arrival pattern of high-priority customers at subsystems C_{11} & C_{12}, mean queue length and variance increase with high speed when $\lambda_{1H} = 9$ & $\lambda_{2H} = 8$. Average time spent by the customer in the system increases higher than before when $\lambda_{1H} = 12$. Table 3 and 4 results in practical conclusion increased number of arrivals of customers at any server increase queue length and waiting time. Also, the arrivals of low-priority customers do not affect the utilization of servers by high-priority customers. Table5 shows the change in traffic intensity, variance and queue lengths with a change in service rate for high-priority customers at C_{11}, and from the results, it is clear that an increase in service rate for high-priority customers decreases traffic intensity γ_1 & γ_7 at C_{11} and traffic intensities $\gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6$ remains unaffected at other servers. Queue lengths, fluctuation in queues and time spent by a customer in the system decrease. Thus, practically and mathematically, it is true that while increasing service rates, the customers are served rapidly, and as a result, the length of queues and average Waiting time decreases. The same outcome is shown in Table 6-11.

7. Conclusion

The present study is the analytical study of priority bi-serial queues at both subsystems centrally connected to a common subsystem. The present model is developed to find various queue behaviors such as server utilization, queue lengths and variance, and we can check the numerical behavior of the model with variations in various input parameters. This analytical study has various daily life applications in networking systems, supermarkets, administrations, industries etc. The validity of the study can be checked by considering the particular case, if we take parallel service channels instead of biserial channels at exit level subsystem results given by Saini A. and Gupta Deepak [15] and if we remove priority on entry biserial subsystems, then the results coincide with Gupta Deepak [3]. Thus, this model is useful to increase customer satisfaction and optimum utilization of the server.

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