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Original Article

h-Normal Spaces in General Topology

Hamant kumar[1], Poonam Sharma[2]

[1]*Department of Mathematics, Veerangana Avantibai Government Degree College, Atrauli-Aligarh, U. P. (India)*
[2]*Department of Mathematics, HIMT College, Greater Noida, U. P. (India)*

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Abstract *- In this paper, we introduce and study a new class of sets, called h generalized closed sets and relationships among closed, g-closed, gh-closed and hg-closed sets are investigated. Further, we introduce a new class of normal spaces, called h-normal spaces and obtain a characterization of h-normal spaces. Moreover, we define the forms of generalized h-closed, h-generalized closed and some h-generalized continuous functions. By utilizing these functions, we study properties of the forms of generalized h-closed functions and preservation theorems for h-normal spaces.*

Keywords - *h-open, gh-closed and hg-closed sets, h-normal spaces, h-closed, gh-closed and h-gh-closed functions.*

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1. Introduction

In 1970, Levine [15] initiated the investigation of g-closed sets in topological spaces; since then, many modifications of g-closed sets have been defined and investigated by a large number of topologists. In 1973, Carnahan [5] introduced some properties related to compactness in topological spaces. In 1978, Maheshwari and Prasad [16] introduced s-normal spaces and obtained their characterizations. In 2000, Dontchev and Noiri [6] studied the concept of quasi-normal spaces and obtained their properties.

In 2008, Kalantan [11] introduced the notion of π-normal spaces and obtained their characterizations. In 2018, Kumar [12] introduced the concept of softly normal spaces and obtained their properties. In 2019, Kumar [13, 14] introduced the notions of silky normal and β^*g-normal spaces and obtained their characterizations. In 2021, Abbas [1] introduced the concept of h-open sets and obtained their properties. In 2022, Abdullah [2] introduced the concept of generalized h-closed sets and studied their properties.

In this paper, we introduce and study a new class of sets, called h generalized closed sets and relationships among closed, g-closed, gh-closed and hg-closed sets are investigated. Further, we introduce a new class of normal spaces, called h-normal spaces and obtain a characterization and preservation theorems of h-normal spaces.

2. Preliminaries

In what follows, spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated and $f : (X, \Im) \to (Y, \sigma)$ (or simply $f: X \to Y$) denotes a function f of a space $(X, \Im)$ into a space (Y, σ). Let A be a subset of a space X. The closure and the interior of A are denoted by cl(A) and int(A), respectively. A subset A of the topological space $(X, \Im)$ is said to be an h-open [1] set if for every non-empty set U in X, $U \neq X$ and $U \in \Im$, such that $A \subset int(A \cup U)$. The complement of the h-open set is called h-closed. The collection of all h-open (resp. h-closed) sets is denoted by h-O(X) (resp. h-C(X)).

2.1. Definition. A subset A of a topological space $(X, \Im)$ is said to be
(1) **g-closed** [**15**] if $cl(A) \subset U$ whenever $A \subset U$ and $U \in \Im$.
(2) **generalized h-closed** [2] (briefly gh-closed) if $h\text{-}cl(A) \subset U$ whenever $A \subset U$ and $U \in \Im$.
(3) **h-generalized-closed** (briefly hg-closed) if $h\text{-}cl(A) \subset U$ whenever $A \subset U$ and $U \in h\text{-}O(X)$.

2.2. Remark. We have the following implications for the properties of subsets:

closed	→	g-closed
↓		↓
h-closed → hg-closed	→	gh-closed

Where none of the implications are reversible.

2.3. Example. Let X = {a, b, c} and $\Im$ = {ϕ, X, {a}, {a, b}, {a, c}}. Then the subset A = {a} is h-closed as well as gh-closed but not g-closed in (X, $\Im$).

2.4. Example. Let X = {a, b, c, d} and $\Im$ = {ϕ, X, {a}, {a, b}, {a, c}, {a, b, c}}. Then the subset A = {a} is h-closed but not closed in (X, $\Im$).

2.5. Example. Let X = {a, b, c} and $\Im$ = {ϕ, X, {a}}. Then the subset A = {a} is h-closed as well as gh-closed in (X, $\Im$). But A = {b} is gh-closed but not h-closed in (X, $\Im$).

3. h-Normal Spaces

3.1. Definition. A space X is said to be **h-normal** if, for any pair of disjoint closed sets A and B of X, there exist disjoint h-open sets U and V such that A ⊂ U and B ⊂ V.

3.2. Remark. The following diagram holds for a topological space (X, $\Im$):

normal → h-normal

Where none of the implications are reversible.

3.3. Example. Let X = {a, b, c} and $\Im$ = {ϕ, X, {a}, {a, b}, {a, c}}. Consider the disjoint closed sets A = {b} and B = {c}. Then there exist disjoint h-open sets U = {b} and V = {c} such that A ⊂ U and B ⊂ V. Hence the topological space (X, $\Im$) is h-normal but not normal, since U and V are not open sets.

3.4. Example. Let X = {a, b, c, d} and $\Im$ = {ϕ, X, {a}, {a, b}, {a, c}, {a, b, c}}. Consider the disjoint closed sets A = ϕ and B = {d}. Then there exist disjoint h-open sets U = {a} and V = {b, c, d} such that A ⊂ U and B ⊂ V. Hence the topological space (X, $\Im$) is h-normal but not normal since the set V is not open.

3.5. Example. Let X = {a, b, c} and $\Im$ = {ϕ, X, {a}}. Consider the disjoint closed sets A = ϕ and B = {b, c}. Then there exist disjoint h-open sets U = {a} and V = {b, c} such that A ⊂ U and B ⊂ V. Hence the topological space (X, $\Im$) is h-normal but not normal since V is not an open set.

3.6. Example. Let X = {a, b, c, d} and $\Im$ = {ϕ, X, {b, d}, {a, b, d}, {b, c, d}}. Consider the disjoint closed sets A = ϕ and B = {c}. Then there exist disjoint h-open sets U = {b} and V = {a, c, d} such that A ⊂ U and B ⊂ V. Hence the topological space (X, $\Im$) is h-normal but not normal since the sets U and V are not open.

3.7. Theorem. For a space X, the following are equivalent:
(1) X is h-normal,
(2) For every pair of open sets U and V whose union is X, there exist h-closed sets A and B such that A ⊂ U, B ⊂ V and A ∪ B = X,
(3) For every closed set H and every open set K containing H, there exists an h-open set U such that H ⊂ U ⊂ h-cl(U) ⊂ K.
Proof. (1) ⇒ (2): Let U and V be a pair of open sets in an h-normal space X such that X = U ∪ V. Then X − U, X − V are disjoint closed sets. Since X is h-normal, there exist disjoint h-open sets U_1 and V_1 such that X − U ⊂ U_1 and X − V ⊂ V_1. Let A = X − U_1, B = X − V_1. Then A and B are h-closed sets such that A ⊂ U, B ⊂ V and A ∪ B = X.

(2) ⇒ (3): Let H be a closed set and K be an open set containing H. Then X − H, and K are open sets whose union is X. Then by (2), there exist h-closed sets M_1 and M_2 such that M_1 ⊂ X − H and M_2 ⊂ K and M_1 ∪ M_2 = X. Then H ⊂ X − M_1, X − K ⊂

$X - M_2$ and $(X - M_1) \cap (X - M_2) = \phi$. Let $U = X - M_1$ and $V = X - M_2$. Then U and V are disjoint h-open sets such that $H \subset U \subset X - V \subset K$. As $X - V$ is an h-closed set, we have $h\text{-}cl(U) \subset X - V$ and $H \subset U \subset h\text{-}cl(U) \subset K$.

(3) ⇒ (1): Let H_1 and H_2 be any two disjoint closed sets of X. Put $K = X - H_2$, then $H_2 \cap K = \phi$. $H_1 \subset K$, where K is an open set. Then by (3), there exists an h-open set U of X such that $H_1 \subset U \subset h\text{-}cl(U) \subset K$. It follows that $H_2 \subset X - h\text{-}cl(U) = V$, say, then V is h-open and $U \cap V = \phi$. Hence H_1 and H_2 are separated by h-open sets U and V. Therefore, X is h-normal.

4. Some Related Generalized Functions with h-Normal Spaces

4.1. Definition. A function $f: X \to Y$ is called
(1) **R-map** [**5**] if $f^{-1}(V)$ is regular open in X for every regular open set V of Y,
(2) **completely continuous** [**3**] if $f^{-1}(V)$ is regular open in X for every open set V of Y,
(3) **rc-continuous** [**10**] if for each regular closed set F in Y, $f^{-1}(F)$ is regular closed in X.

4.2. Definition. A function $f: X \to Y$ is called
(1) **strongly h-open** if $f(U) \in h\text{-}O(Y)$ for each $U \in h\text{-}O(X)$,
(2) **strongly h-closed** if $f(U) \in h\text{-}C(Y)$ for each $U \in h\text{-}C(X)$,
(3) **almost h-irresolute** if for each x in X and each h-neighbourhood V of f(x), $h\text{-}cl(f^{-1}(V))$ is a h-neighbourhood of x.

4.3. Theorem. A function $f: X \to Y$ is strongly h-closed if and only if for each subset A in Y and for each h-open set U in X containing $f^{-1}(A)$, there exists an h-open set V containing A such that $f^{-1}(V) \subset U$.

Proof. (⇒): Suppose that f is strongly h-closed. Let A be a subset of Y and $U \in h\text{-}O(X)$ containing $f^{-1}(A)$. Put $V = Y - f(X - U)$, then V is an h-open set of Y such that $A \subset V$ and $f^{-1}(V) \subset U$.

(⇐): Let K be any h-closed set of X. Then $f^{-1}(Y - f(K)) \subset X - K$ and $X - K \in h\text{-}O(X)$. There exists an h-open set V of Y such that $Y - f(K) \subset V$ and $f^{-1}(V) \subset X - K$. Therefore, we have $f(K) \supset Y - V$ and $K \subset f^{-1}(Y - V)$. Hence, we obtain $f(K) = Y - V$, and f(K) is h-closed in Y . This shows that f is strongly h-closed.

4.4. Lemma. For a function $f: X \to Y$, the following are equivalent:
(1) f is almost h-irresolute,
(2) $f^{-1}(V) \subset \text{-}int(h\text{-}cl(f^{-1}(V)))$ for every $V \in h\text{-}O(Y)$.

4.5. Theorem. A function $f: X \to Y$ is almost -irresolute if and only if $f(h\text{-}cl(U)) \subset h\text{-}cl(f(U))$ for every $U \in h\text{-}O(X)$.

Proof. (⇒): Let $U \in h\text{-}O(X)$. Suppose $y \notin h\text{-}cl(f(U))$. Then there exists $V \in h\text{-}O(Y)$ such that $V \cap f(U) = \phi$. Hence, $f^{-1}(V) \cap U = \phi$. Since $U \in h\text{-}O(X)$, we have $h\text{-}int(h\text{-}cl(f^{-1}(V))) \cap h\text{-}cl(U) = \phi$. Then by Lemma 4.4, $f^{-1}(V) \cap h\text{-}cl(U) = \phi$ and hence $V \cap f(h\text{-}cl(U)) = \phi$. This implies that $y \notin f(h\text{-}cl(U))$.

(⇐): If $V \in h\text{-}O(Y)$, then $M = X - h\text{-}cl(f^{-1}(V)) \in h\text{-}O(X)$. By hypothesis, $f(h\text{-}cl(M)) \subset h\text{-}cl(f(M))$ and hence $X - h\text{-}int(h\text{-}cl(f^{-1}(V))) = h\text{-}cl(M) \subset f^{-1}(h\text{-}cl(f(M))) \subset f^{-1}(h\text{-}cl(f(X - f^{-1}(V)))) \subset f^{-1}(h\text{-}cl(Y - V)) = f^{-1}(Y - V) = X - f^{-1}(V)$. Therefore, $f^{-1}(V) \subset h\text{-}int(h\text{-}cl(f^{-1}(V)))$. By Lemma 4.4, f is almost h-irresolute.

4.6. Theorem. If $f: X \to Y$ is a strongly h-open continuous almost h-irresolute function from an h-normal space X onto a space Y, then Y is h-normal.

Proof. Let A be a closed subset of Y and B be an open set containing A. Then by continuity of f, $f^{-1}(A)$ is closed, and $f^{-1}(B)$ is an open set of X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is h-normal, there exists an h-open set U in X such that $f^{-1}(A) \subset U \subset h\text{-}cl(U) \subset f^{-1}(B)$ by Theorem 3.7. Then, $f(f^{-1}(A)) \subset f(U) \subset f(h\text{-}cl(U)) \subset f(f^{-1}(B))$. Since f is a strongly h-open, almost h-irresolute surjection, we obtain $A \subset f(U) \subset h\text{-}cl(f(U)) \subset B$. Then again, by Theorem 3.7, the space Y is h-normal.

4.7. Theorem. If $f: X \to Y$ is a strongly h-closed continuous function from an h-normal space X onto a space Y, then Y is h-normal.

Proof. Let M_1 and M_2 be disjoint closed sets. Then $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are closed sets. Since X is h-normal, then there exist disjoint h-open sets U and V such that $f^{-1}(M_1) \subset U$ and $f^{-1}(M_2) \subset V$. By Theorem 4.3, there exist h-open sets A and B such that $M_1 \subset A$, $M_2 \subset B$, $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Also, A and B are disjoint. Thus, Y is h-normal.

5. Some Generalized h-Closed Functions

5.1. Definition. A function f: X → Y is said to be
(1) **h-closed** [**1**] if f(A) is h-closed in Y for each closed set A of X,
(2) **hg-closed** if f(A) is hg-closed in Y for each closed set A of X,
(3) **gh-closed** if f(A) is gh-closed in Y for each closed set A of X.

5.2. Definition. A function f: X → Y is said to be
(1) **quasi h-closed** if f(A) is closed in Y for each A ∈ h-C(X),
(2) **h-hg-closed** if f(A) is hg-closed in Y for each A ∈ h-C(X),
(3) **h-gh-closed** if f(A) is gh-closed in Y for each A ∈ h-C(X),
(4) **almost gh-closed** if f(A) is gh-closed in Y for each A ∈ R-C(X).

5.3. Definition. A function f: X → Y is said to be h-gh-continuous if f^{-1}(K) is gh-closed in X for every K ∈ hC(Y).

5.4. Definition. A function f: X → Y is said to be h-irresolute [1] if f^{-1}(V) ∈ h-O(X) for every V ∈ h-O(Y).

5.5. Theorem. Let f: X → Y and g: Y → Z be functions. Then
(1) the composition gof: X → Z is h-gh-closed if f is h-gh-closed and g is continuous h-gh-closed.
(2) the composition gof: X → Z is h-gh-closed if f is strongly h-closed and g is h-gh-closed.
(3) the composition gof: X → Z is h-gh-closed if f is quasi h-closed and g is gh-closed.

5.6. Theorem. Let f: X → Y and g: Y → Z be functions, and let the composition gof: X → Z be h-gh-closed. If f is an h-irresolute surjection, then g is h-hg-closed.

Proof. Let K ∈ h-C(Y). Since f is h-irresolute and surjective, f^{-1}(K) ∈ h-C(X) and (gof)(f^{-1}(K)) = g(K). Hence, g(K) is gh-closed in Z, and hence g is h-gh-closed.

5.7. Lemma. A function f: X → Y is h-gh-closed if and only if for each subset B of Y and each U ∈ h-O(X) containing f^{-1}(B), there exists a g h-open set V of Y such that B ⊂ V and f^{-1}(V) ⊂ U.

Proof. (⇒): Suppose that f is h-gh-closed. Let B be a subset of Y and U ∈ h-O(X) containing f^{-1}(B). Put V = Y − f(X − U), then V is a gh-open set of Y such that B ⊂ V and f^{-1}(V) ⊂ U.

(⇐): Let K be any h-closed set of X. Then f^{-1}(Y − f(K)) ⊂ X − K and X − K ∈ h-O(X). There exists a gh-open set V of Y such that Y − f(K) ⊂ V and f^{-1}(V) ⊂ X − K. Therefore, we have f(K) ⊃ Y − V and K ⊂ f^{-1}(Y − V). Hence, we obtain f(K) = Y − V, and f(K) is gh-closed in Y . This shows that f is h-gh-closed.

5.8. Theorem. If f: X → Y is continuous h-gh-closed, then f(H) is gh-closed in Y for each gh-closed set H of X.
Proof. Let H be any g-closed set of X and V, an open set of Y containing f(H). Since f^{-1}(V) is an open set of X containing H, h-cl(H) ⊂ f^{-1}(V) and hence f(h-cl(H)) ⊂ V . Since f is h-gh-closed and h-cl(H) ∈ hC(X), we have h-cl(f(H)) ⊂ h-cl(f(h-cl(H))) ⊂ V . Therefore, f(H) is gh-closed in Y.

5.9. Remark. Every h-irresolute function is h-gh-continuous but not conversely.

5.10. Theorem. A function f: X → Y is h-gh-continuous if and only if f^{-1}(V) is gh-open in X for every V ∈ h-O(Y).

5.11. Theorem. If f: X → Y is closed h-gh-continuous, then f^{-1}(K) is gh-closed in X for each gh-closed set K of Y.
Proof. Let K be a gh-closed set of Y and U an open set of X containing f^{-1}(K). Put V = Y − f(X − U), then V is open in Y, K ⊂ V, and f^{-1}(V) ⊂ U. Therefore, we have h-cl(K) ⊂ V and hence f^{-1}(K) ⊂ f^{-1}(h-cl(K)) ⊂ f^{-1}(V) ⊂ U. Since f is h-gh-continuous, f^{-1}(h-cl(K)) is gh-closed in X and hence h-cl(f^{-1}(K)) ⊂ h-cl(f^{-1}(h-cl(K))) ⊂ U. This shows that f^{-1}(K) is gh-closed in X.

5.12. Corollary. If f: X → Y is closed h-irresolute, then f^{-1}(K) is gh-closed in X for each gh-closed set K of Y.

5.13. Theorem. If f: X → Y is an open h-gh-continuous bijection, then f^{-1}(K) is gh-closed in X for every gh-closed set K of Y.

Proof. Let K be a gh-closed set of Y and U an open set of X containing $f^{-1}(K)$. Since f is an open surjective, $K = f(f^{-1}(K)) \subset f(U)$ and f(U) is open. Therefore, $h\text{-}cl(K) \subset f(U)$. Since f is injective, $f^{-1}(K) \subset f^{-1}(h\text{-}cl(K)) \subset f^{-1}(f(U)) = U$. Since f is h-gh-continuous, $f^{-1}(h\text{-}cl(K))$ is gh-closed in X and hence $h\text{-}cl(f^{-1}(K)) \subset h\text{-}cl(f^{-1}(h\text{-}cl(K))) \subset U$. This shows that $f^{-1}(K)$ is gh-closed in X.

5.14. Theorem. Let $f: X \to Y$ and $g: Y \to Z$ be functions, and let the composition $gof: X \to Z$ be h-gh-closed. If g is an open h-gh-continuous bijection, then f is h-gh-closed.
Proof. Let $H \in h\text{-}C(X)$. Then (gof)(H) is gh -closed in Z and $g^{-1}((gof)(H)) = f(H)$. By Theorem 5.13, f(H) is g h-closed in Y, and hence f is h-gh-closed.

5.15. Theorem. Let $f: X \to Y$ and $g: Y \to Z$ be functions, and let the composition $gof: X \to Z$ be h-gh-closed. If g is a closed h-gh-continuous injection, then f is h-gh-closed.
Proof. Let $H \in hC(X)$. Then (gof)(H) is gh-closed in Z and $g^{-1}((gof)(H)) = f(H)$. By Theorem 5.11, f(H) is gh-closed in Y, and hence f is h-gh-closed.

6. Characterizations and Preservation Theorems of h-Normal Spaces

6.1. Theorem. For a topological space X, the following are equivalent:
(a) X is h-normal,
(b) for any pair of disjoint closed sets A and B of X, there exist disjoint gh-open sets U and V of X such that $A \subset U$ and $B \subset V$,
(c) for each closed set A and each open set B containing A, there exists a gh-open set U such that $cl(A) \subset U \subset h\text{-}cl(U) \subset B$,
(d) for each closed A and each g-open set B containing A, there exists an h-open set U such that $A \subset U \subset h\text{-}cl(U) \subset int(B)$,
(e) for each closed A and each g-open set B containing A, there exists a gh-open set G such that $A \subset G \subset h\text{-}cl(G) \subset int(B)$,
(f) for each g-closed set A and each open set B containing A, there exists an h-open set U such that $cl(A) \subset U \subset h\text{-}cl(U) \subset B$,
(g) for each g-closed set A and each open set B containing A, there exists a gh-open set G such that $cl(A) \subset G \subset h\text{-}cl(G) \subset B$.

Proof. (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c): Since every h-open set is gh-open, it is obvious.

(d) $\Rightarrow$ (e) $\Rightarrow$ (c) and (f) $\Rightarrow$ (g) $\Rightarrow$ (c): Since every closed (resp. open) set is g-closed (resp. g-open), it is obvious.

(c) $\Rightarrow$ (e): Let A be a closed subset of X and B be a g-open set such that $A \subset B$. Since B is g-open, and A is closed, $A \subset int(A)$. Then, there exists a gh-open set U such that $A \subset U \subset h\text{-}cl(U) \subset int(B)$.

(e) $\Rightarrow$ (d): Let A be any closed subset of X and B be a g-open set containing A. Then there exists a gh-open set G such that $A \subset G \subset h\text{-}cl(G) \subset int(B)$. Since G is gh-open, $A \subset h\text{-}int(G)$. Put $U = h\text{-}int(G)$, then U is h-open and $A \subset U \subset h\text{-}cl(U) \subset int(B)$.

c) $\Rightarrow$ (g): Let A be any g-closed subset of X and B be an open set such that $A \subset B$. Then $cl(A) \subset B$. Therefore, there exists a gh-open set U such that $cl(A) \subset U \subset h\text{-}cl(U) \subset B$.

(g) $\Rightarrow$ (f): Let A be any g-closed subset of X and B be an open set containing A. Then there exists a gh-open set G such that $cl(A) \subset G \subset h\text{-}cl(G) \subset B$. Since G is gh-open and $cl(A) \subset G$, we have $cl(A) \subset h\text{-}int(G)$, put $U = h\text{-}int(G)$, then U is h-open and $cl(A) \subset U \subset h\text{-}cl(U) \subset B$.

6.2. Theorem. If $f: X \to Y$ is a continuous quasi h-closed surjection, and X is h-normal, then Y is normal.
Proof. Let M_1 and M_2 be any disjoint closed sets of Y. Since f is continuous, $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are disjoint closed sets of X. Since X is h-normal, there exist disjoint $U_1, U_2 \in h\text{-}O(X)$ such that $f^{-1}(M_i) \subset U_i$ for i = 1, 2. Put $V_i = Y - f(X - U_i)$; then V_i is open in Y, $M_i \subset V_i$ and $f^{-1}(V_i) \subset U_i$ for i = 1, 2. Since $U_1 \cap U_2 = \phi$ and f is surjective; we have $V_1 \cap V_2 = \phi$. This shows that Y is normal.

6.3. Lemma. A subset A of a space X is gh-open if and only if $F \subset h\text{-}int(A)$ whenever F is closed and $F \subset A$.

6.4. Theorem. Let $f: X \to Y$ be a closed h-gh-continuous injection. If Y is h-normal, then X is h-normal.
Proof. Let N_1 and N_2 be disjoint closed sets of X; since f is a closed injection, $f(N_1)$ and $f(N_2)$ are disjoint closed sets of Y . By the h-normality of Y, there exist disjoint $V_1, V_2 \in h\text{-}O(Y)$ such that $f(N_i) \subset V_i$ for i = 1, 2. Since f is h-gh-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint gh-open sets of X and $N_i \subset f^{-1}(V_i)$ for i = 1, 2. Now, put $U_i = h\text{-}int(f^{-1}(V_i))$ for i = 1, 2. Then, $U_i \in h\text{-}O(X)$, $N_i \subset U_i$ and $U_1 \cap U_2 = \phi$. This shows that X is h-normal.

6.5. Corollary. If f: $X \to Y$ is a closed h-irresolute injection, and Y is h-normal, then X is h-normal.
Proof. This is an immediate consequence since every h-irresolute function is h-gh-continuous.

6.6. Lemma. A function f: $X \to Y$ is almost gh-closed if and only if for each subset B of Y and each U $\in$ R-O(X) containing $f^{-1}(B)$, there exists a gh-open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

6.7. Lemma. If f: $X \to Y$ is almost gh-closed, then for each closed set M of Y and each U $\in$ R-O(X) containing $f^{-1}(M)$, there exists V $\in$ h-O(Y) such that $M \subset V$ and $f^{-1}(V) \subset U$.

6.8. Theorem. Let f: $X \to Y$ be a continuous, almost gh-closed surjection. If X is normal, then Y is h-normal.
Proof. Let M_1 and M_2 be any disjoint, closed sets of Y. Since f is continuous, $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are disjoint closed sets of X. By the normality of X, there exist disjoint open sets U_1 and U_2 such that $f^{-1}(M_i) \subset U_i$, where i = 1, 2. Now, put $G_i = int(cl(U_i))$ for i = 1, 2, then $G_i \in$ R-O(X), $f^{-1}(M_i) \subset U_i \subset G_i$ and $G_1 \cap G_2 = \phi$. By Lemma 6.7, there exists $V_i \in$ h-O(Y) such that $M_i \subset V_i$ and $f^{-1}(V_i) \subset G_i$, where i = 1, 2. Since $G_1 \cap G_2 = \phi$ and f is surjective, we have $V_1 \cap V_2 = \phi$. This shows that Y is h-normal.

6.9. Corollary. If f: $X \to Y$ is a continuous h-closed surjection, and X is normal, then Y is h-normal.

7. Conclusion

In this paper, we introduce and study a new class of sets, called h generalized closed sets and relationships among closed, g-closed, gh-closed and hg-closed sets are investigated. Further, we introduce a new class of normal spaces, called h-normal spaces and obtain a characterization of h-normal spaces. Moreover, we define the forms of generalized h-closed, h-generalized closed and some h-generalized continuous functions. By utilizing these functions, we study properties of the forms of generalized h-closed functions and preservation theorems for h-normal spaces. This idea can be extended to bitopological, ordered topological, ordered bitopological and fuzzy topological spaces etc.

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