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Original Article

A New Fractional Integral Transform and its Applications

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Abstract

This article proposes a novel fractional integral transform, termed as the fractional Yang integral transform and investigates some of its fundamental properties. The duality relationship between fractional Yang integral transform and fractional Laplace integral transform is derived. Some fractional differential equations of electric circuits are effectively solved using the newly developed fractional Yang integral transform.

1 Introduction

There are several integral transforms that are extensively used in science and engineering domains. Integral transforms such as the Fourier, Laplace, Hankel, Mellin and so on are widely used to solve differential equations, and there are several publications on the theory and applications of integral transforms . Consider a set $\mathscr{A}$ of functions $\phi(t)$ of exponential order, expressed as

$$\mathscr{A} = \Big\{ \phi(t) \in L^1([0,\infty)); \exists\, N, p_1 \; and/or \; p_2 > 0 \in \mathbb{R} \; such \; that \\ |\phi(t)| < N e^{\frac{|t|}{p_i}}, if \; t \in \; (-1)^i \times [0,\infty), i = 1,2, \Big\},$$

where N must be a finite constant, while p_1 , p_2 may be finite and need not exist simultaneously. Xion-Jun Yang[22] introduced Yang transform and Kharde Uttam Dattu[6] investigated some of its fundamental properties. Yang integral transform for the function $\phi(t) \in \mathscr{A}$, denoted by $Y\{\phi(t)\}$ or $T(u)$, is defined as

$$Y\{\phi(t)\} = T(u) = \int_0^\infty e^{-\frac{t}{u}} \phi(t) dt, \;\; t > 0, \tag{1}$$

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[2]Key Words: Modified Riemann Liouville derivatives, Mittag leffler function, Yang transform, Fractional Yang transform

assuming that the integral exists for some $u \in C$. The function $\phi(t)$ in (1) is normally continuous and continuously differentiable. The question is what happens when it is continuous but only has fractional derivative of order γ, $0<\gamma<1$. There are two possibilities. In the first instance, $\phi(t)$ may have both a continuous derivative and a fractional derivative. In the second instance, $\phi(t)$ may have fractional derivative of order γ, $0<\gamma<1$ but no derivative and in this situation, Yang transform does not work. For the second instance, we have to find an alternative.

Recently, as extension of the classical integral transforms, various fractional integral transforms, including the Fractional Sadik transform[3], Fractional Laplace transform[10], Fractional Natural transform[18], Fractional Fourier transform[11], Fractional Sumudu transform [9], Fractional tarig transform[12] and Fractional Elzaki transform[16] have been introduced.

The main objective of this article is to solve fractional differential equations of electric circuits using fractional Yang integral transform. This work is organized as follows. Section 2 provides an overview of fractional derivatives. In section 3, fractional Yang integral transform is defined and some of its fundamental properties are discussed. Finally, in section 4, fractional differential equations of LC, RL, RC and RLC circuits are investigated.

2 Background on fractional derivatives

2.1 Fractional derivatives via fractional difference

Definition 2.1. *[10] Let $\phi : \mathbb{R} \longrightarrow \mathbb{R}$, $t \longrightarrow \phi(t)$ stand for a continuous (but not necessarily differentiable) function, and let $h>0$ stand for a constant discretization span. Then the forward operator, denoted by $FW(h)$, is defined by the equality*

$$FW(h)\phi(t) = \phi(t+h).$$

Now, the fractional difference of order γ, $0<\gamma<1$, of $\phi(t)$ is defined by the expression

$$\begin{aligned}\Delta^{\gamma}\phi(t) &= (FW-1)^{\gamma}\phi(t),\\ &= \sum_{k=0}^{\infty}(-1)^k\binom{\gamma}{k}\phi[t+(\gamma-k)h]\end{aligned}$$

and the fractional derivative of $\phi(t)$ of order γ is defined by the limit

$$\phi^{\gamma}(t) = \lim_{h\downarrow 0}\frac{\Delta^{\gamma}[\phi(t)-\phi(0)]}{h^{\gamma}}.$$

2.2 Modified Riemann Liouville derivative

Definition 2.2. *[10] The fractional derivative of order γ of a function $\phi(t)$ in the Riemann Liouville sense is given by*

$$D^{\gamma}\phi(t) = \frac{1}{\Gamma(n-\gamma)}\left(\frac{d}{dt}\right)^n\int_0^x (x-t)^{n-\gamma-1}\phi(t)dt, \quad \gamma>0.$$

By this definition, the fractional derivative of order γ of a constant function is nonzero. In order to circumvent this drawback, we have proposed the following alternative.

Definition 2.3. *[10] Let $\phi : \mathbb{R} \longrightarrow \mathbb{R}$, $t \longrightarrow \phi(t)$ stand for a continuous function but not necessarily differentiable.*
(i)Suppose that $\phi(t)$ is a constant k. Then the fractional derivative with order γ of $\phi(t)$ is given by

$$D_t^{\gamma} k = \begin{cases} k\Gamma^{-1}(1-\gamma)t^{-\gamma}, & \gamma \leq 0, \\ \\ 0, & \gamma > 0. \end{cases}$$

(ii)When $\phi(t)$ is not a constant, we define

$$\phi(t) = \phi(0) + (\phi(t) - \phi(0))$$

and its fractional derivative is defined by the expression

$$\phi^{\gamma}(t) = D_t^{\gamma}\phi(0) + D_t^{\gamma}(\phi(t) - \phi(0)),$$

in which, for negative γ, one has

$$D_t^{\gamma}(\phi(t) - \phi(0)) = \frac{1}{\Gamma(-\gamma)} \int_0^t (t-\xi)^{-\gamma-1}\phi(\xi)d\xi,$$

and for positive γ, one has

$$D_t^{\gamma}(\phi(t) - \phi(0)) = D_t^{\gamma}\phi(t) = D_t^{\gamma}\Big(\phi^{(\gamma-1)})(t)\Big).$$

For $n \leq \gamma < n+1$, we define

$$\phi^{(\gamma)}(t) = (\phi^{(\gamma-n)}(t))^{(n)},$$

which is referred as the modified Riemann Liouville derivative.

2.3 Integral with respect to $(dt)^{\gamma}$

Definition 2.4. *[10] If $\phi(t)$ is a continuous function, then the fractional differential equation*

$$dz = \phi(t)(dt)^{\gamma}, \quad t \geq 0, \quad z(0) = 0, \quad 0 < \gamma \leq 1, \tag{2}$$

has a solution $z(t)$ given by

$$z(t) = \int_0^t \phi(\xi)(d\xi)^{\gamma} \quad = \gamma \int_0^t (t-\xi)^{\gamma-1}\phi(\xi)d\xi.$$

3 Main results

3.1 Fractional Yang integral transform

Definition 3.1. *Let $\phi(t)$ be a function defined for all $t \geq 0$. Then the fractional Yang integral transform of order $0 < \gamma \leq 1$ of $\phi(t)$ is defined by the following expression.*

$$Y_{\gamma}\{\phi(t)\} = T_{\gamma}(u) = \int_0^{\infty} E_{\gamma}\left(-\frac{t^{\gamma}}{u^{\gamma}}\right)\phi(t)(dt)^{\gamma},$$
$$= \lim_{M\uparrow\infty} \int_0^{M} E_{\gamma}\left(-\frac{t^{\gamma}}{u^{\gamma}}\right)\phi(t)(dt)^{\gamma},$$

where $u \in C$ and $E_{\gamma}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\gamma k+1)}$.

Similarly, the double fractional Yang integral transform is defined as follows.

Definition 3.2. *Let $\phi(x,t)$ be a function defined for all $x,t \geq 0$. Then the double fractional Yang integral transform of order $0 < \gamma \leq 1$ of $\phi(x,t)$ is defined as*

$$Y_\gamma^2\{\phi(x,t)\} = T_\gamma^2(u,v) = \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right) \phi(x,t)(dx)^\gamma(dt)^\gamma,$$
$$= \lim_{M,N\uparrow\infty} \int_0^M \int_0^N E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right) \phi(x,t)(dx)^\gamma(dt)^\gamma,$$

where $u,v \in C$ and $E_\gamma(x) = \sum_{k=0}^\infty \frac{x^k}{\Gamma(\gamma k+1)}$.

Remark 3.3. *From the definition(3.2), we note that $T_\gamma^2(u,0) = T_\gamma^2(u)$ and $T_\gamma^2(0,v) = T_\gamma^2(v)$, where $T_\gamma(.)$ denotes the fractional Yang integral transform.*

3.2 The duality relationship between fractional Yang and fractional Laplace integral transforms

Theorem 3.4. *If $F_\gamma(u)$ denotes the fractional Laplace integral transform of order $0 < \gamma \leq 1$ of a function $\phi(t)$ and $T_\gamma(u)$ denotes the fractional Yang integral transform of order $0 < \gamma \leq 1$ of the same function $\phi(t)$, then*

$$T_\gamma(u) = F_\gamma\left(\frac{1}{u}\right).$$

Proof. By the definition (3.1), we have

$$T_\gamma(u) = Y_\gamma\{\phi(t)\} = \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t)(dt)^\gamma = F_\gamma\left(\frac{1}{u}\right).$$

□

3.3 The duality relationship between fractional double Yang and fractional double Laplace integral transforms

Theorem 3.5. *If $F_\gamma^2(u,v)$ denotes the fractional double Laplace integral transform of order $0 < \gamma \leq 1$ of a function $\phi(x,t)$ and $T_\gamma^2(u,v)$ denotes the fractional Yang integral transform of order $0 < \gamma \leq 1$ of the same function $\phi(x,t)$, then*

$$T_\gamma^2(u,v) = F_\gamma^2\left(\frac{1}{u},\frac{1}{v}\right).$$

Proof. By the definition (3.2), we have

$$T_\gamma^2(u,v) = Y_\gamma^2\{\phi(x,t)\} = \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(x,t)(dx)^\gamma(dt)^\gamma = F_\gamma\left(\frac{1}{u},\frac{1}{v}\right).$$

□

3.4 Basic properties of fractional Yang integral and fractional double Yang integral transforms

Theorem 3.6. ***(Linear property)***
(i) If $Y_\gamma\{\phi(t)\} = T_\gamma(u)$ and $Y_\gamma\{\chi(t)\} = S_\gamma(u)$, then

$$Y_\gamma\{c_1\phi(t) + c_2\chi(t)\} = c_1T_\gamma(u) + c_2S_\gamma(u),$$

where c_1 and c_2 are real numbers.

Proof. By the definition (3.1), we have

$$\begin{aligned}
Y_\gamma\{c_1\phi(t) + c_2\chi(t)\} &= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)[c_1\phi(t) + c_2\chi(t)](dt)^\gamma, \\
&= c_1\int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t)(dt)^\gamma + c_2\int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\chi(t)(dt)^\gamma, \\
&= c_1Y_\gamma\{\phi(t)\} + c_2Y_\gamma\{\chi(t)\}, \\
Y_\gamma\{c_1\phi(t) + c_2\chi(t)\} &= c_1T_\gamma(u) + c_2S_\gamma(u).
\end{aligned}$$

□

(ii) Let $\phi(x,t)$ and $\chi(x,t)$ be the functions of the variables x and t. If $Y_\gamma^2\{\phi(x,t)\} = T_\gamma^2(u,v)$ and $Y_\gamma^2\{\chi(x,t)\} = S_\gamma^2(u,v)$, then

$$Y_\gamma^2\{c_1\phi(x,t) + c_2\chi(x,t)\} = c_1T_\gamma^2(u,v) + c_2S_\gamma^2(u,v),$$

where c_1 and c_2 are real numbers.

Proof. By the definition (3.2), we have

$$\begin{aligned}
Y_\gamma^2\{c_1\phi(x,t) + c_2\chi(x,t)\} &= \int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)[c_1\phi(x,t) + c_2\chi(x,t)](dx)^\gamma(dt)^\gamma, \\
&= c_1\int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(x,t)(dx)^\gamma(dt)^\gamma \\
&\quad + c_2\int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\chi(x,t)(dx)^\gamma(dt)^\gamma, \\
Y_\gamma^2\{c_1\phi(x,t) + c_2\chi(x,t)\} &= c_1T_\gamma^2(u,v) + c_2S_\gamma^2(u,v).
\end{aligned}$$

□

Theorem 3.7. ***(Scaling property)***
(i) If $Y_\gamma\{\phi(t)\} = T_\gamma(u)$, then

$$Y_\gamma\{\phi(ct)\} = \frac{1}{c^\gamma}T_\gamma(cu), \text{ where } c \text{ is a real number.}$$

Proof. By the definition (3.1), we have

$$\begin{aligned}
Y_\gamma\{\phi(ct)\} &= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(ct)(dt)^\gamma, \\
&= \lim_{M\uparrow\infty}\int_0^M E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(ct)(dt)^\gamma, \\
&= \gamma\lim_{M\uparrow\infty}\int_0^M (M-t)^{\gamma-1}E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(ct)dt,
\end{aligned}$$

By changing the variable $ct = v$, we have

$$
\begin{aligned}
Y_\gamma\{\phi(ct)\} &= \gamma \lim_{M\uparrow\infty} \int_0^{cM} \left(M - \frac{v}{c}\right)^{\gamma-1} E_\gamma\left(-\frac{v^\gamma}{c^\gamma u^\gamma}\right)\phi(v)\frac{dv}{c},\\
&= \frac{\gamma}{c^\gamma} \lim_{M\uparrow\infty} \int_0^{cM} (cM - v)^{\gamma-1} E_\gamma\left(-\frac{v^\gamma}{c^\gamma u^\gamma}\right)\phi(v)dv,\\
&= \frac{1}{c^\gamma} \lim_{M\uparrow\infty} \int_0^{cM} E_\gamma\left(-\frac{v^\gamma}{c^\gamma u^\gamma}\right)\phi(v)(dv)^\gamma,\\
Y_\gamma\{\phi(ct)\} &= \frac{1}{c^\gamma} T_\gamma(cu).
\end{aligned}
$$

□

(ii) If $Y_\gamma^2\{\phi(x,t)\} = T_\gamma^2(u,v)$, then

$$
Y_\gamma^2\{\phi(ax,bt)\} = \frac{1}{(ab)^\gamma} T_\gamma^2(au,bv), \;\; \textit{where a and b are real numbers.}
$$

Proof. By the definition (3.2), we have

$$
\begin{aligned}
Y_\gamma^2\{\phi(ax,bt)\} &= \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(ax,bt)(dx)^\gamma(dt)^\gamma,\\
&= \lim_{M,N\uparrow\infty} \int_0^M \int_0^N E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(ax,bt)(dx)^\gamma(dt)^\gamma,\\
&= \gamma^2 \lim_{M,N\uparrow\infty} \int_0^M \int_0^N (M-x)^{\gamma-1}(N-t)^{\gamma-1} E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(ax,bt)dx\;\; dt,
\end{aligned}
$$

By changing the variables $ax = y$ and $bt = z$, we have

$$
\begin{aligned}
Y_\gamma^2\{\phi(ax,bt)\} &= \gamma^2 \lim_{M,N\uparrow\infty} \int_0^{aM} \int_0^{bN} \left(M - \frac{y}{a}\right)^{\gamma-1} \left(N - \frac{z}{b}\right)^{\gamma-1} E_\gamma\left(-\frac{y^\gamma}{a^\gamma u^\gamma}\right) E_\gamma\left(-\frac{z^\gamma}{b^\gamma v^\gamma}\right)\phi(y,z)\frac{dy}{a}\frac{dz}{b},\\
&= \frac{\gamma^2}{(ab)^\gamma} \lim_{M,N\uparrow\infty} \int_0^{aM} \int_0^{bN} (aM-y)^{\gamma-1}(bN-z)^{\gamma-1} E_\gamma\left(-\frac{y^\gamma}{a^\gamma u^\gamma}\right) E_\gamma\left(-\frac{z^\gamma}{b^\gamma v^\gamma}\right)\phi(y,z)dy\; dz,\\
&= \frac{1}{(ab)^\gamma} \lim_{M,N\uparrow\infty} \int_0^{aM} \int_0^{bN} (aM-y)^{\gamma-1}(bN-z)^{\gamma-1} E_\gamma\left(-\frac{y^\gamma}{a^\gamma u^\gamma}\right) E_\gamma\left(-\frac{z^\gamma}{b^\gamma v^\gamma}\right)\\
&\qquad \phi(y,z)(dy)^\gamma(dz)^\gamma,\\
Y_\gamma^2\{\phi(ax,bt)\} &= \frac{1}{(ab)^\gamma} T_\gamma^2(au,bv).
\end{aligned}
$$

□

Theorem 3.8. ***(Shifting property)***
(i) If $Y_\gamma\{\phi(t)\} = T_\gamma(u)$, then

$$
Y_\gamma\{E_\gamma(a^\gamma t^\gamma)\phi(t)\} = T_\gamma\left(\frac{u}{1-au}\right), \;\; \textit{where a is a real number.}
$$

Proof. By the definition (3.1), we have

$$Y_\gamma\{E_\gamma(a^\gamma t^\gamma)\phi(t)\} = \int_0^\infty E_\gamma\left(\frac{-t^\gamma}{u^\gamma}\right)E_\gamma(at)^\gamma\phi(t)(dt)^\gamma,$$

$$Y_\gamma\{E_\gamma(a^\gamma t^\gamma)\phi(t)\} = \lim_{M\uparrow\infty}\int_0^M E_\gamma\left(\frac{-t^\gamma}{u^\gamma}\right)E_\gamma(at)^\gamma\phi(t)(dt)^\gamma,$$

$$= \gamma\lim_{M\uparrow\infty}\int_0^M (M-t)^{\gamma-1}E_\gamma\left[-\left(\frac{1-au}{u}\right)t\right]^\gamma\phi(t)(dt),$$

By changing the variable $\left(\frac{1-au}{u}\right)t = y$, we have

$$Y_\gamma\{E_\gamma(a^\gamma t^\gamma)\phi(t)\} = \gamma\lim_{M\uparrow\infty}\int_0^{\left(\frac{1-au}{u}\right)M}\left[M-\left(\frac{yu}{1-au}\right)\right]^{\gamma-1}E_\gamma(-y)^\gamma\phi\left(\frac{uy}{1-au}\right)\frac{udy}{1-au},$$

$$= \left(\frac{u}{1-au}\right)^\gamma\lim_{M\uparrow\infty}\int_0^{\left(\frac{1-au}{u}\right)M}E_\gamma(-y^\gamma)\phi\left(\frac{uy}{1-au}\right)(dy)^\gamma,$$

$$Y_\gamma\{E_\gamma(a^\gamma y^\gamma)\phi(t)\} = T_\gamma\left(\frac{u}{1-au}\right).$$

□

(ii) If $Y_\gamma^2\{\phi(x,t)\} = T_\gamma^2(u,v)$*, then*

$Y_\gamma^2\{E_\gamma(ax)^\gamma E_\gamma(bt)^\gamma\phi(x,t)\} = T_\gamma^2\left(\frac{u}{1-au},\frac{v}{1-bv}\right)$, *where a and b are a real numbers.*

Proof. By the definition (3.2), we have

$$Y_\gamma^2\{E_\gamma(ax)^\gamma E_\gamma(bt)^\gamma\phi(x,t)\}$$

$$= \int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)[E_\gamma(ax)^\gamma E_\gamma(bt)^\gamma\phi(x,t)](dx)^\gamma(dt)^\gamma,$$

$$= \lim_{M.N\uparrow\infty}\int_0^M\int_0^N E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma(ax)^\gamma E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)E_\gamma(bt)^\gamma\phi(x,t)(dx)^\gamma(dt)^\gamma,$$

$$= \gamma^2\lim_{M,N\uparrow\infty}\int_0^M\int_0^N(M-x)^{\gamma-1}(N-t)^{\gamma-1}E_\gamma\left[-\left(\frac{1-au}{u}\right)x\right]^\gamma E_\gamma\left[-\left(\frac{1-bv}{v}\right)t\right]^\gamma\phi(x,t)dx\,dt.$$

By changing the variables $\left(\frac{1-au}{u}\right)x = \tau$ and $\left(\frac{1-bv}{v}\right)t = \delta$, we have

$$= \gamma^2\lim_{M,N\uparrow\infty}\int_0^{\left(\frac{1-au}{u}\right)M}\int_0^{\left(\frac{1-bv}{v}\right)N}\left[M-\left(\frac{\tau u}{1-au}\right)\right]^{\gamma-1}\left[N-\left(\frac{\delta v}{1-bv}\right)\right]^{\gamma-1}E_\gamma(-\tau)^\gamma E_\gamma(-\delta)^\gamma$$
$$\phi\left(\frac{\tau u}{1-au},\frac{\delta v}{1-bv}\right)\frac{ud\tau}{1-au}\frac{vd\delta}{1-bv},$$

$$= \frac{\gamma^2}{(1-au)^{\gamma-1}(1-bv)^{\gamma-1}}\lim_{M,N\uparrow\infty}\int_0^{\left(\frac{1-au}{u}\right)M}\int_0^{\left(\frac{1-bv}{v}\right)N}[M(1-au)-\tau u]^{\gamma-1}[N(1-bv)-\delta v]^{\gamma-1}$$
$$E_\gamma(-\tau)^\gamma E_\gamma(-\delta)^\gamma\phi\left(\frac{\tau u}{1-au},\frac{\delta v}{1-bv}\right)\frac{ud\tau}{1-au}\frac{vd\delta}{1-bv},$$

$$= \left(\frac{u}{1-au}\right)^\gamma\left(\frac{v}{1-bv}\right)^\gamma\lim_{M,N\uparrow\infty}\int_0^{\left(\frac{1-au}{u}\right)M}\int_0^{\left(\frac{1-bv}{v}\right)N}E_\gamma(-\tau)^\gamma E_\gamma(-\delta)^\gamma\phi\left(\frac{\tau u}{1-au},\frac{\delta v}{1-bv}\right)(d\tau)^\gamma(d\delta)^\gamma,$$

$$Y_\gamma^2\{E_\gamma(ax^\gamma)E_\gamma(bt^\gamma)\phi(x,t)\} = T_\gamma^2\left(\frac{u}{1-au}, \frac{v}{1-bv}\right).$$

□

Theorem 3.9. ***(Second shifting property)***
(i)If $Y_\gamma\{\phi(t)\} = T_\gamma(u)$, *then*

$$Y_\gamma\{\phi(t-a)\} = E_\gamma\left(-\frac{a^\gamma}{u^\gamma}\right)T_\gamma(u), \text{ where } a \text{ is a real number.}$$

Proof. By the definition (3.1), we have

$$\begin{aligned}
Y_\gamma\{\phi(t-a)\} &= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t-a)(dt)^\gamma,\\
&= \lim_{M\uparrow\infty}\int_0^M E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t-a)(dt)^\gamma,\\
&= \gamma\lim_{M\uparrow\infty}\int_0^M (M-t)^{\gamma-1}E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t-a)dt.
\end{aligned}$$

By changing the variable $(t-a) = y$, we have

$$\begin{aligned}
Y_\gamma\{\phi(t-a)\} &= \gamma\lim_{M\uparrow\infty}\int_{-a}^{M-a}(M-a-y)^{\gamma-1}E_\gamma\left(-\frac{(y+a)^\gamma}{u^\gamma}\right)\phi(y)dy,\\
&= \gamma\lim_{M\uparrow\infty}\int_0^{M-a}(M-a-y)^{\gamma-1}E_\gamma\left(-\frac{y^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{a^\gamma}{u^\gamma}\right)\phi(y)dy,\\
&= E_\gamma\left(-\frac{a^\gamma}{u^\gamma}\right)\lim_{M\uparrow\infty}\int_0^{M-a}E_\gamma\left(-\frac{y^\gamma}{u^\gamma}\right)\phi(y)(dy)^\gamma,\\
Y_\gamma\{\phi(t-a)\} &= E_\gamma\left(-\frac{a^\gamma}{u^\gamma}\right)T_\gamma(u).
\end{aligned}$$

□

(ii) If $Y_\gamma^2\{\phi(x,t)\} = T_\gamma^2(u,v)$, *then*

$$Y_\gamma^2\{\phi(x-a,t-b)\} = E_\gamma\left(-\left(\frac{a^\gamma}{u^\gamma}+\frac{b^\gamma}{v^\gamma}\right)\right)T_\gamma^2(u,v), \text{ where } a \text{ and } b \text{ are real numbers.}$$

Proof. By the definition (3.2), we have

$$\begin{aligned}
Y_\gamma^2\{\phi(x-a,t-b)\} &= \int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(x-a,t-b)(dx)^\gamma(dt)^\gamma,\\
&= \lim_{M,N\uparrow\infty}\int_0^M\int_0^N E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(x-a,t-b)(dx)^\gamma(dt)^\gamma,\\
&= \gamma^2\lim_{M,N\uparrow\infty}\int_0^M\int_0^N (M-x)^{\gamma-1}(N-t)^{\gamma-1}E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(x-a,t-b)dx\,dt.
\end{aligned}$$

By changing the variables $x-a=y$ and $t-b=z$, we have

$$=\gamma^2 \lim_{M,N\uparrow\infty}\int_{-a}^{M-a}\int_{-b}^{N-b}(M-a-y)^{\gamma-1}(N-b-z)^{\gamma-1}E_\gamma\left(-\left(\frac{y+a}{u}\right)^\gamma\right)E_\gamma\left(-\left(\frac{z+b}{v}\right)^\gamma\right)\phi(y,z)dy\,dz,$$

$$=\gamma^2 \lim_{M,N\uparrow\infty}\int_{0}^{M-a}\int_{0}^{N-b}(M-a-y)^{\gamma-1}(N-b-z)^{\gamma-1}E_\gamma\left(-\left(\frac{y+a}{u}\right)^\gamma\right)E_\gamma\left(-\left(\frac{z+b}{v}\right)^\gamma\right)\phi(y,z)dy\,dz,$$

$$=E_\gamma\left(-\left(\frac{a^\gamma}{u^\gamma}+\frac{b^\gamma}{v^\gamma}\right)\right)\lim_{M,N\uparrow\infty}\int_{0}^{M-a}\int_{0}^{N-b}E_\gamma\left(-\frac{y^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{z^\gamma}{v^\gamma}\right)\phi(y,z)(dy)^\gamma(dz)^\gamma.$$

$$Y_\gamma^2\{\phi(x-a,t-b)\}=E_\gamma\left(-\left(\frac{a^\gamma}{u^\gamma}+\frac{b^\gamma}{v^\gamma}\right)\right)T_\gamma^2(u,v).$$

□

Theorem 3.10. ***(Fractional Yang integral transform for fractional derivatives)***
If $Y_\gamma\{\phi(t)\}=T_\gamma(u)$, then

$$Y_\gamma\{\phi^{(\gamma)}(t)\}=\frac{T_\gamma(u)}{u^\gamma}-\Gamma(\gamma+1)\phi(0),\quad 0<\gamma\le 1.$$

Proof. By the definition (3.1), we have

$$Y_\gamma\{\phi^{(\gamma)}(t)\}=\int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi^{(\gamma)}(t)(dt)^\gamma.$$

Since $\int_a^b u^{(\gamma)}(t)v(t)(dt)^\gamma=\Gamma(\gamma+1)[u(t)v(t)]_a^b-\int_a^b u(t)v^{(\gamma)}(t)(dt)^\gamma$, *we have*

$$\begin{aligned}Y_\gamma\{\phi^{(\gamma)}(t)\}&=\Gamma(\gamma+1)\left[E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t)\right]_0^\infty+\frac{1}{u^\gamma}\int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\phi(t)(dt)^\gamma,\\&=-\Gamma(\gamma+1)\phi(0)+\frac{1}{u^\gamma}T_\gamma(u),\\Y_\gamma\{\phi^{(\gamma)}(t)\}&=\frac{T_\gamma(u)}{u^\gamma}-\Gamma(\gamma+1)\phi(0).\end{aligned}$$

□

Remark 3.11. *By extending the theorem (3.10), we obtain*

$$Y_\gamma\{\phi^{(n\gamma)}(t)\}=\frac{T_\gamma(u)}{u^{n\gamma}}-\Gamma(\gamma+1)\sum_{k=0}^{n-1}\frac{\phi^{(\gamma k)}(0)}{u^{(n-k-1)\gamma}}.$$

Theorem 3.12. *If $Y_\gamma^2\{\phi(x,t)\}=T_\gamma^2(u,v)$, then*

$$Y_\gamma^2\left\{\frac{\partial^\gamma}{\partial x^\gamma}\phi(x,t)\right\}=\frac{T_\gamma^2(u,v)}{u^\gamma}-\Gamma(\gamma+1)T_\gamma^2(0,v).$$

Proof. By the definition (3.2), we have

$$
\begin{aligned}
Y_\gamma^2\left\{\frac{\partial^\gamma}{\partial x^\gamma}\phi(x,t)\right\} &= \int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\frac{\partial^\gamma}{\partial x^\gamma}\phi(x,t)(dx)^\gamma(dt)^\gamma,\\
&= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\left[\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)\frac{\partial^\gamma}{\partial x^\gamma}\phi(x,t)(dx)^\gamma\right](dt)^\gamma,\\
&= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\left\{\Gamma(\gamma+1)\left[\phi(x,t)E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)\right]_0^\infty - \right.\\
&\qquad\left.\int_0^\infty \phi(x,t)\frac{\partial^\gamma}{\partial x^\gamma}E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)(dx)^\gamma\right\}(dt)^\gamma,\\
&= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\left\{\Gamma(\gamma+1)\left[\phi(x,t)E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)\right]_0^\infty + \right.\\
&\qquad\left.\frac{1}{u^\gamma}\int_0^\infty \phi(x,t)E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)(dx)^\gamma\right\}(dt)^\gamma,\\
&= -\int_0^\infty E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\left\{\Gamma(\gamma+1)\phi(0,t)+\right.\\
&\qquad\left.\frac{1}{u^\gamma}\int_0^\infty \phi(x,t)E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)(dx)^\gamma\right\}(dt)^\gamma,\\
&= -\int_0^\infty E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\Gamma(\gamma+1)\phi(0,t)(dt)^\gamma+\\
&\qquad\frac{1}{u^\gamma}\int_0^\infty\int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right)\phi(x,t)(dx)^\gamma(dt)^\gamma,\\
&= -\Gamma(\gamma+1)Y_\gamma^2\{\phi(0,t)\}+\frac{Y_\gamma^2\{\phi(x,t)\}}{u^\gamma},\\
&= \frac{Y_\gamma^2\{\phi(x,t)\}}{u^\gamma}-\Gamma(\gamma+1)Y_\gamma^2\{\phi(0,t)\},\\
Y_\gamma^2\left\{\frac{\partial^\gamma}{\partial x^\gamma}\phi(x,t)\right\} &= \frac{T_\gamma^2(u,v)}{u^\gamma}-\Gamma(\gamma+1)T_\gamma^2(0,v).
\end{aligned}
$$

□

In the same way, we can obtain fractional double Yang transform of fractional partial derivatives with respect to t, that is,

$$Y_\gamma^2\left\{\frac{\partial^\gamma}{\partial t^\gamma}\phi(x,t)\right\} = \frac{T_\gamma^2(u,v)}{v^\gamma}-\Gamma(\gamma+1)T_\gamma^2(u,0).$$

Theorem 3.13. ***(Fractional Yang integral transform for fractional integration)***
If $Y_\gamma\{\phi(t)\} = T_\gamma(u)$ *, then*

$$Y_\gamma\left\{\int_0^x \phi(t)(dt)^\gamma\right\} = u^\gamma\Gamma(\gamma+1)T_\gamma(u).$$

Proof. By the theorem (3.10), we have

$$Y_\gamma\{\phi^{(\gamma)}(t)\} = \frac{Y_\gamma\{\phi(t)\}}{u^\gamma} - \Gamma(\gamma+1)\phi(0) \text{ and so}$$
$$\frac{Y_\gamma\{\phi(t)\}}{u^\gamma} = Y_\gamma\{\phi^{(\gamma)}(t)\} + \Gamma(\gamma+1)\phi(0).$$

If $h(t) = \int_0^t \phi(t)(dt)^\gamma$, *then* $h(0) = 0$.

$$\begin{aligned} Now,\ u^{-\gamma} Y_\gamma\Big\{\int_0^t \phi(t)(dt)^\gamma\Big\} &= Y_\gamma\{h^{(\gamma)}(t)\}, \\ &= Y_\gamma\Big\{D_t^\gamma \int_0^t \phi(t)(dt)^\gamma\Big\}, \\ &= Y_\gamma\{\Gamma(\gamma+1)\phi(t)\}, \\ &= \Gamma(\gamma+1)Y_\gamma\{\phi(t)\}, \\ Y_\gamma\left\{\int_0^t \phi(t)(dt)^\gamma\right\} &= u^\gamma \Gamma(\gamma+1) T_\gamma(u). \end{aligned}$$

□

3.5 Convolution theorem

Theorem 3.14. *If the convolution with order γ of the functions $\phi(t)$ and $\psi(t)$ is given by the formula*

$$\Big(\phi(t) * \psi(t)\Big)_\gamma = \int_0^t \phi(t-\tau)\psi(\tau)(d\tau)^\gamma,$$

then one has the equality

$$Y_\gamma\Big\{\Big(\phi(t) * \psi(t)\Big)_\gamma\Big\} = Y_\gamma\{\phi(t)\} Y_\gamma\{\psi(t)\}.$$

Proof. By the definition (3.1), we have

$$\begin{aligned} Y_\gamma\Big\{\Big(\phi(t) * \psi(t)\Big)_\gamma\Big\} &= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\Big(\phi(t) * \psi(t)\Big)_\gamma (dt)^\gamma, \\ &= \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right)\int_0^t \phi(t-\tau)\psi(\tau)(d\tau)^\gamma (dt)^\gamma, \\ &= \int_0^\infty E_\gamma\left(-\frac{(t-\tau)^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{\tau^\gamma}{u^\gamma}\right)\int_0^t \phi(t-\tau)\psi(\tau)(d\tau)^\gamma (dt)^\gamma, \end{aligned}$$

By changing the variables $t-\tau = y$ and $\tau = x$, we have

$$\begin{aligned} Y_\gamma\Big\{\Big(\phi(t) * \psi(t)\Big)_\gamma\Big\} &= \int_0^\infty E_\gamma\left(-\frac{y^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)\int_0^\infty \phi(y)\psi(x)(dy)^\gamma (dx)^\gamma, \\ &= \int_0^\infty E_\gamma\left(-\frac{y^\gamma}{u^\gamma}\right)\phi(y)(dy)^\gamma \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right)\psi(x)(dx)^\gamma, \\ Y_\gamma\Big\{\Big(\phi(t) * \psi(t)\Big)_\gamma\Big\} &= Y_\gamma\{\phi(t)\} Y_\gamma\{\psi(t)\}. \end{aligned}$$

□

3.6 Double convolution theorem

Theorem 3.15. *If the double convolution with order γ of the functions $\phi(x,t)$ and $\psi(x,t)$ is given by the formula*

$$(\phi(x,t) ** \psi(x,t))_\gamma = \int_0^x \int_0^t \phi(x-\tau, t-\theta)\psi(\tau\theta)(d\tau)^\gamma (d\theta)^\gamma,$$

then

$$Y_\gamma^2\{(\phi(x,t) ** \psi(x,t))_\gamma\} = Y_\gamma^2\{\phi(x,t\} Y_\gamma^2\{\psi(x,t)\}.$$

Proof. By the definition (3.2), we have

$$\begin{aligned}
Y_\gamma^2\{(\phi(x,t) ** \psi(x,t))_\gamma\} &= \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right) (\phi(x,t) ** \psi(x,t))_\gamma (dx)^\gamma (dt)^\gamma, \\
&= \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right) \\
&\qquad \int_0^x \int_0^t \phi(x-\tau, t-\theta)\psi(\tau,\theta)(d\tau)^\gamma (d\theta)^\gamma (dx)^\gamma (dt)^\gamma, \\
&= \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{(x-\tau)^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{\tau^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{(t-\theta)^\gamma}{v^\gamma}\right) E_\gamma\left(-\frac{\theta^\gamma}{v^\gamma}\right) \\
&\qquad \int_0^x \int_0^t \phi(x-\tau, t-\theta)\psi(\tau,\theta)(d\tau)^\gamma (d\theta)^\gamma (dx)^\gamma (dt)^\gamma.
\end{aligned}$$

By changing the variables $x-\tau = p$, $t-\theta = r$, $\tau = q$ and $\theta = s$ we have

$$\begin{aligned}
Y_\gamma^2\{(\phi(x,t) ** \psi(x,t))_\gamma\} &= \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{p^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{q^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{r^\gamma}{v^\gamma}\right) E_\gamma\left(-\frac{s^\gamma}{v^\gamma}\right) \\
&\qquad \int_0^\infty \int_0^\infty \phi(p,q)\psi(r,s)(dp)^\gamma (dq)^\gamma (dr)^\gamma (ds)^\gamma, \\
&= \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{p^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{r^\gamma}{u^\gamma}\right) \phi(p,r)(dp)^\gamma (dr)^\gamma \\
&\qquad \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{q^\gamma}{v^\gamma}\right) E_\gamma\left(-\frac{s^\gamma}{v^\gamma}\right) \psi(q,s)(dq)^\gamma (ds)^\gamma, \\
Y_\gamma^2\{(\phi(x,t)\psi(x,t))_\gamma\} &= Y_\gamma^2\{\phi(x,t)\} Y_\gamma^2\{\psi(x,t)\}.
\end{aligned}$$

□

3.7 Inversion theorem for fractional Yang integral transform

Theorem 3.16. *The inversion formula for fractional Yang integral transform of $\phi(t)$, that is,*

$$T_\gamma(u) = \int_0^\infty E_\gamma\left(-\frac{t^\gamma}{u^\gamma}\right) \phi(t)(dt)^\gamma,$$

is given by

$$\phi(t) = \frac{1}{(M_\gamma)^\gamma} \int_{-i\infty}^{+i\infty} E_\gamma(t^\gamma u^\gamma)\, T_\gamma\left(\frac{1}{u}\right) (du)^\gamma,$$

where $E_\gamma(i(M_\gamma)^\gamma) = 1$ defines M_γ as the period of the complex-valued mittag leffler function.

Proof. The inversion formula for fractional Laplace transform of $\phi(t)$, i.e.,

$$F_\gamma(u) = \int_0^\infty E_\gamma(-u^\gamma t^\gamma)\phi(t)(dt)^\gamma, \;\; 0 < \gamma \leq 1,$$

is given by

$$\phi(t) = \frac{1}{(M_\gamma)^\gamma} \int_{-i\infty}^{+i\infty} E_\gamma(u^\gamma t^\gamma) F_\gamma(u)(du)^\gamma.$$

Through the duality relation between the fractional Yang and the fractional Laplace integral transforms, the requisite inversion formula is derived. □

3.8 Inversion theorem for fractional double Yang integral transform

Theorem 3.17. *The inversion formula for fractional double Yang integral transform of $\phi(x,t)$, that is,*

$$T_\gamma^2(u,v) = \int_0^\infty \int_0^\infty E_\gamma\left(-\frac{x^\gamma}{u^\gamma}\right) E_\gamma\left(-\frac{t^\gamma}{v^\gamma}\right) \phi(x,t)(dx)^\gamma (dt)^\gamma,$$

is given by

$$\phi(x,t) = \frac{1}{(M_\gamma)^{2\gamma}} \int_{-i\infty}^{+i\infty} \int_{-i\infty}^{+i\infty} E_\gamma(x^\gamma u^\gamma) E_\gamma(t^\gamma v^\gamma) T_\gamma^2\left(\frac{1}{u}, \frac{1}{v}\right)(du)^\gamma (dv)^\gamma,$$

where $E_\gamma(i(M_\gamma)^\gamma) = 1$ defines M_γ as the period of the complex-valued mittag leffler function.

Proof. The inversion formula for fractional double Laplace transform of $\phi(x,t)$, i.e.,

$$F_\gamma(u,v) = \int_0^\infty E_\gamma(-u^\gamma x^\gamma) E_\gamma(-v^\gamma t^\gamma)\phi(x,t)(dx)^\gamma (dt)^\gamma, \;\; 0 < \gamma < 1$$

is given by

$$\phi(x,t) = \frac{1}{(M_\gamma)^\gamma} \int_{-i\infty}^{+i\infty} E_\gamma(u^\gamma x^\gamma) E_\gamma(v^\gamma t^\gamma) F_\gamma^2(u,v)(du)^\gamma (dv)^\gamma.$$

Through the duality relation between the fractional double Yang and the fractional double Laplace integral transforms, the requisite inversion formula is derived. □

4 Applications

In this section, the proposed fractional Yang integral transform is employed in solving some fractional differential equations of LC, RL, RC and RLC electric circuits.

Example 4.1. ***LC Circuit***
Consider the fractional differential equation of an LC circuit with only charged capacitor and inductor[1][2],

$$I^{(\gamma)}(t) + \omega_0^2 I(t) = 0, \qquad \gamma \in (1,2), \tag{3}$$

with regard to the initial constraints $I(0) = I_0$ and $I^\gamma(0) = 0$, where $\omega_0^2 = \frac{1}{LC}$.

Applying fractional Yang integral transform on both sides of (3), we have

$$Y_\gamma\left\{I^{(\gamma)}(t)+\omega_0^2 I(t)\right\}=0,$$

$$\begin{aligned}\frac{T_\gamma(u)}{u^{2\gamma}}-\Gamma(\gamma+1)\frac{I(0)}{u^\gamma}-\Gamma(\gamma+1)I^\gamma(0)+\omega_0^2T_\gamma(u)&=0,\\ \frac{T_\gamma(u)}{u^{2\gamma}}-\Gamma(\gamma+1)\frac{I(0)}{u^\gamma}+\omega_0^2T_\gamma(u)&=0,\\ T_\gamma(u)\left[\frac{1}{u^{2\gamma}}+\omega_0^2\right]&=\Gamma(\gamma+1)\frac{I(0)}{u^\gamma},\\ T_\gamma(u)&=\Gamma(\gamma+1)I(0)\left(\frac{u^\gamma}{1+\omega_0^2u^{2\gamma}}\right).\end{aligned} \tag{4}$$

We obtain the desired solution through the following complex inversion formula for fractional Yang integral transform of (4).

$$I(t)=\frac{1}{(M\gamma)^\gamma}\int_{-i\infty}^{+i\infty}I(0)cos(\omega_0t)E_\gamma(x^\gamma u^\gamma)T_\gamma\left(\frac{1}{u}\right)(du)^\gamma.$$

Example 4.2. ***RL Circuit***

Consider the fractional differential equation of an RL circuit with only resistor, inductor and a non-variant voltage source

$$LI^{(\gamma)}(t)+RI(t)=V, \qquad \gamma\in(0,1), \tag{5}$$

with regard to the initial constraint $I(0)=I_0$ and V is the constant voltage source, where $R/L=\eta$ and $V/L=\rho$.

Applying the fractional Yang integral transform on both sides of (5), we attain

$$\begin{aligned}Y_\gamma\{I^\gamma(t)+\eta I(t)\}&=Y_\gamma\{\rho\},\\ \frac{T_\gamma(u)}{u^\gamma}-\Gamma(\gamma+1)I(0)+\eta T_\gamma(u)&=\rho\Gamma(\gamma+1)u^\gamma,\\ \frac{T_\gamma(u)}{u^\gamma}-\Gamma(\gamma+1)I_0+\eta T_\gamma(u)&=\rho\Gamma(\gamma+1)u^\gamma,\\ \frac{T_\gamma(u)}{u^\gamma}+\eta T_\gamma(u)&=\rho\Gamma(\gamma+1)u^\gamma+\Gamma(\gamma+1)I_0,\\ T_\gamma(u)\left[\frac{1}{u^\gamma}+\eta\right]&=\rho\Gamma(\gamma+1)u^\gamma+\Gamma(\gamma+1)I_0,\\ T_\gamma(u)&=\Big(\rho\Gamma(\gamma+1)u^\gamma+\Gamma(\gamma+1)I_0\Big)\left(\frac{u^\gamma}{1+\eta u^\gamma}\right),\\ T_\gamma(u)&=\rho\Gamma(\gamma+1)\left(\frac{u^{2\gamma}}{1+\eta u^\gamma}\right)+\Gamma(\gamma+1)I_0\left(\frac{u^\gamma}{1+\eta u^\gamma}\right).\end{aligned} \tag{6}$$

We obtain the desired solution through the following complex inversion formula for fractional Yang integral transform of (6).

$$I(t)=\frac{1}{(M\gamma)^\gamma}\int_{-i\infty}^{+i\infty}\rho\Gamma(\gamma+1)\left(\frac{u^{2\gamma}}{1+\eta u^\gamma}\right)+\Gamma(\gamma+1)I_0\left(\frac{u^\gamma}{1+\eta u^\gamma}\right)E_\gamma(t^\gamma u^\gamma)T_\gamma\left(\frac{1}{u}\right)_g(du)^\gamma.$$

Example 4.3. ***RC Circuit***
Consider the fractional differential equation of an RC circuit with only charged capacitor and resistor

$$CV^{(\gamma)}(t)+\frac{1}{R}V(t)=0, \qquad \gamma\in(0,1), \tag{7}$$

with regard to the initial constraint$V(0)=V_0$.

Applying the fractional Yang integral transform on both sides of (7), we attain

$$\begin{aligned} Y_\gamma\{V^\gamma(t)+\frac{1}{RC}V(t)\} &= 0,\\ \frac{T_\gamma(u)}{u^\gamma}-\Gamma(\gamma+1)V(0)+\frac{1}{RC}T_\gamma(u) &= 0,\\ \frac{T_\gamma(u)}{u^\gamma}-\Gamma(\gamma+1)V_0+\frac{1}{RC}T_\gamma(u) &= 0,\\ \frac{T_\gamma(u)}{u^\gamma}+\frac{1}{RC}T_\gamma(u) &= \Gamma(\gamma+1)V_0,\\ T_\gamma(u)\left[\frac{1}{u^\gamma}+\frac{1}{RC}\right] &= \Gamma(\gamma+1)V_0,\\ T_\gamma\Big(u\Big) &= \Gamma(\gamma+1)V_0\left(\frac{RCu^\gamma}{RC+u^\gamma}\right), \end{aligned} \tag{8}$$

We obtain the desired solution through the following complex inversion formula for fractional Yang integral transform of (8).

$$V(t)=\frac{1}{(M\gamma)^\gamma}\int_{-i\infty}^{+i\infty}\Gamma(\gamma+1)V_0\left(\frac{RCu^\gamma}{RC+u^\gamma}\right)E_\gamma(t^\gamma u^\gamma)T_\gamma\left(\frac{1}{u}\right)(du)^\gamma.$$

Example 4.4. ***RLC Circuit***
Consider the fractional differential equation of an RLC circuit with resistor, Inductor and charged capacitor,

$$LI^{(\gamma)}(t)+RI+\frac{1}{C}\int_0^t I(t)(dt)^\gamma=E(t), \qquad \gamma\in(0,1), \tag{9}$$

with regard to the initial constraints $I(0)=0$ *and* $Q(0)=0$.

Applying fractional Yang integral transform on both sides of (9), we have

$$Y_\gamma\left\{LI^\gamma(t)+RI+\frac{1}{C}\int_0^t I(t)(dt)^\gamma\right\}=Y_\gamma\{E(t)\},$$

$$L\left[\frac{T_\gamma(u)_I}{u^\gamma}-\Gamma(\gamma+1)I(0)\right]+RT_\gamma(u)_I+\frac{1}{C}u^\gamma\Gamma(\gamma+1)T_\gamma(u)_I=T_\gamma(u)_E,$$

$$T_\gamma(u)_I\left[L\frac{1}{u^\gamma}+R+\frac{1}{C}u^\gamma\Gamma(\gamma+1)\right]=T_\gamma(u)_E,$$

$$T_\gamma(u)_I\left[L+Ru^\gamma+\frac{1}{C}u^{2\gamma}\Gamma(\gamma+1)\right]=u^\gamma T_\gamma(u)_E,$$

$$T_\gamma(u)_I=\frac{u^\gamma}{L+Ru^\gamma+\frac{1}{C}u^{2\gamma}\Gamma(\gamma+1)}T_\gamma(u)_E,$$

$$T_\gamma(u)_I=\frac{u^\gamma}{L\left[1+\frac{Ru^\gamma}{L}+\frac{1}{CL}u^{2\gamma}\Gamma(\gamma+1)\right]}T_\gamma(u)_E,$$

$$T_\gamma\left(\frac{1}{u}\right)_I=\frac{u^\gamma}{L\left[u^{-2\gamma}+\frac{Ru^{-\gamma}}{L}+\frac{1}{CL}\Gamma(\gamma+1)\right]}T_\gamma\left(\frac{1}{u}\right)_E,$$

$$T_\gamma\left(\frac{1}{u}\right)_I=\frac{u^\gamma}{L\left[u^{2\gamma}+\frac{Ru^{\gamma}}{L}+\frac{1}{CL}\Gamma(\gamma+1)\right]}T_\gamma\left(\frac{1}{u}\right)_E, \quad (10)$$

The characteristic equation of (10) is given by

$$\frac{u^\gamma}{u^{2\gamma}+\frac{Ru^\gamma}{L}+\frac{1}{CL}\Gamma(\gamma+1)}=0. \quad (11)$$

The characteristic equation (10) has the complex roots, ie., $u^\gamma=-k\pm$ in, with the negative real part, where $k=\frac{R}{CL}$ and $n^2=\frac{\Gamma(\gamma+1)}{CL}-\frac{R^2}{4L^2}$ and so the system is stable.
We obtain the desired solution through the following complex inversion formula for fractional Yang integral transform on (10).

$$I(t)=\frac{1}{L(M\gamma)^\gamma}\int_{-i\infty}^{+i\infty}E_\gamma(-t^\gamma u^\gamma)\frac{u^\gamma}{u^{2\gamma}+\frac{Ru^\gamma}{L}+\frac{1}{CL}\Gamma(\gamma+1)}\}T_\gamma\left(\frac{1}{u}\right)_E(du)^\gamma.$$

5 Conclusion

In this article, a new fractional integral transform and some of its properties were discussed. The newly developed fractional Yang integral transform is now a very useful tool because we may proceed with fractional Yang integral transform or the existing Yang transform based on our convenience and issue circumstance. It is also observed that the fractional Yang integral transform is simple and effective approach in solving fractional differential equations of electric circuits.

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