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Original Article

CASP-CUSUM Schemes based on Truncated Half-Logistic Distribution

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Abstract *- This paper is on the study of continuous acceptance sampling plans established primarily for the acceptance or rejection of bulk finished products. Several methods can be used for quality control. Some methods are commonly used for destructive testing where 100% verification is not possible, such as in the production of cookies, marbles, batteries, light bulbs, etc. In this paper, we optimize the CASP-CUSUM scheme under the assumption that the continuous variable under consideration follows a truncated Half-Logistic Distribution. The Half-Logistic Distribution is a continuous distribution and it is extending well-known distributions as well as provides great flexibility to model specific real data and it is very easy in mathematical properties. We propose an optimization of the CASP-CUSUM scheme based on the numerical results obtained by varying the parameter values of the Half-Logistic Distribution.*

Keywords *- CASP – CUSUM Schemes, Optimal Truncated Point, optimal lower and upper truncated Point, Truncated Half-Logistic Distribution.*

1. Introduction

Quality relates to one or more desirable characteristics that a product or service should possess. Quality has become one of the most important consumer decision factors in the selection of competing products and services. Quality improvement methods can be applied to any area within a company or organization, including the manufacturing process, development, engineering design, finance and accounting, marketing, distribution and logistics, customer service, and field service of products. In order to tackle the development of advanced technologies, the reliability of products has become a significant matter of concern. It factors with respect to failure avoidance rather than the probability of failure. Product failure occurs when the product is not able to perform its objective function and does not meet its requirements. Thus truncation of a product is the capability to fulfill intended tasks for a specified performance period.

One of the most commonly used functions of inspection tools is the acceptance sampling plan. The acceptance sampling plan uses statistical principles to determine the "sample size" and criteria for accepting or rejecting a product. Industrial uses different kinds of technology and is mainly used in the manufacture of bullets, firecrackers, bulbs and batteries, etc., where 100% inspection is not possible. Sampling provides a reasonable way to ensure that a product meets specified requirements. 100% annual checking is also too expensive in terms of time and profit and cannot ensure 100% obedience. Instead of evaluating every item or product, a specific sample is taken and inspected or tested before making an acceptance or rejection for an entire lot of products.

Acceptance sampling is a statistical measure used in quality control. It allows a company to determine the quality of a batch of products by selecting a specified number for testing. The quality of this designated sample will be viewed as the quality level for the entire group of products. It is a technique which deals with the acceptance or rejection of a lot or process based on the results obtained from a random sample or samples taken randomly from the lot. A point to remember is that the main purpose of acceptance sampling is to decide whether or not the lot is likely to be accepted, not to estimate the quality of the lot. "An individual sampling plan has much the effect of a lone sniper, while the sampling plan scheme can provide a fusillade in the battle for quality improvement", as noted by Ed Schilling.

Akhtar and Sarma[1] studied Continuous acceptance sampling plans based on the truncated negative exponential distribution for Optimizing CASP-CUSUM schemes by solving the integral equation using the Gauss-Chebyshev integration method with the help of a computer program. Lastly, the obtained results were compared at different values of the parameters.

Balakrishna[3] introduced Half-Logistic distribution if a random variable having a half-logistic distribution is obtained by folding the logistic distribution. For this distribution, some recurrence relations are established for the moments and product moments of order statistics. Starting with the first k moments of X, it is noted that one can calculate the first k moments of all order statistics.

C. R. Dias[4] presents some special models of the new class and investigates the shapes and derives explicit expressions for the ordinary and incomplete moments, quantile and generating functions and probability-weighted moments.

Dhanunjaya.S[6] has studied the lifetime of the units by using the Truncated Lindley failure model and to optimize CASP – CUSUM Schemes through Truncated Lindley distribution by solving integral equations by the Method of Gauss – Chebyshev integration determines the probability of acceptance at various hypothetical values of the parameters.

El-Gohary[7] proposed Generalized Gompertz distribution; it has an increasing or constant, or decreasing bathtub curve failure rate depending upon the shape parameter. This property makes GGD very useful in survival analysis. Some statistical properties such as moments, mode, and quantiles, Failure rate function are derived.

Venkatesulu.G[13] considered life tests experiments are carried out to determine an optimal truncated point. Truncated distributions are employed in many practical situations where there is a constraint on the lower and upper limits of the variable under study.

2. Half-Logistic Distribution

Definition: If X belongs to Half-Logistic with parameter λ > 0 cumulative distribution function (c.d.f.) of HL distribution can be obtained as

$$\mathrm{F}(\mathrm{x};\mathrm{a},\mathrm{b},\lambda) = \frac{2\lambda e^{-\lambda x}}{(1+e^{-\lambda x})^2} \tag{2.1}$$

The Half-Logistic distribution is formed by using the absolute transformation of the logistic distribution, therefore, having much importance in statistics, physics, hydrology and logistic regression. The Probability models are frequently used for the prediction of lifetime products in various fields of applied sciences. These models are also used to explain the failure rate and survival rate of a certain product. Therefore, many generalizations are formed by adding additional shape parameters to increase the flexibility of these probabilistic models.

2.1. Truncated Half-Logistic Distribution

It is the ratio of the probability density function of the Half-Logistic distribution to their corresponding cumulative distribution function at point B. The random variable X is said to follow Truncated Half-Logistic distribution as

$$f_{B}(x) = \frac{\frac{2\lambda e^{-\lambda x}}{(1+e^{-\lambda x})^2}}{\frac{1-e^{-\lambda B}}{1+e^{-\lambda B}}} \tag{2.2}$$

Where a > 0, b > 0, λ > 0 are the shape parameters.

3. Description of CASP- CUSUM Schemes

Beattie has suggested the method for constructing the continuous acceptance sampling plans. The scheme suggested by him consists of chosen decision interval, namely, the "Return interval" with the length h' above the decision line is taken. We plot on the chart the sum $S_m = \sum(X_i - k \quad) X_i's (i = 1,2,3 \ldots\ldots)$ is distributed independently, and k is the reference value. If the sum lies in the area of the normal chart, the product is accepted, and if it lies on the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the subsequent point is to be plotted at the highest, i.e., h+h.'
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.
   When the CUSUM falls in the return chart, a network or a change of specification may be engaged rather than outright rejected.

   The procedure, in brief, is given underneath.

   A. Start plotting the CUSUM at 0.
   B. The product is accepted $S_m = \sum(X_i - k) < h$; when $S_m < 0$, return cumulative to 0.
   C. When h < S_m < h+h', the product is rejected: when S_m crosses h, i.e., when S_m > h+h' and continues rejecting the product until S_m > h+h' returns cumulative to h+h.'

The type-C OC function, which is defined as the probability of acceptance of an item as a function of incoming quality when the sampling rate is the same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0)+L'(0)} \tag{3.1}$$

Where L (0) = Average Run Length in acceptance zone and
L' (0) = Average Run Length in rejection zone.
Page E.S.[11] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1-P(0)} \tag{3.2}$$

$$L'(0) = \frac{N'(0)}{1-P'(0)} \tag{3.3}$$

Where P (0) =Probability for the test starting from zero on the normal chart,
N (0) = ASN for the test starting from zero on the normal chart,
P' (0) = probability for the test on the return chart and
N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities
P (0), N (0), and P' (0), N' (0) as follows:

$$P(z) = F(k-z) + \int_0^h P(y)f(y+k-z)dy, \tag{3.4}$$

$$N(z) = 1 + \int_0^h N(y)f(y+k-z)dy, \tag{3.5}$$

$$P'(z) = \int_{k_1+z}^{B} f(y)dy + \int_0^h P'(y)f(-y+k+z)dy \tag{3.6}$$

$$N'(z) = 1 + \int_0^h N'(y)\,f(-y+k+z)dy, \tag{3.7}$$

$$F(x) = 1 + \int_A^h f(x)dx:$$

$$F(k-z) = 1 + \int_A^{k_1-z} f(y)\,dy$$

and z is the distance of the preliminary test in the normal chart from zero.

4. Computation of ARL's AND P (A)

We expanded computer programs to solve the equations (3.4), (3.5), (3.6) & (3.7), and we got the following results given in Tables (4.1) to (4.30).

Table 4.1 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 1, h = 0.01, h'= 0.01

B	L(0)	L'(0)	P(A)
0.5	1.9736	1.0303342	0.6570063
0.4	2.0246	1.0379221	0.6610914
0.3	2.1369	1.0509106	0.6703327
0.2	2.5074	1.0780401	0.6993308
0.1	17.5613	1.1689932	0.9375881

Table 4.2 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 1, h = 0.02, h'= 0.02

B	L(0)	L'(0)	P(A)
0.6	2.0397	1.0520990	0.6597171
0.5	2.1022	1.0625944	0.6642460
0.4	2.2205	1.0788779	0.6730031
0.3	2.5062	1.1073642	0.6935530
0.2	3.8027	1.1694901	0.7647915

Table 4.3 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 1, h = 0.03, h'= 0.03

B	L(0)	L'(0)	P(A)
0.6	2.1431	1.0802691	0.6648598
0.5	2.2477	1.0969690	0.6720266
0.4	2.4559	1.1232454	0.6861715
0.3	3.0214	1.1703144	0.7208005
0.2	7.6781	1.2781163	0.8572921

Table 4.4 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 1, h = 0.04, h'= 0.04

B	L(0)	L'(0)	P(A)
0.6	2.2568	1.1100100	0.6703123
0.5	2.4137	1.1336727	0.6804223
0.4	2.7444	1.1714677	0.7008370
0.3	3.7901	1.2409474	0.7533438
0.2	1991.9415	1.4092407	0.9992930

Table 4.5 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 1, h = 0.05, h'= 0.05

B	L(0)	L'(0)	P(A)
0.7	2.2600	1.1201972	0.6685966
0.6	2.3827	1.1414566	0.6761091
0.5	2.6049	1.1729493	0.6895215
0.4	3.1059	1.2240682	0.7173049
0.3	5.0610	1.3207558	0.7930434

Table 4.6 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 1, h = 0.01, h'= 0.01

B	L(0)	L'(0)	P(A)
0.6	4.5035	1.0232244	0.8148596
0.5	4.6071	1.0270911	0.8177032
0.4	4.8065	1.0331422	0.8230814
0.3	5.2884	1.0436623	0.8351781
0.2	7.3116	1.0657955	0.8727778

Table 4.7 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 1, h = 0.02, h'= 0.02

B	L(0)	L'(0)	P(A)
0.6	4.8554	1.0475421	0.8225390
0.5	5.0988	1.0556760	0.8284696
0.4	5.6039	1.0685331	0.8398592
0.3	7.0703	1.0912676	0.8662918
0.2	24.7612	1.1407456	0.9559590

Table 4.8 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 1, h = 0.03, h'= 0.03

B	L(0)	L'(0)	P(A)
0.7	5.0239	1.0643786	0.8251749
0.6	5.2597	1.0730314	0.8305591
0.5	5.6948	1.0858793	0.8398565
0.4	6.6860	1.1064055	0.8580143
0.3	10.4855	1.1433663	0.9016787

Table 4.9 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 1, h = 0.04, h'= 0.04

B	L(0)	L'(0)	P(A)
0.7	5.3679	1.0876999	0.8315108
0.6	5.7292	1.0997773	0.8389552
0.5	6.4322	1.1178406	0.8519422
0.4	8.2379	1.1470257	0.8777796
0.3	19.6740	1.2006149	0.9424844

Table 4.10 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 1, h = 0.05, h'= 0.05

B	L(0)	L'(0)	P(A)
0.7	5.7567	1.1120536	0.8381005
0.6	6.2810	1.1278740	0.8477668
0.5	7.3680	1.1517154	0.8648178
0.4	10.6501	1.1906991	0.8994414
0.3	128.9552	1.2638034	0.9902948

Table 4.11 Values of ARL's AND TYPE-C OC CURVES when λ = 3, k = 1, h = 0.01, h'= 0.01

B	L(0)	L'(0)	P(A)
0.6	12.4236	1.0240337	0.9238505
0.5	13.0197	1.0271856	0.9268744
0.4	14.2863	1.0323073	0.9326107
0.3	18.0609	1.0414805	0.9454790
0.2	70.1609	1.0611800	0.9851004

Table 4.12 Values of ARL's AND TYPE-C OC CURVES when λ = 3, k = 1, h = 0.02, h'= 0.02

B	L(0)	L'(0)	P(A)
0.7	14.1307	1.0449135	0.9311452
0.6	15.0467	1.0492175	0.9348148
0.5	16.8832	1.0558468	0.9411424
0.4	21.7796	1.0667089	0.9533094
0.3	57.2689	1.0864439	0.9813823

Table 4.13 Values of ARL's AND TYPE-C OC CURVES when λ = 3, k = 1, h = 0.03, h'= 0.03

B	L(0)	L'(0)	P(A)
0.8	15.8395	1.0643218	0.9370368
0.7	16.9209	1.0688728	0.9405844
0.6	18.9587	1.0756340	0.9463103
0.5	23.7480	1.0861049	0.9562655
0.4	44.2959	1.1034113	0.9756954

Table 4.14 Values of ARL's AND TYPE-C OC CURVES when λ = 3, k = 1, h = 0.04, h'= 0.04

B	L(0)	L'(0)	P(A)
0.9	17.7062	1.0832105	0.9423499
0.8	18.8805	1.0875857	0.9455339
0.7	20.9778	1.0939236	0.9504377
0.6	25.4172	1.1033747	0.9583955
0.5	39.3267	1.1180949	0.9723550

Table 4.15 Values of ARL's AND TYPE-C OC CURVES when λ = 3, k = 1, h = 0.05, h'= 0.05

B	L(0)	L'(0)	P(A)
0.9	21.1340	1.1061561	0.9502632
0.8	23.2610	1.1118585	0.9543812
0.7	27.4162	1.1201409	0.9607469
0.6	38.1057	1.1325399	0.9711368
0.5	108.9889	1.1519670	0.9895409

Table 4.16 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 2, h = 0.01, h'= 0.01

B	L(0)	L'(0)	P(A)
0.6	4.5279	1.0253789	0.8153561
0.5	4.6571	1.0303342	0.8188416
0.4	4.9075	1.0379221	0.8254240
0.3	5.5289	1.0509106	0.8402826
0.2	8.4861	1.0780401	0.8872837

Table 4.17 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 2, h = 0.02, h'= 0.02

B	L(0)	L'(0)	P(A)
0.6	4.9157	1.0520990	0.8237038
0.5	5.2284	1.0625944	0.8310927
0.4	5.8975	1.0788779	0.8453530
0.3	8.0431	1.1073642	0.8789825
0.2	1632.3413	1.1694901	0.9992841

Table 4.18 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 2, h = 0.03, h'= 0.03

B	L(0)	L'(0)	P(A)
0.7	5.0661	1.0687660	0.8257878
0.6	5.3724	1.0802691	0.8325863
0.5	5.9517	1.0969690	0.8443712
0.4	7.3652	1.1232454	0.8676738
0.3	14.5413	1.1703144	0.9255128

Table 4.19 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 2, h = 0.04, h'= 0.04

B	L(0)	L'(0)	P(A)
0.7	5.4382	1.0938698	0.8325388
0.6	5.9183	1.1100100	0.8420653
0.5	6.8969	1.1336727	0.8588309
0.4	9.7661	1.1714677	0.8928955
0.3	70.6144	1.2409474	0.9827300

Table 4.20 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 2, h = 0.05, h'= 0.05

B	L(0)	L'(0)	P(A)
0.8	5.4693	1.1049460	0.8319272
0.7	5.8666	1.1201972	0.8396683
0.6	6.5822	1.1414566	0.8522124
0.5	8.1850	1.1729493	0.8746574
0.4	14.4053	1.2240682	0.9216818

Table 4.21 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 2, h = 0.01, h'= 0.01

B	L(0)	L'(0)	P(A)
0.8	36.6673	1.0187007	0.9729687
0.7	39.1720	1.0205847	0.9746076
0.6	43.7962	1.0232244	0.9771701
0.5	54.4532	1.0270911	0.9814873
0.4	98.3399	1.0331422	0.9896034

Table 4.22 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 2, h = 0.02, h'= 0.02

B	L(0)	L'(0)	P(A)
0.9	47.4780	1.0352302	0.9786609
0.8	53.4272	1.0381074	0.9809400
0.7	65.4605	1.0420263	0.9843311
0.6	100.3337	1.0475421	0.9896674
0.5	844.7839	1.0556760	0.9987519

Table 4.23 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 2, h = 0.03, h'= 0.03

B	L(0)	L'(0)	P(A)
1.1	55.4392	1.0478579	0.9814495
1.0	61.5715	1.0504247	0.9832259
0.9	72.5755	1.0537832	0.9856880
0.8	97.0487	1.0582601	0.9892132
0.7	191.8455	1.0643786	0.9944825

Table-4.24 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 2, h = 0.04, h'= 0.04

B	L(0)	L'(0)	P(A)
1.2	71.4737	1.0620973	0.9853576
1.1	81.8197	1.0648344	0.9871528
1.0	101.4961	1.0683703	0.9895835
0.9	151.3566	1.0730062	0.9929606
0.8	492.3849	1.0792021	0.9978130

Table 4.25 Values of ARL's AND TYPE-C OC CURVES when λ = 2, k = 2, h = 0.05, h'= 0.05

B	L(0)	L'(0)	P(A)
1.4	86.0475	1.0738931	0.9876736
1.3	96.7284	1.0760716	0.9889977
1.2	115.2862	1.0788326	0.9907289
1.1	154.1920	1.0823637	0.9930294
1.0	281.2453	1.0869329	0.9961501

Table 4.26 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 3, h = 0.01, h'= 0.01

B	L(0)	L'(0)	P(A)
0.7	11.9447	1.0219035	0.9211897
0.6	12.4491	1.0253789	0.9239022
0.5	13.3739	1.0303342	0.9284701
0.4	15.4587	1.0379221	0.9370825
0.3	23.1243	1.0509106	0.9565294

Table 4.27 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 3, h = 0.02, h'= 0.02

B	L(0)	L'(0)	P(A)
0.8	12.9699	1.0394588	0.9258025
0.7	13.7600	1.0448024	0.9294280
0.6	15.1656	1.0520990	0.9351265
0.5	18.2072	1.0625944	0.9448571
0.4	28.5993	1.0788779	0.9636474

Table 4.28 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 3, h = 0.03, h'= 0.03

B	L(0)	L'(0)	P(A)
0.8	14.6375	1.0603966	0.9324498
0.7	16.2028	1.0687660	0.9381201
0.6	19.3479	1.0802691	0.9471187
0.5	28.3280	1.0969690	0.9627198
0.4	176.1987	1.1232454	0.9936655

Table 4.29 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 3, h = 0.04, h'= 0.04

B	L(0)	L'(0)	P(A)
0.9	15.2283	1.0734247	0.9341527
0.8	16.7803	1.0822072	0.9394147
0.7	19.6668	1.0938698	0.9473106
0.6	26.6208	1.1100100	0.9599719
0.5	62.8863	1.1336727	0.9822919

Table 4.30 Values of ARL's AND TYPE-C OC CURVES when λ = 1, k = 3, h = 0.05, h'= 0.05

B	L(0)	L'(0)	P(A)
1.0	15.6382	1.0846835	0.9351377
0.9	17.0998	1.0935217	0.9398943
0.8	19.6352	1.1049460	0.9467244
0.7	24.9614	1.1201972	0.9570502
0.6	42.4169	1.1414566	0.9737948

5. Numerical Results And Conclusion

At the hypothetical values of the parameters λ, k, h and h' are given at the top of each Table, we determine optimum truncated point B at which P (A) the probability of accepting an item is greatest and also obtained ARL's values which represent the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C OC Curve, i.e. P (A), are given in columns I, II, III, and IV, respectively.

Table 5.1 Consolidated Table from the Tables (4.1) to (4.30)

B	λ	k	h	h'	P(A)
0.1	1	1	0.01	0.01	0.9375881
0.2	1	1	0.02	0.02	0.7647915
0.2	1	1	0.03	0.03	0.8572921
0.2	***1***	***1***	***0.04***	***0.04***	***0.9992930***
0.3	1	1	0.05	0.05	0.7930434
0.2	2	1	0.01	0.01	0.8727778
0.2	2	1	0.02	0.02	0.9559590
0.3	2	1	0.03	0.03	0.9016787
0.3	2	1	0.04	0.04	0.9424844
0.3	2	1	0.05	0.05	0.9902948
0.2	3	1	0.01	0.01	0.9851004
0.3	3	1	0.02	0.02	0.9813823
0.4	3	1	0.03	0.03	0.9756954
0.5	3	1	0.04	0.04	0.9723550
0.5	3	1	0.05	0.05	0.9895409
0.2	1	2	0.01	0.01	0.8872837
0.2	1	2	0.02	0.02	0.9992841
0.3	1	2	0.03	0.03	0.9255128
0.3	1	2	0.04	0.04	0.9827300
0.4	1	2	0.05	0.05	0.9216818
0.4	2	2	0.01	0.01	0.9896034
0.5	2	2	0.02	0.02	0.9987519
0.7	2	2	0.03	0.03	0.9944825
0.8	2	2	0.04	0.04	0.9978130
1.0	2	2	0.05	0.05	0.9961501
0.3	1	3	0.01	0.01	0.9565294
0.4	1	3	0.02	0.02	0.9636474
0.4	1	3	0.03	0.03	0.9936655
0.5	1	3	0.04	0.04	0.9822919
0.6	1	3	0.05	0.05	0.9737948

From the above Tables 4.1 to 4.30, we made the subsequent conclusions.

1. From Table 4.1 to 4.30, it is observed that the values of P (A) are increased as the value of the truncated point decreases; thus, the truncated point of the random variable and the various parameters for CASP-CUSUM are correlated.
2. From Table 4.1 to 4.30, we observe that it is possible to maximize the truncated point B by increasing the value of *k*.
3. From Table 4.1 to 4.30, it is observed that at the maximum level of probability of acceptance.

P (A) the truncated point 'B' from 5.0 to 0.2 as the value of h changes from 0.01 to 0.05.

4. From Table 4.1 to 4.30, it can be observed that the value of L(0) and P(A) is increased as the value of the Truncated point decreases; thus, the Truncated point of the random variable and the various parameters for CASP-CUSUM are related.
5. From Table 4.1 to 4.30, it was observed that the truncated point 'B' changes from 5.0 to 0.2, and P (A) are as$h \rightarrow 0.04$ maximum, i.e. 0.9992930. Thus truncated point B and k are inversely related, and hand P(A) are positively related.
6. From Table 4.1 to 4.30, it is observed that the optimal truncated point changes from 0.1 to 0.2 as$h \rightarrow 0.04$
7. From Table 4.1 to 4.30, it is observed that the values of Maximum Probabilities increased as the increased values of 'λ' at constant values of the constraint.
8. The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM based on the above Table 4.1 to 4.30 are observed in the next Table.

By observing Table- 5.1, we can conclude that the optimum CASP-CUSUM Schemes, which have the values of ARL and P(A), reach their maximum, i.e**.,** 0.04, 0.9992930 correspondingly, is

$$\begin{bmatrix} B = 0.2 \\ \lambda = 1 \\ k = 1 \\ h = 0.04 \\ h' = 0.04 \end{bmatrix}$$

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