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Original Article

Analysis of Bi-Tandem Priority Queue System in Stochastic Environment

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Abstract *- This paper makes an effort to analyze Priority Queue Network model having two Bi-Tandem parallel servers linked in series to a central server. The different queue characteristics are derived using statistical techniques and the law of calculus. Time - independent solution of the system is derived by using Generating Function and Partial Generating Functions techniques. Poisson law is assumed to be followed by the arrival rates and services. The Performance of the model have been studied in stochastic environment. Mathematical formulation of the present model ensures implementation of proposed model in various real-world problems.*

Keywords *- Bi-Tandem, Numerical illustration, Stochastic environment, Traffic intensity, Variance.*

1. Introduction

Queuing theory have wide range of applications that helps a person in making right decision in complex queuing system encountered in daily life. Queuing theory deals with the study of queues that occurs in real world situations and arise when the pace of arrival exceeds the rate of service. Several Researchers in the past have been carried out their work in the investigations of various queuing model characteristics. The concept of bi-tandem queues was first introduced by Maggu [12]. Singh [11] investigated certain serial and biserial channels' queuing networks. Tandem queues in a fuzzy environment were examined by Nagoorgani and Retha [8]. The passing behavior of a system of parallel biserial queues in a queue network studied by Singh, T.P. et al [7]. Kumar et al [9] investigated the study-state behavior of a queuing network with two biserial subsystems linked with a common channel in stochastic environment. Singh and Pardeep [6] discussed serial queues in fuzzy environment with blocking. Seema et al [5] has analyzed different queuing network performance indicators under uncertain circumstances. Sameer [4] made an attempt to analyze a harmonious system of biserial and parallel servers in fuzzy environment. Saini V and Gupta D [1,2] examined a queue network with feedback on three subsystems providing revisit service at most twice to any of the servers.

In this paper, the pre-emptive priority service rule and bi-series on parallel subsystems are applied on a queue network design with parallel and biserial channels connected to a shared server in series given by Gupta et al [7]. Various Queue characteristics is derived from the model's steady-state solution. Numerical and Graphical analysis of the proposed model has been done.

2. Practical Situation

In an administration, the classification in clearance of file is find at different levels such as at district level and several secretariat level offices, after that file goes at C.M. office for final approval. Few files appearing in the system with priority discrimination at biserial district level service channels for clearance while others enter through secretariat level biserial service channels. After that file go to the C.M. office for final processing.

3. Model Description

In the present queuing system, three subsystems C_1, C_2 and C_3 are there. The subsystems C_1 and C_2 have biserial service channels C_{11} & C_{12} and C_{21} & C_{22} respectively. The parallel subsystems C_1 and C_2 linked to a common subsystem C_3. Customers with arrival rates of λ_{1L}, λ_{1H} & λ_{2L}, λ_{2H} will initially arrive to subsystem C_1 before service channels C_{11} and C_{12}. Customer will shift probabilistically to service channel C3 or C12 after receiving service at C_{11} such that $\alpha_{12} + \alpha_{15} = 1$. From server C_{12}, customer either visit to C_{11} or C_3 with moving probabilities α_{21} & α_{25} with condition $\alpha_{21} + \alpha_{25} =1$.

Those who will arrive with arrival rates λ_1 & λ_2 at service channels C_{21} & C_{22}, after availing the service of service channel C_{21} may either go C_{22} or C_3 with leaving probabilities α_{34} & α_{35}, $\alpha_{34} + \alpha_{35} = 1$. And from server C_{22}, customer either visit to C_{21} with α_{43} or C_3 with α_{45}. Finally, customer exit the system with leaving probability α_5 after successful completion of the service.

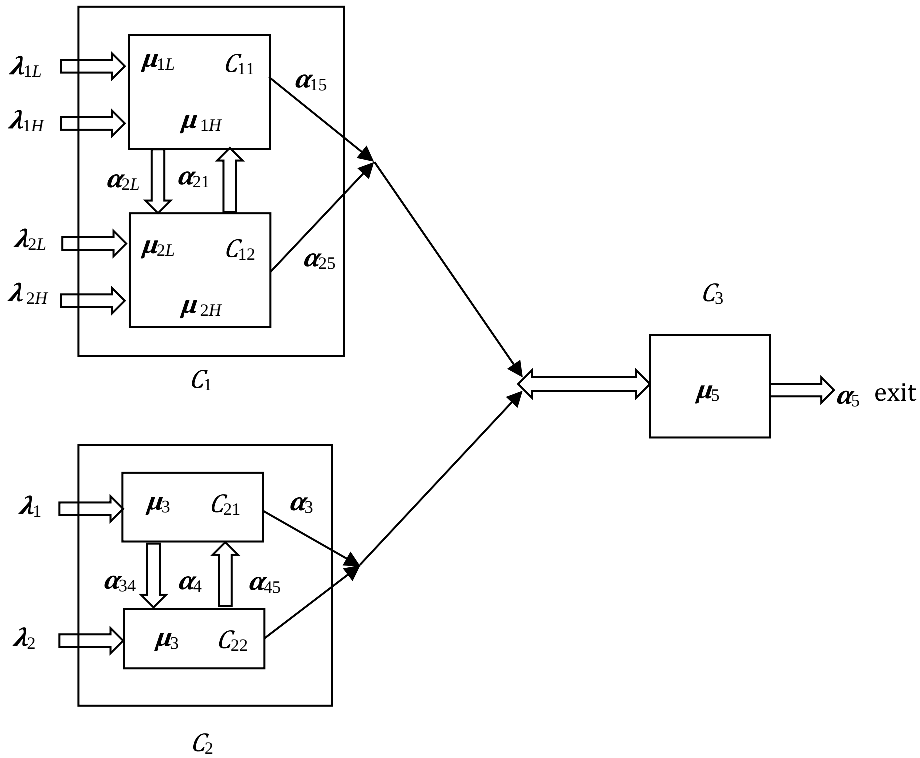


Fig. 1 Suggested model

1.1. Notations

Table 1. Notations

Service channel	C_{11}	C_{12}	C_{21}	C_{22}	C_3
No. of items	m_{1L} m_{1H}	m_{2L} m_{2H}	m_3	m_4	m_5
Rate of Arrival	λ_{1L} λ_{1H}	λ_{2L} λ_{2H}	λ_1	λ_2	$\lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H} + \lambda_1 + \lambda_2$
Service Rate	μ_{1L} μ_{1H}	μ_{2L} μ_{2H}	μ_3	μ_4	μ_5
Probability of customers moving from one server to another	C_{11} to C_{12} α_{12} C_{11} to C_3 α_{15}	C_{12} to C_{11} α_{21} C_{12} to C_3 α_{25}	C_{21} to C_3 α_{35} C_{21} to C_{22} α_{34}	C_{22} to C_{21} α_{43} C_{22} to C_3 α_{45}	C_3 α_5

2. Mathematical Modelling

Let us, define Joint Probability function $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4,m_5}(t)$ of $m_{1L}, m_{1H}, m_{2L}, m_{2H}, m_3, m_4, m_5 \geq 0$ number of customers in front of servers S_{11}, S_{12}, S_{21}, S_{22}, S_3 respectively.

The differential difference equations of bi-tandem priority model in time- independent state is defined as

$(\lambda_{1L}+ \lambda_{1H}+ \lambda_{2L}+ \lambda_{2H} + \lambda_1+ \lambda_2 + \mu_{1H} + \mu_{2H} +\mu_3 + \mu_4 + \mu_5) P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4,m_5} = \lambda_{1L} P_{m_{1L}-1,m_{1H},m_{2L},m_{2H},m_3,m_4,m_5} + \lambda_{1H} P_{m_{1L},m_{1H}-1,m_{2L},m_{2H},m_3,m_4,m_5} + \lambda_{2L} P_{m_{1L},m_{1H},m_{2L-1},m_{2H},m_3,m_4,m_5} + \lambda_{2H} P_{m_{1L},m_{1H},m_{2L},m_{2H-1},m_3,m_4,m_5} + \lambda_1$

$P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3-1,m_4,m_5}$ + λ_2 $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4-1,m_5}$ +μ_{1H} α_{12} $P_{m_{1L},m_{1H+1},m_{2L},m_{2H}-1,m_3,m_4,m_5}$+ μ_{1H} α_{15} $P_{m_{1L},m_{1H+1},m_{2L},m_{2H},m_3,m_4,m_5-1}$ +μ_{2H} α_{21} $P_{m_{1L},m_{1H}-1,m_{2L},m_{2H}+1,m_3,m_4,m_5}$ + μ_{2H} α_{25} $P_{m_{1L},m_{1H},m_{2L},m_{2H}+1,m_3,m_4,m_5-1}$ +$\mu_3\alpha_{34}$ $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3+1,m_4-1,m_5}$+ μ_3 α_{35} $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3+1,m_4,m_5-1}$+ μ_4 α_{43} $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_{3-1},m_4+1,m_5}$ +μ_4 α_{45} $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_{4+1},m_{5-1}}$+ μ_5 α_5 $P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4,m_5+1}$

$$m_{1L}, m_{1H}, m_{2L}, m_{2H}, m_3, m_4, m_5 > 0 \qquad (A_1)$$

3. Solution Methodology

To solve the steady state equations (A_1) to (A_{128}), we will introduce the generating function and partial generating functions because we are finding the probability distribution function solution. Generating function is as follows:

$$H(Z_1,Z_2,Z_3,Z_4,Z_5,Z_6,Z_7) = \sum_{m_{1L}=0}^{\infty}\sum_{m_{1H}=0}^{\infty}\sum_{m_{2L}=0}^{\infty}\sum_{m_{2H}=0}^{\infty}\sum_{m_3=0}^{\infty}\sum_{m_4=0}^{\infty}\sum_{m_5=0}^{\infty} P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4,m_5} Z_1^{m_{1L}} Z_2^{m_{1H}} Z_3^{m_{2L}} Z_4^{m_{2H}} Z_5^{m_3} Z_6^{m_4} Z_7^{m_5}$$

Where, $|Z_1|=1$, $|Z_2|=1$, $|Z_3|=1$, $|Z_4|=1$, $|Z_5|=1$, $|Z_6|=1$, $|Z_7|=1$and partial generating functions are

$$H_{m_{1H},m_{2L},m_{2H},m_3,m_4,m_5}(Z_1) = \sum_{m_{1L}=0}^{\infty} P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4,m_5} Z_1^{m_{1L}} \tag{1}$$

$$H_{m_{2L},m_{2H},m_3,m_4,m_5}(Z_1,Z_2) = \sum_{m_{1H}=0}^{\infty} H_{m_{1H},m_{2L},m_{2H},m_3,m_4,m_5}(Z_1) Z_2^{m_{1H}} \tag{2}$$

$$H_{m_{2H},m_3,m_4,m_5}(Z_1,Z_2,Z_3) = \sum_{m_{2L}=0}^{\infty} H_{m_{2L},m_{2H},m_3,m_4,m_5}(Z_1,Z_2) Z_3^{m_{2L}} \tag{3}$$

$$H_{m_3,m_4,m_5}(Z_1,Z_2,Z_3,Z_4) = \sum_{m_{2H}=0}^{\infty} H_{m_{2H},m_3,m_4,m_5}(Z_1,Z_2,Z_3) Z_4^{m_{2H}} \tag{4}$$

$$H_{m_4,m_5}(Z_1,Z_2,Z_3,Z_4,Z_5) = \sum_{m_3=0}^{\infty} H_{m_3,m_4,m_5}(Z_1,Z_2,Z_3,Z_4) Z_5^{m_3} \tag{5}$$

$$H_{m_5}(Z_1,Z_2,Z_3,Z_4,Z_5,Z_6) = \sum_{m_4=0}^{\infty} H_{m_4,m_5}(Z_1,Z_2,Z_3,Z_4,Z_5) Z_6^{m_4} \tag{6}$$

$$H(Z_1,Z_2,Z_3,Z_4,Z_5,Z_6,Z_7) = \sum_{m_5=0}^{\infty} H_{m_5}(Z_1,Z_2,Z_3,Z_4,Z_5,Z_6) Z_7^{m_5} \tag{7}$$

We may obtain the solution by solving above equations using equations (1) to (7).

$$\mathrm{H}(Z_1,Z_2,Z_3,Z_4,Z_5,Z_6,Z_7) = \frac{\begin{array}{c} H_1\left[\mu_{1H}\left(1-\frac{\alpha_{12}Z_4}{Z_2}-\frac{\alpha_{15}Z_7}{Z_2}\right)-\mu_{1L}\left(1-\frac{\alpha_{12}Z_3}{Z_1}-\frac{\alpha_{15}Z_7}{Z_1}\right)\right]+\mu_3\left(1-\frac{\alpha_{34}Z_6}{Z_5}-\frac{\alpha_{35}Z_7}{Z_5}\right)H_3+ \\ H_2\left[\mu_{2H}\left(1-\frac{\alpha_{21}Z_2}{Z_4}-\frac{\alpha_{25}Z_7}{Z_4}\right)-\mu_{2L}\left(1-\frac{\alpha_{21}Z_1}{Z_3}-\frac{\alpha_{25}Z_7}{Z_3}\right)\right]+\mu_4\left(1-\frac{\alpha_{43}Z_5}{Z_6}-\frac{\alpha_{45}Z_7}{Z_6}\right)H_4+\mu_5\left(1-\frac{\alpha_5}{Z_7}\right)H_5 \\ +\mu_{1L}\left(1-\frac{\alpha_{12}Z_3}{Z_1}-\frac{\alpha_{15}Z_7}{Z_1}\right)H_6+\mu_{2L}\left(1-\frac{\alpha_{21}Z_1}{Z_3}-\frac{\alpha_{25}Z_7}{Z_3}\right)H_7 \end{array}}{\begin{array}{c} \lambda_{1L}(1-Z_1)+\lambda_{1H}(1-Z_2)+\lambda_{2L}(1-Z_3)+\lambda_{2H}(1-Z_4)+\mu_{1H}\left(1-\frac{\alpha_{12}Z_4}{Z_2}-\frac{\alpha_{15}Z_7}{Z_2}\right)+ \\ \mu_3\left(1-\frac{\alpha_{34}Z_6}{Z_5}-\frac{\alpha_{35}Z_7}{Z_5}\right)+\mu_{2H}\left(1-\frac{\alpha_{34}Z_6}{Z_5}-\frac{\alpha_{35}Z_7}{Z_5}\right)+\mu_4\left(1-\frac{\alpha_{43}Z_5}{Z_6}-\frac{\alpha_{45}Z_7}{Z_6}\right)+\mu_5\left(1-\frac{\alpha_5}{Z_7}\right) \end{array}} \tag{8}$$

Here for convenience, we denote

$H_1=H_0(Z_1,Z_3,Z_4,Z_5,Z_6,Z_7)$, $H_2=H_0(Z_1,Z_2,Z_3,Z_5,Z_6,Z_7)$, $H_3=H_0(Z_1,Z_2,Z_3,Z_4,Z_6,Z_7)$, $H_4=H_0(Z_1,Z_2,Z_3,Z_4,Z_5,Z_7)$,

$H_5=H_0(Z_1,Z_2,Z_3,Z_4,Z_5,Z_6)$, $H_6=H_{0,0}(Z_3,Z_4,Z_5,Z_6,Z_7)$, $H_7=H_{0,0}(Z_1,Z_2,Z_5,Z_6,Z_7)$

Since, equation (8) is in indeterminate form, Thus, by using the L'Hospital rule, we arrive at the following conclusions.

$$-\lambda_{1L} = -\mu_{1L}H_1 + \mu_{1L}H_6 + \mu_{2L}\alpha_{21}H_2 - \mu_{2L}\alpha_{21}H_7 \tag{9}$$

$$-\lambda_{1H} + \mu_{1H} - \mu_{2H}\alpha_{21} = \mu_{1H}H_1 - \mu_{2H}\alpha_{21}H_2 \tag{10}$$

$$-\lambda_{2L} = \mu_{1L}\alpha_{12}H_1 - \mu_{1L}\alpha_{12}H_6 - \mu_{2L}H_2 + \mu_{2L}H_7 \tag{11}$$

$$-\lambda_{2H} - \mu_{1H}\alpha_{12} + \mu_{2H} = -\mu_{1H}\alpha_{12}H_1 + \mu_{2H}H_2 \tag{12}$$

$$-\mu_3\alpha_{34} + \mu_4 - \lambda_2 = -\mu_3\alpha_{34}H_3 + \mu_4H_4 \tag{13}$$

$$-\mu_4\alpha_{43} + \mu_3 - \lambda_1 = \mu_3H_3 - \mu_4\alpha_{43}H_4 \tag{14}$$

$$-\mu_{1H}\alpha_{15} - \mu_{2H}\alpha_{25} + \mu_5 - \mu_3\alpha_{35} - \mu_4\alpha_{45} = \mu_5H_5 - \mu_{1L}\alpha_{15}H_6 - \mu_{2L}\alpha_{25}H_7 - \mu_{1H}\alpha_{15}H_1 + \mu_{1L}\alpha_{15}H_1 - \mu_{2H}\alpha_{25}H_2 + \mu_{2L}\alpha_{25}H_2 - \mu_3\alpha_{35}H_3 - \mu_4\alpha_{45}H_4 \tag{15}$$

Solve equations (9) to (15), we get

$$H_1 = 1 - \frac{\lambda_{1H} + \lambda_{2H}\alpha_{21}}{\mu_{1H}(1-\alpha_{12}\alpha_{21})} \tag{16}$$

$$H_2 = 1 - \frac{\lambda_{2H} + \lambda_{1H}\alpha_{12}}{\mu_{2H}(1-\alpha_{12}\alpha_{21})} \tag{17}$$

$$H_3 = 1 - \frac{\lambda_1 + \lambda_2\alpha_{43}}{\mu_3(1-\alpha_{34}\alpha_{43})} \tag{18}$$

$$H_4 = 1 - \frac{\lambda_2 + \lambda_1\alpha_{34}}{\mu_4(1-\alpha_{34}\alpha_{43})} \tag{19}$$

$$H_5 = 1 - \frac{\alpha_{35}(\lambda_1 + \lambda_2\alpha_{43}) + \alpha_{45}(\lambda_2 + \lambda_1\alpha_{34})}{\mu_5(1-\alpha_{34}\alpha_{43})} - \frac{\alpha_{15}\,[(\lambda_{1H} + \lambda_{2H}\alpha_{21}) + (\lambda_{1L} + \lambda_{2L}\alpha_{21})] + \alpha_{25}\,[(\lambda_{2H} + \lambda_{1H}\alpha_{12}) + (\lambda_{2L} + \lambda_{1L}\alpha_{12})]}{\mu_5(1-\alpha_{12}\alpha_{21})} \tag{20}$$

$$H_6 = 1 - \frac{\mu_{1L}(\lambda_{1H} + \lambda_{2H}\alpha_{21}) + \mu_{1H}(\lambda_{1L} + \lambda_{2L}\alpha_{21})}{\mu_{1L}\mu_{1H}(1-\alpha_{12}\alpha_{21})} \tag{21}$$

$$H_7 = 1 - \frac{\mu_{2L}(\lambda_{2H} + \lambda_{1H}\alpha_{12}) + \mu_{2H}(\lambda_{2L} + \lambda_{1L}\alpha_{12})}{\mu_{2L}\mu_{2H}(1-\alpha_{12}\alpha_{21})} \tag{22}$$

The model's equilibrium solution is,

$$P_{m_{1L},m_{1H},m_{2L},m_{2H},m_3,m_4,m_5} = (1-H_1)^{m_{1H}}(1-H_2)^{m_{2H}}(1-H_3)^{m_3}(1-H_4)^{m_4}(1-H_5)^{m_5}(1-H_6)^{m_{1L}}(1-H_7)^{m_{2L}}H_1H_2H_3H_4H_5H_6H_7$$

$$=\gamma_1^{m_{1H}}\gamma_2^{m_{2H}}\gamma_3^{m_3}\gamma_4^{m_4}\gamma_5^{m_5}\gamma_6^{m_{1L}}\gamma_7^{m_{2L}}(1-\gamma_1)(1-\gamma_2)(1-\gamma_3)(1-\gamma_4)(1-\gamma_5)(1-\gamma_6)(1-\gamma_7)$$

And utilization of server is represented by

$$\gamma_1 = \frac{\lambda_{1H} + \lambda_{2H}\alpha_{21}}{\mu_{1H}(1-\alpha_{12}\alpha_{21})} \tag{23}$$

$$\gamma_2 = \frac{\lambda_{2H} + \lambda_{1H}\alpha_{12}}{\mu_{2H}(1-\alpha_{12}\alpha_{21})} \tag{24}$$

$$\gamma_3 = \frac{\lambda_1 + \lambda_2\alpha_{43}}{\mu_3(1-\alpha_{34}\alpha_{43})} \tag{25}$$

$$\gamma_4 = \frac{\lambda_2 + \lambda_1\alpha_{34}}{\mu_4(1-\alpha_{34}\alpha_{43})} \tag{26}$$

$$\gamma_5 = \frac{\alpha_{35}(\lambda_1 + \lambda_2\alpha_{43}) + \alpha_{45}(\lambda_2 + \lambda_1\alpha_{34})}{\mu_5(1-\alpha_{34}\alpha_{43})} + \frac{\alpha_{15}\,[(\lambda_{1H} + \lambda_{2H}\alpha_{21}) + (\lambda_{1L} + \lambda_{2L}\alpha_{21})] + \alpha_{25}\,[(\lambda_{2H} + \lambda_{1H}\alpha_{12}) + (\lambda_{2L} + \lambda_{1L}\alpha_{12})]}{\mu_5(1-\alpha_{12}\alpha_{21})} \tag{27}$$

$$\gamma_6 = \frac{\mu_{1L}(\lambda_{1H} + \lambda_{2H}\alpha_{21}) + \mu_{1H}(\lambda_{1L} + \lambda_{2L}\alpha_{21})}{\mu_{1L}\mu_{1H}(1-\alpha_{12}\alpha_{21})} \tag{28}$$

$$\gamma_7 = \frac{\mu_{2L}(\lambda_{2H} + \lambda_{1H}\alpha_{12}) + \mu_{2H}(\lambda_{2L} + \lambda_{1L}\alpha_{12})}{\mu_{2L}\mu_{2H}(1-\alpha_{12}\alpha_{21})} \tag{29}$$

The solution of model exist if $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \leq 1$

4. Performance Measure of Queue

a) Expected Length of Queue

$$L = L_{q1L} + L_{q1H} + L_{q2L} + L_{q2H} + L_{q3} + L_{q4} + L_{q5}$$

$$Where \quad L_{q1L} = \frac{\gamma_7}{(1-\gamma_7)}, L_{q1H} = \frac{\gamma_1}{(1-\gamma_1)}, L_{q2L} = \frac{\gamma_6}{(1-\gamma_6)}, L_{q2H} = \frac{\gamma_2}{(1-\gamma_2)},$$

$$L_{q3} = \frac{\gamma_3}{(1-\gamma_3)}, L_{q4} = \frac{\gamma_4}{(1-\gamma_4)}, L_{q5} = \frac{\gamma_5}{(1-\gamma_5)}$$

b) Variance in queue length

$$Var = V_{n1L} + V_{n1H} + V_{n2L} + V_{n2H} + V_3 + V_4 + V_5$$

$$Where \quad V_{n1L} = \frac{\gamma_7}{(1-\gamma_7)^2}, V_{n1H} = \frac{\gamma_1}{(1-\gamma_1)^2}, V_{n2L} = \frac{\gamma_6}{(1-\gamma_6)^2}, V_{n2H} = \frac{\gamma_2}{(1-\gamma_2)^2},$$

$$V_3 = \frac{\gamma_3}{(1-\gamma_3)^2}, V_4 = \frac{\gamma_4}{(1-\gamma_4)^2}, V_5 = \frac{\gamma_5}{(1-\gamma_5)^2}$$

c) Average waiting time

$E = \frac{L}{\lambda}$, $\lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H} + \lambda_1 + \lambda_2$

5. Numerical Illustration and Results

The queuing model with two servers that have biserial subsystems connected in series to a common server has been investigated in this work. A comprehensive discussion of the model and its accompanying equations are presented in sections 2 and 4. The details of various input parameters used for computing several queue characteristics are presented in Table 1. The Table 2,3,4,5,6,7 represents numerically various queue characteristics for different arrival rates and Table 8,9,10,11,12,13 & 14 shows queues behavior w.r.t various service rates.

Fig. 2$(a_1, a_2, a_3, a_4, a_5, a_6)$ express the mean and partial length of Queues at different servers for various arrival rates. It is clear from results while increases arrival rates , Queue length increases which confirms the basic conditions. In Fig. 4 and 6 $(a_1, a_2, a_3, a_4, a_5, a_6)$ the variance and expected waiting times are plotted w.r.t different arrival rates. It shows on increasing arrival rates, these queue characteristics also increases. Fig. 3$(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ represents the mean length of Queues for various service rates . Result represents that on increasing service rates at different servers the mean length of Queues decreases. In Fig. 5 and 7 $(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$shows variance and expected waiting times for different service rates. The values of these queue parameters are decreases on increasing service rates at different servers.

The following values of parameters are used in computation of results

$\lambda_{1H} = 6,\ \lambda_{2H} = 5,\ \lambda_{1L} = 3,\ \lambda_{2L} = 4$, λ_1=4, λ_2=5

$\mu_{1L} = 18,\ \mu_{1H} = 25,\ \mu_{2H} = 27,\ \mu_{2L} = 20,\ \mu_3 = 28,\ \mu_4 = 26,\ \mu_5 = 30$

$\alpha_{12} = .6, \alpha_{21} = .5, \alpha_{15} = .4, \alpha_{25} = .5,\ \alpha_{35} = .3,\ \alpha_{34} = .7, \alpha_{45} = .8,$

$\alpha_{43} = .2, \alpha_5 = .9$

Table 2. Various Queue Characteristics w.r.t arrival rate λ_{1H}

λ_{1H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	Var	E(w)
4	0.3714	0.3915	0.2076	0.3488	0.8333	0.7682	0.8058	14.5	68.9405	0.5798
4.5	0.4	0.4074	0.2076	0.3488	0.85	0.7968	0.8216	16.3	86.4606	0.6411
5	0.4285	0.4232	0.2076	0.3488	0.8666	0.8253	0.8375	18.7	111.692	0.7179
5.5	0.4571	0.4391	0.2076	0.3488	0.8833	0.8539	0.8534	21.7	149.499	0.8178
6	0.4857	0.455	0.2076	0.3488	0.9	0.8825	0.8693	25.8	210.074	0.9537
6.5	0.5142	0.4708	0.2076	0.3488	0.9166	0.9111	0.8851	31.7	2702.56	1.1534
7	0.5428	0.4867	0.2076	0.3488	0.9333	0.9396	0.901	41.6	3441.05	1.4871
7.5	0.5714	0.5026	0.2076	0.3488	0.95	0.9682	0.9169	63.7	15016.5	2.2361
8	0.6	0.5185	0.2076	0.3488	0.9666	0.9968	0.9328	358	814997	12.334

Table 3. Various Queue Characteristics w.r.t arrival rate λ_{2H}

λ_{2H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
3	0.4285	0.3492	0.2076	0.349	0.8333	0.8253	0.7634	15.04	74.179	0.6016
3.5	0.4428	0.3756	0.2076	0.349	0.85	0.8396	0.7899	16.86	95.03	0.6611
4	0.4571	0.4021	0.2076	0.349	0.8666	0.8539	0.8164	19.11	117.18	0.735
4.5	0.4714	0.4285	0.2076	0.349	0.8833	0.8682	0.8428	21.97	154.03	0.8291
5	0.4857	0.455	0.2076	0.349	0.9	0.8825	0.8693	25.75	210.07	0.9537
5.5	0.5	0.4814	0.2076	0.349	0.9166	0.8968	0.8957	31.02	1699	1.1281
6	0.5142	0.5079	0.2076	0.349	0.9333	0.9111	0.9222	39.03	5177.3	1.3939
6.5	0.5285	0.5343	0.2076	0.349	0.95	0.9253	0.9486	52.96	10400	1.8582
7	0.5428	0.5608	0.2076	0.349	0.9666	0.9396	0.9756	87.19	27656	3.0065

Table 4. Various Queue Characteristics w.r.t arrival rate λ_{1L}

λ_{1L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
1	0.4857	0.455	0.2076	0.3488	0.8333	0.7238	0.7835	13.8	60.85	0.5528
1.5	0.4857	0.455	0.2076	0.3488	0.85	0.7634	0.805	15.6	77.2	0.6118
2	0.4857	0.455	0.2076	0.3488	0.8666	0.8031	0.8264	17.9	101.7	0.6893
2.5	0.4857	0.455	0.2076	0.3488	0.8833	0.8428	0.8478	21.1	143.1	0.7958
3	0.4857	0.455	0.2076	0.3488	0.9	0.8825	0.8693	25.8	210.1	0.9537
3.5	0.4857	0.455	0.2076	0.3488	0.9166	0.9222	0.8907	33.6	2945	1.222
4	0.4857	0.455	0.2076	0.3488	0.9333	0.9619	0.9121	52.3	10181	1.8668

Table 5. Various Queue Characteristics w.r.t arrival rate λ_{2L}

λ_{2L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
2	0.4857	0.455	0.2076	0.3488	0.8333	0.8031	0.7264	14.3	65.07	0.5726
2.5	0.4857	0.455	0.2076	0.3488	0.85	0.823	0.7621	16.1	82.17	0.6314
3	0.4857	0.455	0.2076	0.3488	0.8666	0.8428	0.7978	18.4	107.3	0.6811
3.5	0.4857	0.455	0.2076	0.3488	0.8833	0.8626	0.8335	21.4	146	0.7797
4	0.4857	0.455	0.2076	0.3488	0.9	0.8825	0.8693	25.8	210.1	0.9196
4.5	0.4857	0.455	0.2076	0.3488	0.9166	0.9023	0.905	32.4	3288	1.1355
5	0.4857	0.455	0.2076	0.3488	0.9333	0.9222	0.9407	44.3	6350	1.5292

Table 6. Various Queue Characteristics w.r.t arrival rate λ_1

λ_1	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
2	0.4857	0.455	0.1245	0.2862	0.8333	0.8825	0.8693	21.5	149.7	0.8599
2.5	0.4857	0.455	0.1453	0.3018	0.85	0.8825	0.8693	22.2	157.5	0.8714
3	0.4857	0.455	0.1661	0.3175	0.8666	0.8825	0.8693	23.1	168.8	0.8891
3.5	0.4857	0.455	0.1868	0.3331	0.8833	0.8825	0.8693	24.3	184.9	0.9153
4	0.4857	0.455	0.2016	0.3488	0.9	0.8825	0.8693	25.7	210.1	0.9533
4.5	0.4857	0.455	0.2284	0.3644	0.9166	0.8825	0.8693	27.8	1449	1.0118
5	0.4857	0.455	0.2491	0.3801	0.9333	0.8825	0.8693	30.9	2454	1.1039
5.5	0.4857	0.455	0.2699	0.3957	0.95	0.8825	0.8693	36	3921	1.2623
6	0.4857	0.455	0.2906	0.4114	0.9666	0.8825	0.8693	46.1	8908	1.5892

Table 7. Various Queue Characteristics w.r.t arrival rate λ_2

λ_2	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
3	0.4857	0.455	0.191	0.2593	0.8333	0.8825	0.8693	21.5	149.8	0.8616
3.5	0.4857	0.455	0.1951	0.2817	0.85	0.8825	0.8693	22.3	157.5	0.8727
4	0.4857	0.455	0.1993	0.3041	0.8666	0.8825	0.8693	23.1	168.8	0.8899
4.5	0.4857	0.455	0.2034	0.3264	0.8833	0.8825	0.8693	24.3	184.9	0.9157
5	0.4857	0.455	0.2076	0.3488	0.9	0.8825	0.8693	25.8	210.1	0.9537
5.5	0.4857	0.455	0.2117	0.3711	0.9166	0.8825	0.8693	27.8	1449	1.0114
6	0.4857	0.455	0.2159	0.3935	0.9333	0.8825	0.8693	30.9	2241	1.1032
6.5	0.4857	0.455	0.22	0.4159	0.95	0.8825	0.8693	35.9	3921	1.2612
7	0.4857	0.455	0.2242	0.4382	0.9666	0.8825	0.8693	46	8908	1.5878

Table 8. Various Queue Characteristics w.r.t service rate μ_{1H}

μ_{1H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
22	0.5519	0.455	0.2076	0.3488	0.9	0.9487	0.8693	37.1	3795.4	1.3722
23	0.5279	0.455	0.2076	0.3488	0.9	0.9247	0.8693	30.7	1797.4	1.1371
24	0.5059	0.455	0.2076	0.3488	0.9	0.9027	0.8693	27.6	1106.2	1.0222
25	0.4857	0.455	0.2076	0.3488	0.9	0.8825	0.8693	25.8	210.07	0.9537
26	0.467	0.455	0.2076	0.3488	0.9	0.8638	0.8693	24.5	192.16	0.9078
27	0.4497	0.455	0.2076	0.3488	0.9	0.8465	0.8693	23.6	181.33	0.8749
28	0.4336	0.455	0.2076	0.3488	0.9	0.8304	0.8693	23	174.11	0.8501

Table 9. Various Queue Characteristics w.r.t service rate μ_{2H}

μ_{2H}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
24	0.4857	0.5119	0.2076	0.3488	0.9	0.8225	0.9261	31.9	1875	1.1798
25	0.4857	0.4914	0.2076	0.3488	0.9	0.8225	0.9057	27.9	1175	1.0322
26	0.4857	0.4725	0.2076	0.3488	0.9	0.8225	0.8868	26	214.8	0.9639
27	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8693	24.8	196.5	0.9177
28	0.4857	0.4387	0.2076	0.3488	0.9	0.8225	0.853	23.9	184.9	0.8843
29	0.4857	0.4236	0.2076	0.3488	0.9	0.8225	0.8379	23.2	177.1	0.8591
30	0.4857	0.4095	0.2076	0.3488	0.9	0.8225	0.8238	22.7	171.6	0.8392

Table 10. Various Queue Characteristics w.r.t service rate μ_{1L}

μ_{1L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
15	0.4857	0.455	0.2076	0.3488	0.9	0.9619	0.8693	43.5	7016.4	1.6128
16	0.4857	0.455	0.2076	0.3488	0.9	0.9321	0.8693	32	2172	1.1844
17	0.4857	0.455	0.2076	0.3488	0.9	0.9201	0.8693	29.8	1606.1	1.1023
18	0.4857	0.455	0.2076	0.3488	0.9	0.8825	0.8693	25.8	210.07	0.9537
19	0.4857	0.455	0.2076	0.3488	0.9	0.8616	0.8693	24.5	190.77	0.906
20	0.4857	0.455	0.2076	0.3488	0.9	0.8428	0.8693	23.6	179.92	0.874
21	0.4857	0.455	0.2076	0.3488	0.9	0.8258	0.8693	23	172.91	0.8509

Table 11. Various Queue Characteristics w.r.t service rate μ_{2L}

μ_{2L}	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
17	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.9424	35.5	3014.7	1.3142
18	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.9153	29.9	1448.1	1.1079
19	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8911	27.3	234.46	1.0105
20	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8693	25.8	210.07	0.9537
21	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8495	24.7	196.53	0.9163
22	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8316	24	188.32	0.8901
23	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8152	23.5	182.85	0.8706

Table 12. Various Queue Characteristics w.r.t service rate μ_3

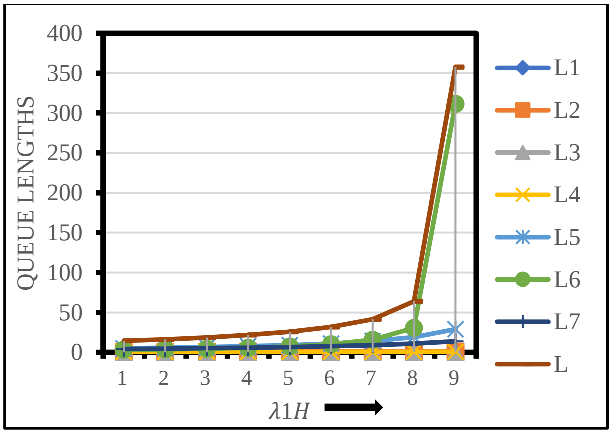
μ_3	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
24	0.4857	0.455	0.2422	0.3488	0.9	0.8225	0.8693	25.8	210.2	0.9558
25	0.4857	0.455	0.2325	0.3488	0.9	0.8225	0.8693	25.8	210.1	0.9552
26	0.4857	0.455	0.2236	0.3488	0.9	0.8225	0.8693	25.8	210.1	0.9546
27	0.4857	0.455	0.2153	0.3488	0.9	0.8225	0.8693	25.8	210.1	0.9541
28	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8693	25.8	210.1	0.9537
29	0.4857	0.455	0.2004	0.3488	0.9	0.8225	0.8693	25.7	210.1	0.9532
30	0.4857	0.455	0.1937	0.3488	0.9	0.8225	0.8693	25.7	210	0.9529
31	0.4857	0.455	0.1875	0.3488	0.9	0.8225	0.8693	25.7	210	0.9525
32	0.4857	0.455	0.1816	0.3488	0.9	0.8225	0.8693	25.7	210	0.9522

Table 13. Various Queue Characteristics w.r.t service rate μ_4

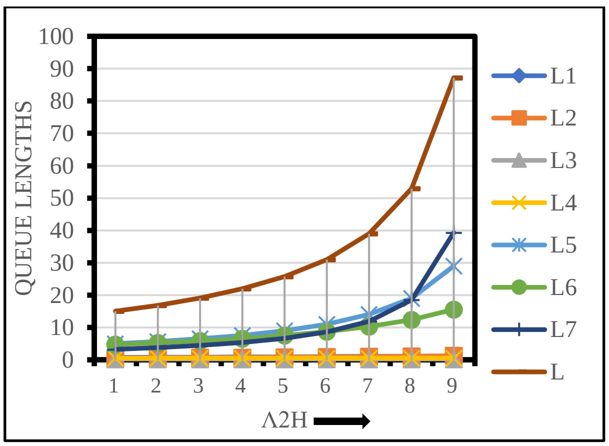
μ_4	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
22	0.4857	0.455	0.2076	0.4122	0.9	0.8225	0.8693	25.9	210.4	0.9598
23	0.4857	0.455	0.2076	0.3943	0.9	0.8225	0.8693	25.9	210.3	0.9579
24	0.4857	0.455	0.2076	0.3779	0.9	0.8225	0.8693	25.8	210.2	0.9563
25	0.4857	0.455	0.2076	0.3627	0.9	0.8225	0.8693	25.8	210.1	0.9549
26	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8693	25.8	210.1	0.9537
27	0.4857	0.455	0.2076	0.3359	0.9	0.8225	0.8693	25.7	210	0.9526
28	0.4857	0.455	0.2076	0.3239	0.9	0.8225	0.8693	25.7	210	0.9516
29	0.4857	0.455	0.2076	0.3127	0.9	0.8225	0.8693	25.7	209.9	0.9507
30	0.4857	0.455	0.2076	0.3023	0.9	0.8225	0.8693	25.6	209.9	0.9499

Table 14. Various Queue Characteristics w.r.t service rate μ_5

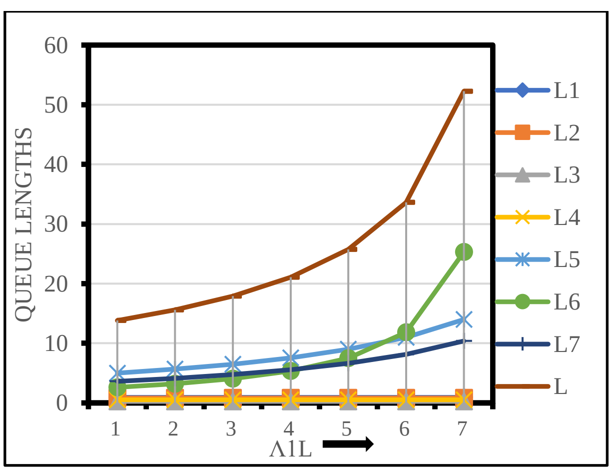
μ_5	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	L	VAR	E(W)
28	0.4857	0.455	0.2076	0.3488	0.9642	0.8225	0.8693	43.8	8155.1	1.6206
29	0.4857	0.455	0.2076	0.3488	0.9309	0.8225	0.8693	30.2	2100.7	1.12
30	0.4857	0.455	0.2076	0.3488	0.9	0.8225	0.8693	25.8	210.07	0.9537
31	0.4857	0.455	0.2076	0.3488	0.8709	0.8225	0.8693	23.5	172.54	0.8704
32	0.4857	0.455	0.2076	0.3488	0.8437	0.8225	0.8693	22.1	154.65	0.8203
33	0.4857	0.455	0.2076	0.3488	0.8181	0.8225	0.8693	21.3	144.87	0.787
34	0.4857	0.455	0.2076	0.3488	0.7941	0.8225	0.8693	20.6	138.85	0.7632



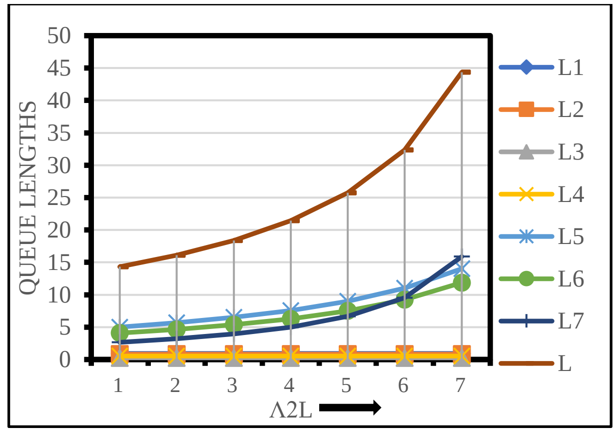
a_1. λ_{1L} **=3,** λ_{2H}**=5,** λ_{2L}**=4,** λ_1**=4,** λ_2**=5**



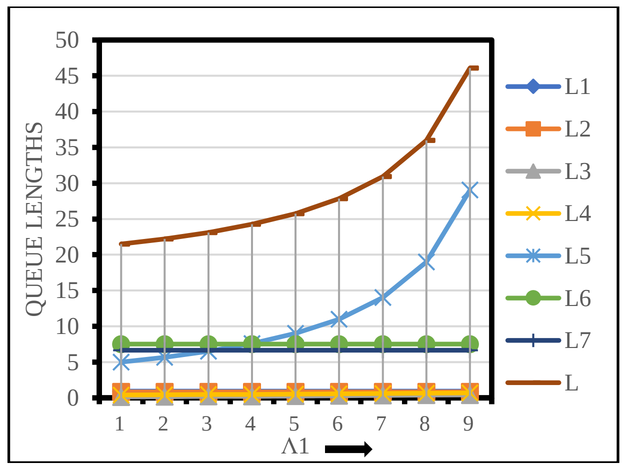
a_2. λ_{1H} **=6,** λ_{1L}**=3,** λ_{2L}**=4,** λ_1**=4,** λ_2**=5**



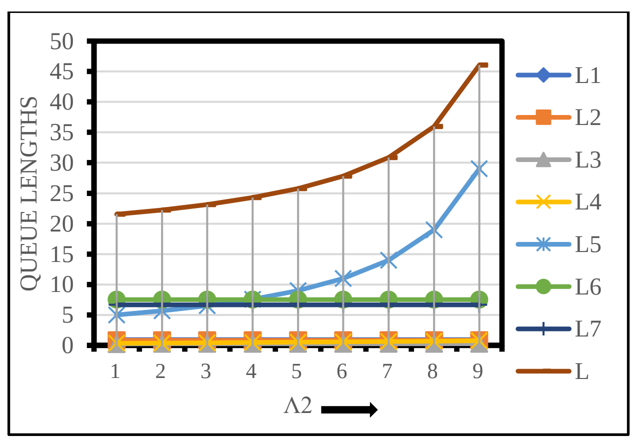
a_3. λ_{1H} **=6,** λ_{2H}**=5,** λ_{2L}**=4,** λ_1**=4,** λ_2**=5**



a_4. λ_{1H}**=6,** λ_{1L}**=3,** λ_{2H}**=5,** λ_1**=4,** λ_2**=5**

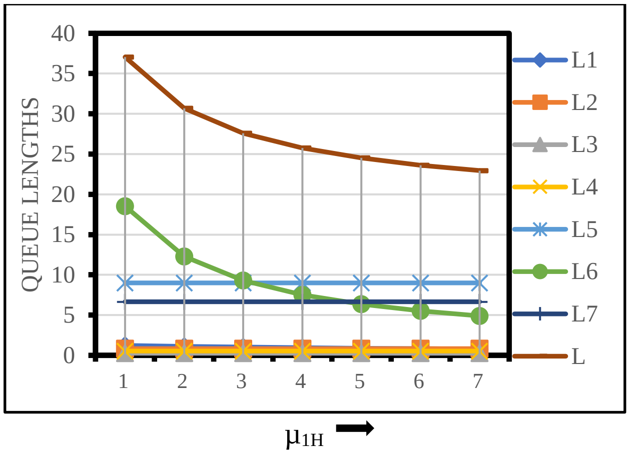


a_5. λ_{1H} **=6,** λ_{1L}**=3,** λ_{2H}**=5,** λ_{2L}**=4,** λ_2**=5**

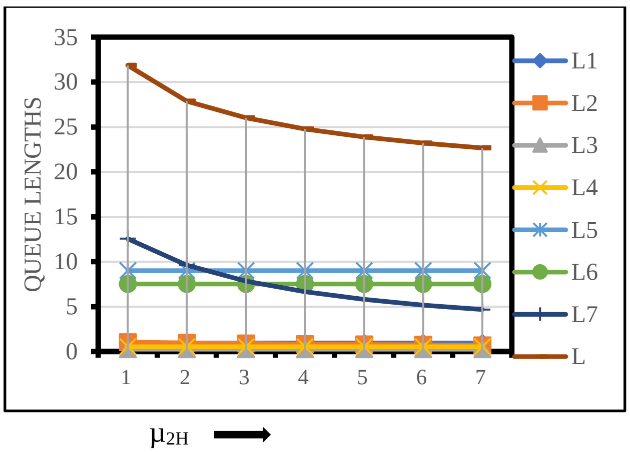


a_6. λ_{1H} **=6,** λ_{1L}**=3,** λ_{2H}**=5,** λ_{2L}**=4,** λ_1**=4**

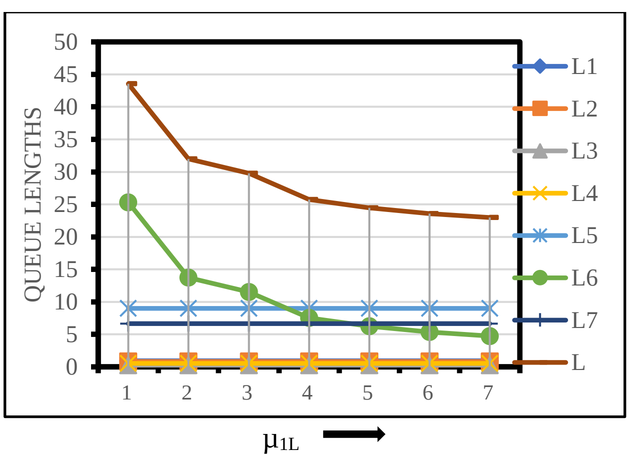
Fig. 2$(a_1, a_2, a_3, a_4, a_5, a_6)$; **Mean Queue Lengths for various arrival rates**



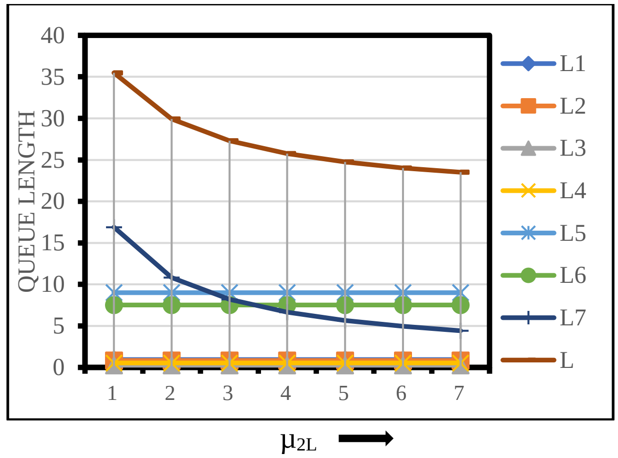
a_1. μ_{1L}=18, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



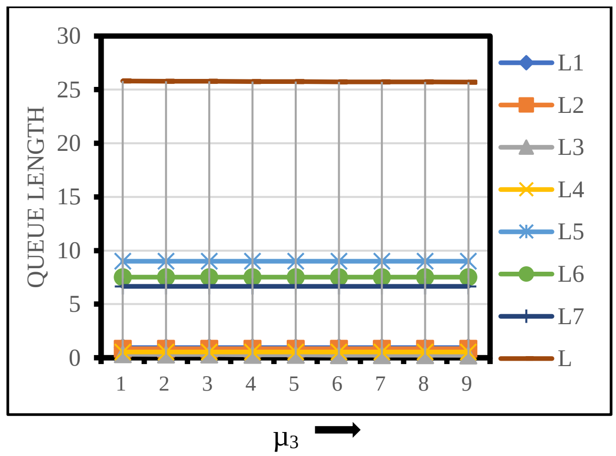
a_2. μ_{1H}=25, μ_{1L}=18, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



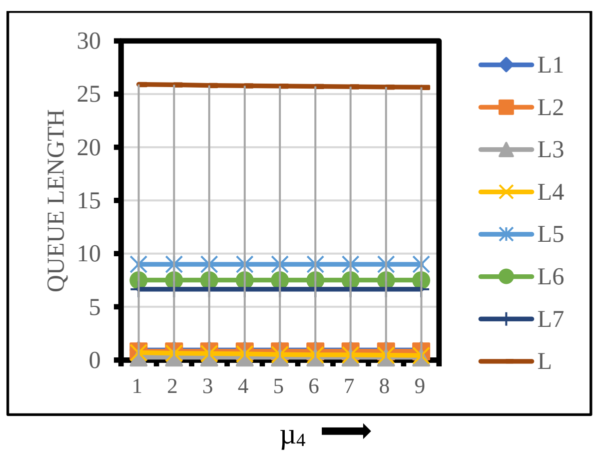
a_3. μ_{1H}=25, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



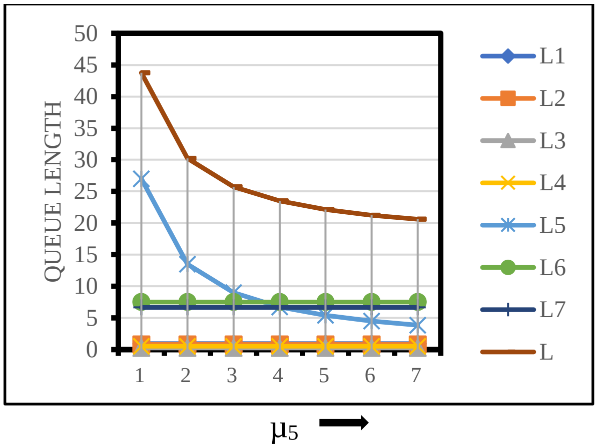
a_4. μ_{1H}=25, μ_{1L}=18, μ_{2H}=27, μ_3=28, μ_4=26, μ_5=30



a_5. μ_{1H}=25, μ_{1L}=18, μ_{2H}=27, μ_{2L}=20, μ_4=26, μ_5=30

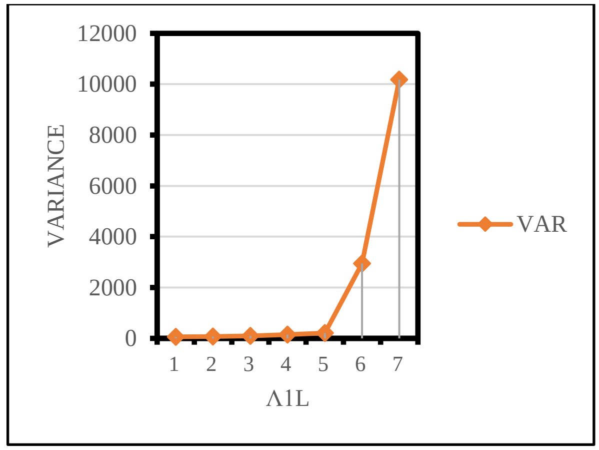


a_6. μ_{1H}=25, μ_{1L}=18, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_5=30

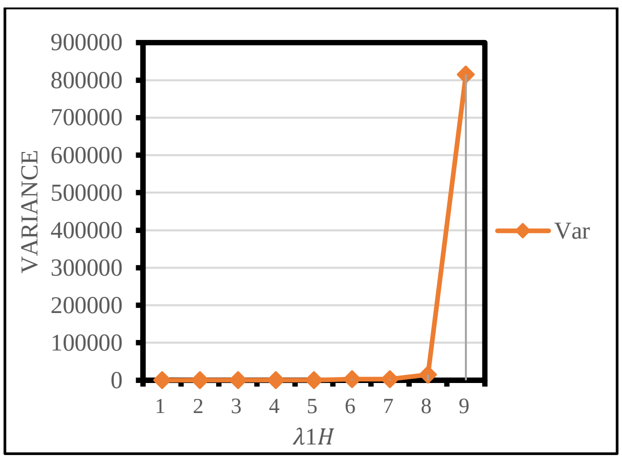


a_7. μ1H=25, μ1L=18, μ2H=27, μ2L=20, μ3=28, μ4=26

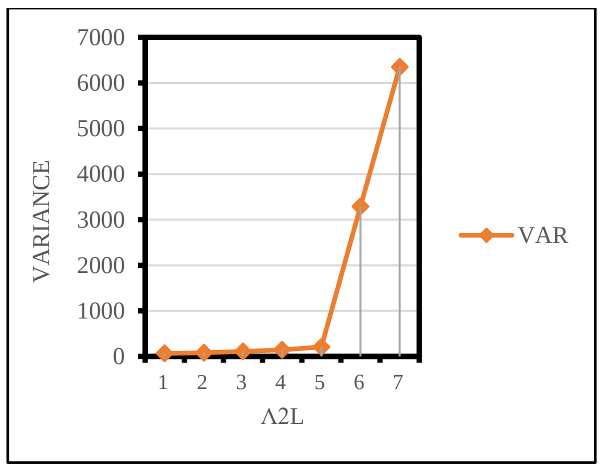
Fig. 3$(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$; Mean Queue Lengths for various service rates



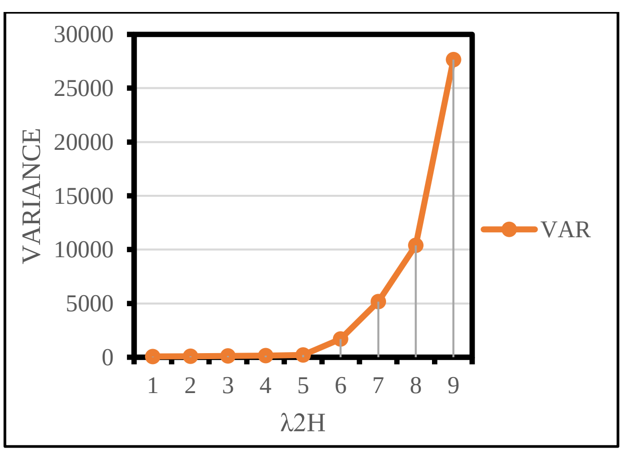
a_3. λ_{1H} =6, λ_{2H}=5, λ_{2L}=4, λ_1=4, λ_2=5



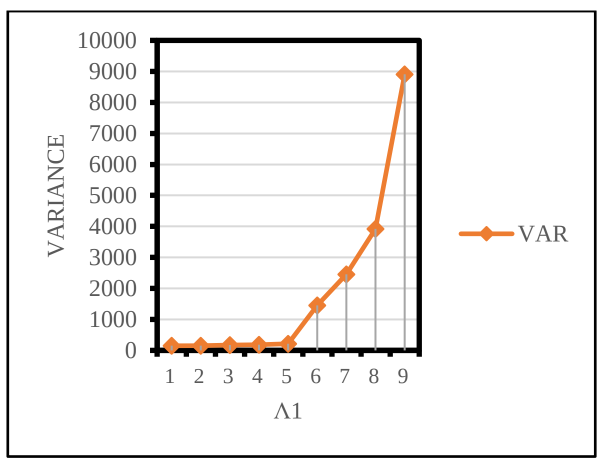
a_1. λ_{1L} =3, λ_{2H}=5, λ_{2L}=4, λ_1=4, λ_2=5



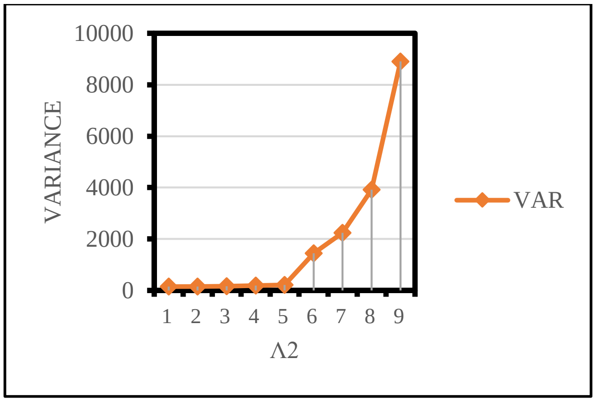
a_4. λ_{1H}=6, λ_{1L}=3, λ_{2H}=5, λ_1=4, λ_2=5



a_2. λ_{1H} =6, λ_{1L}=3, λ_{2L}=4, λ_1=4, λ_2=5

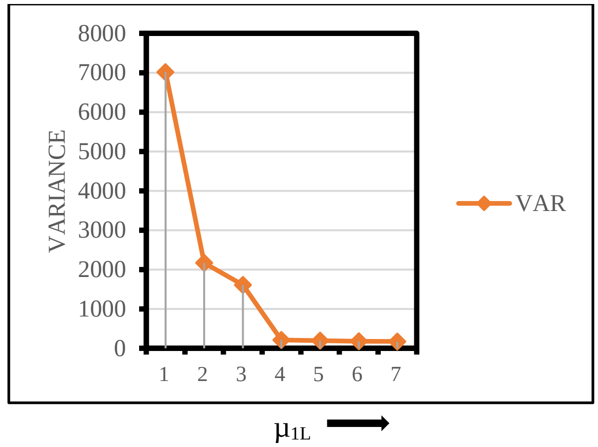


a_5. λ_{1H} =6, λ_{1L}=3, λ_{2H}=5, λ_{2L}=4, λ_2=5

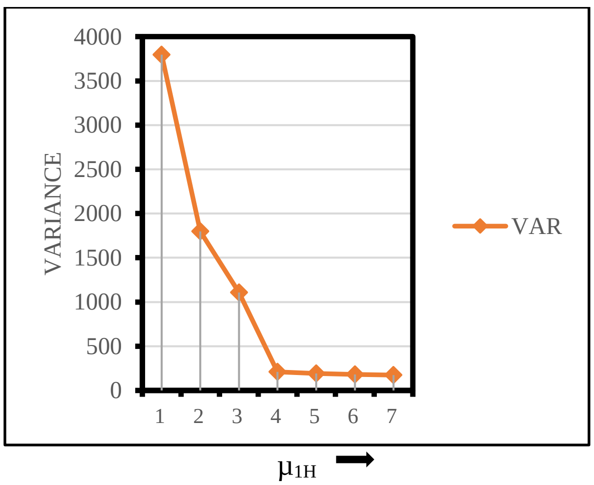


a_6. λ_{1H} =6, λ_{1L}=3, λ_{2H}=5, λ_{2L}=4, λ_1=4

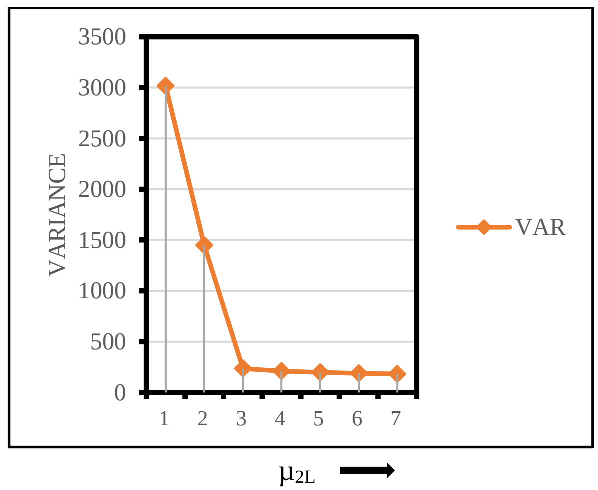
Fig. 4($a_1, a_2, a_3, a_4, a_5, a_6$); Variance in Queues for various arrival rates



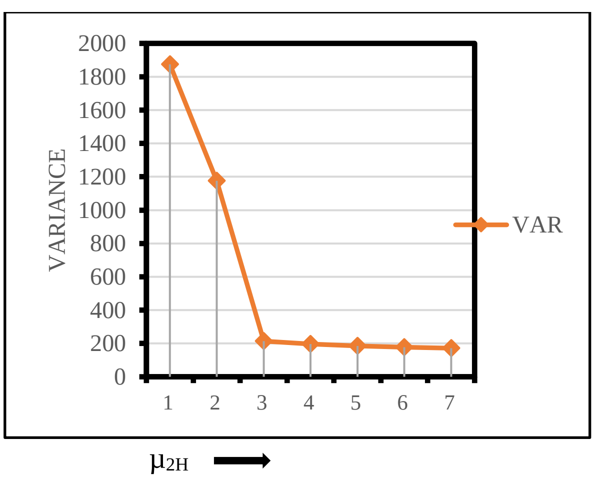
a_3. μ_{1H}=25, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



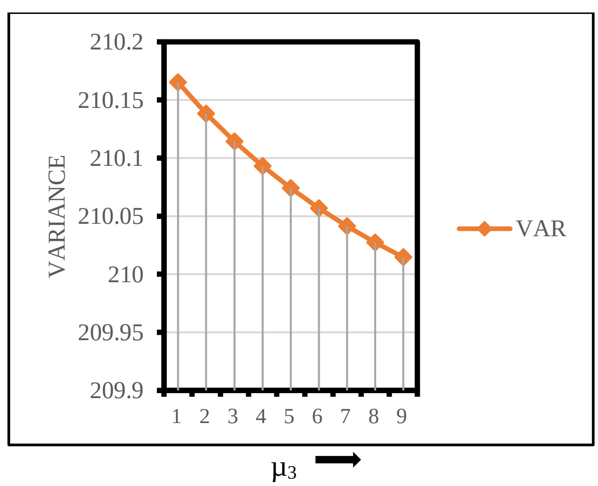
a_1. μ_{1L}=18, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



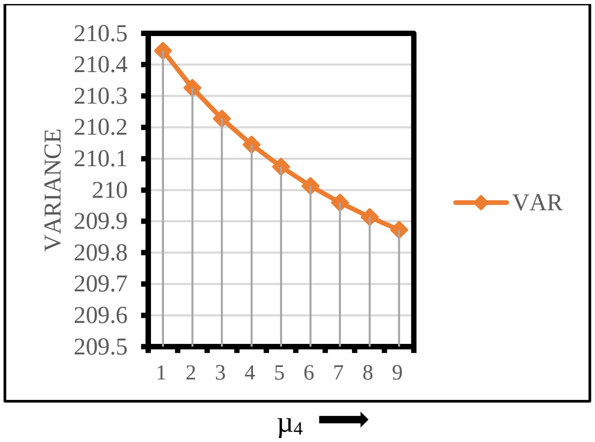
a_4. μ_{1H}=25, μ_{1L}=18, μ_{2H}=27, μ_3=28, μ_4=26, μ_5=30



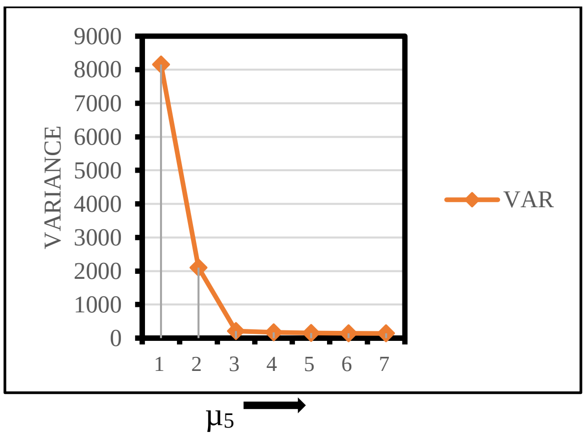
a_2. μ_{1H}=25, μ_{1L}=18, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



a_5. μ_{1H}=25, μ_{1L}=18, μ_{2H}=27, μ_{2L}=20, μ_4=26, μ_5=30

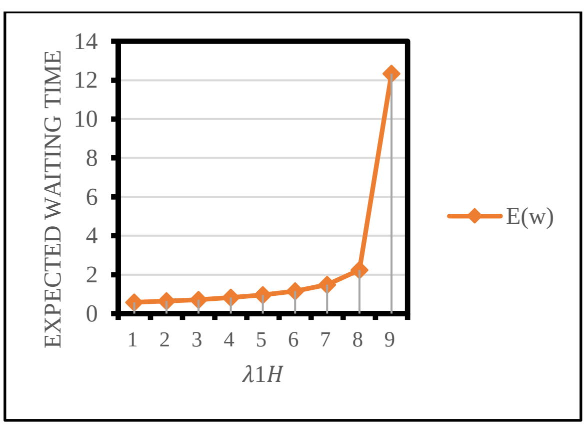


a_6. μ1H=25, μ1L=18, μ2H=27, μ2L=20, μ3=28, μ5=30

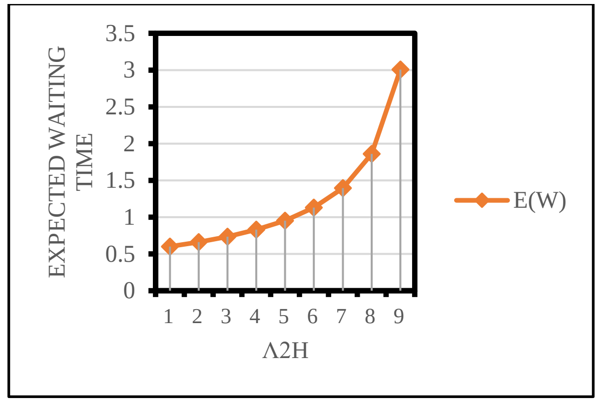


a_7. μ1H=25, μ1L=18, μ2H=27, μ2L=20, μ3=28, μ4=26

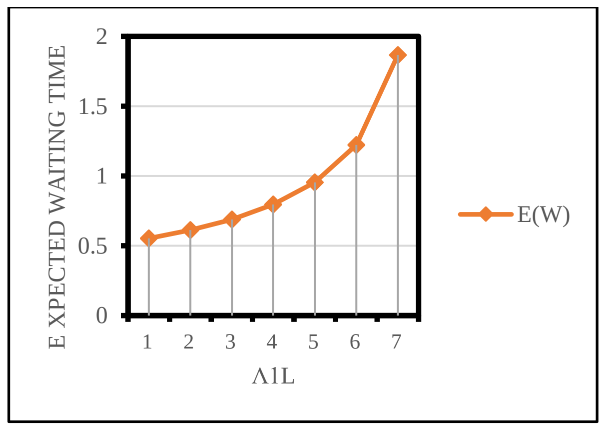
Fig. 5($a_1, a_2, a_3, a_4, a_5, a_6, a_7$); Variance in Queues for various service rates



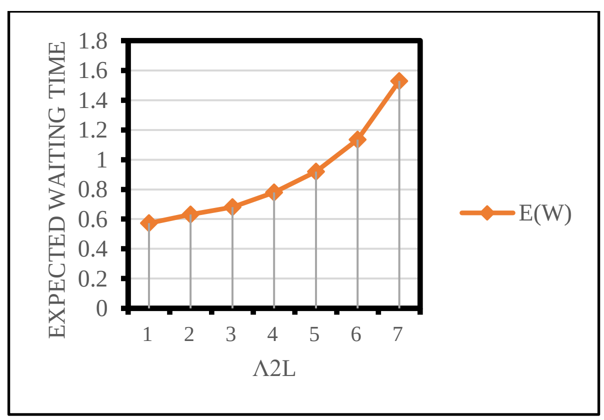
a_1. λ_{1L} =3, λ_{2H}=5, λ_{2L}=4, λ_1=4, λ_2=5



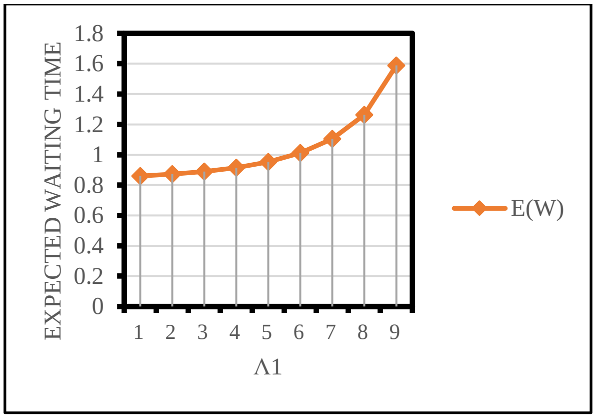
a_2. λ_{1H} =6, λ_{1L}=3, λ_{2L}=4, λ_1=4, λ_2=5



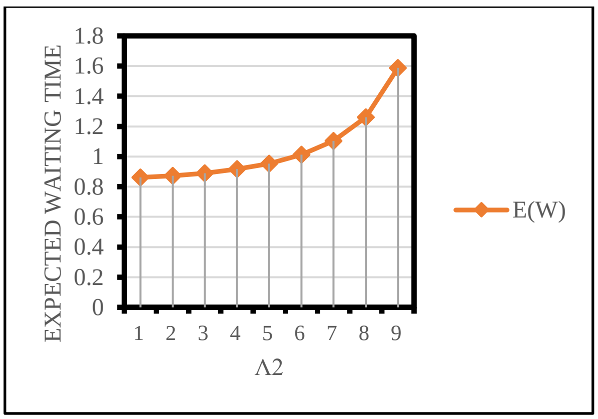
a_3. λ_{1H} =6, λ_{2H}=5, λ_{2L}=4, λ_1=4, λ_2=5



a_4. λ_{1H}=6, λ_{1L}=3, λ_{2H}=5, λ_1=4, λ_2=5

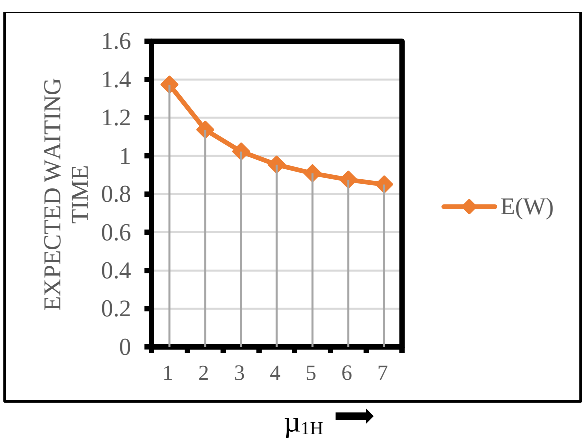


a_5. λ_{1H} =6, λ_{1L}=3, λ_{2H}=5, λ_{2L}=4, λ_2=5

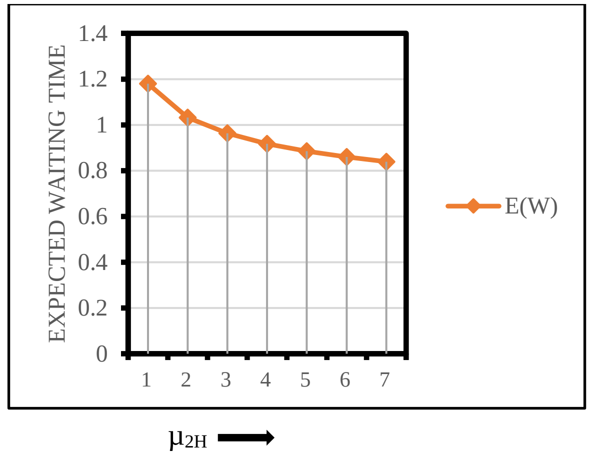


a_6. λ_{1H} =6, λ_{1L}=3, λ_{2H}=5, λ_{2L}=4, λ_1=4

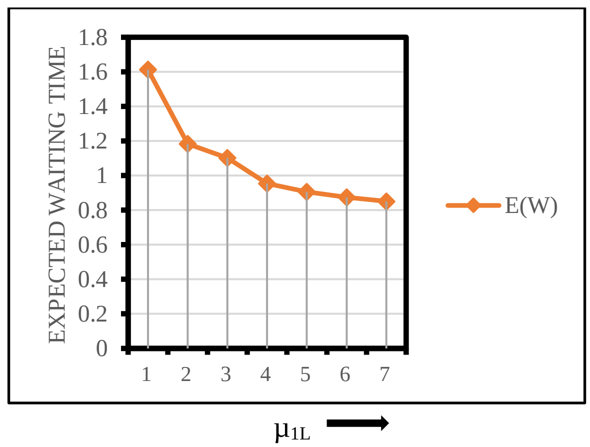
Fig. 6$(a_1, a_2, a_3, a_4, a_5, a_6)$; Expected Waiting time for various arrival rates



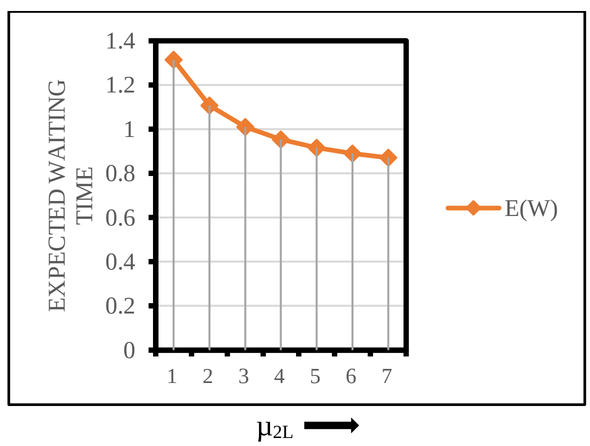
a_1. μ_{1L}=18, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



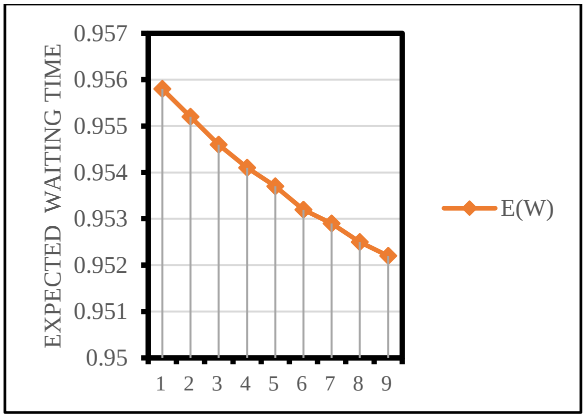
a_2. μ_{1H}=25, μ_{1L}=18, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



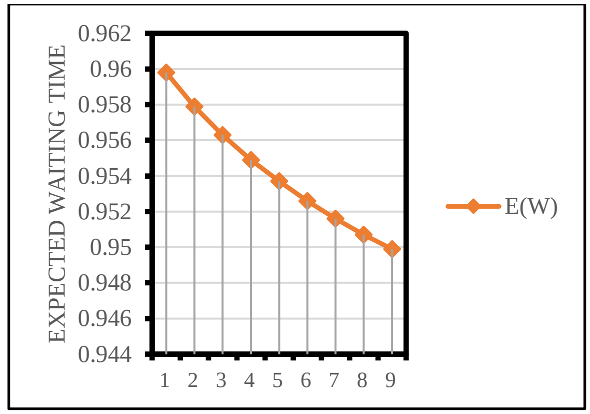
a_3. μ_{1H}=25, μ_{2H}=27, μ_{2L}=20, μ_3=28, μ_4=26, μ_5=30



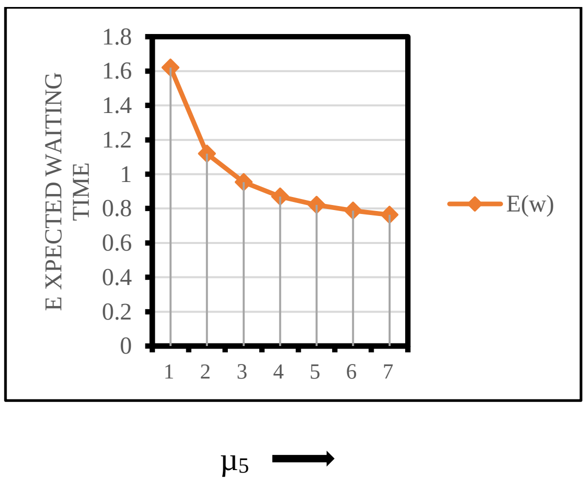
a_4. μ_{1H}=25, μ_{1L}=18, μ_{2H}=27, μ_3=28, μ_4=26, μ_5=30



a_5. μ1H=25, μ1L=18, μ2H=27, μ2L=20, μ4=26, μ5=30



a_6. μ1H=25, μ1L=18, μ2H=27, μ2L=20, μ3=28, μ5=30



a_7. μ1H=25, μ1L=18, μ2H=27, μ2L=20, μ3=28, μ4=26

Fig. 7$(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$; Average Waiting Time for various service rates

6. Conclusion

The current work was devoted to the research of queues behavior of complex Bi-Tandem queuing network model in stochastic environment. The practical implementation of the proposed model can be seen by taking realistic example of administration. Various performance measures like queue lengths, variance, traffic intensities and average waiting time have been computed and graphically represented by using various input parameters. Thus, this model is useful in such type of complex queuing situations to provide better service facilities.

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