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Original Article

Annihilator in a Distributive q-Lattice

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Abstract *- In this paper, we define an annihilator (a: b) in a distributive q-lattice A, and we prove (a:b), which we call an annihilator of 'a' relative to 'b', is an ideal of A. Also, we observe some basic properties of these annihilators.*

Keywords *- Distributive q-lattice, An annihilator (a: b), Ideal element.*

1. Preliminaries

Ivan Chajda [1] defined the concept of q-lattice A and defined distributive q-lattice.

Definition: 1.1

An algebra (A, ∨, ∧) whose binary operations ∨, ∧ satisfy the following is called a q-lattice.

(1) $a_1 \vee b_1 = b_1 \vee a_1$; $a_1 \wedge b_1 = b_1 \wedge a_1$ (commutativity)

(2) $a_1 \vee (b_1 \vee c_1) = (a_1 \vee b_1) \vee c_1$; $a_1 \wedge (b_1 \wedge c_1) = (a_1 \wedge b_1) \wedge c_1$ (associatativity)

(3) $a_1 \vee (a_1 \wedge b_1) = a_1 \vee a_1$; $a_1 \wedge (a_1 \vee b_1) = a_1 \wedge a_1$ (weak- absorption)

(4) $a_1 \vee b_1 = a_1 \vee (b_1 \vee b_1)$; $a_1 \wedge b_1 = a_1 \wedge (b_1 \wedge b_1)$ (weak- idempotence)

(5) $a_1 \vee a_1 = a_1 \wedge a_1$ (equalization)

G.C. Rao et al. [2] defined the concept of ideal in a distributive q-lattice A.

Definition: 1.2

A q-lattice (A, ∨, ∧) is distributive if

$a_1 \vee (b_1 \wedge c_1) = (a_1 \vee b_1) \wedge (a_1 \vee c_1)$ for all $a_1, b_1, c_1 \in A$.

In [2], it is proved that If A be a distributive q-lattice, then

$a_1 \wedge (b_1 \vee c_1) = (a_1 \wedge b_1) \vee (a_1 \wedge c_1)$ for all $a_1, b_1, c_1 \in A$.

Definition: 1.3

A nonempty subset I_1 of a distributive q-lattice A is called an ideal of A if

(1) x_1 and $y_1 \in I_1$ implies $x_1 \vee y_1 \in I_1$

(2) $x_1 \in I_1$ and $a_1 \in A$ implies $a_1 \wedge x_1 \in I_1$

Annihilator in a Distributive q-lattice

G. C. Rao and M. Sambasiva Rao [3] defined ' annihilator ' as an Almost Distributive Lattice (ADLs) and derived some properties

In paper [4], we defined the annihilator in distributive q-lattice A and proved for any ideal I of distributive q-lattice A and a∈ A, the annihilator (a: I) is an ideal of A and derived some properties.

Here we define an annihilator (a: b) in a distributive q-lattice A, and we prove (a:b), which we call an annihilator of 'a' relative to 'b', is an ideal of A. Also, we observe some basic properties of these annihilators. Throughout this paper, we consider A to be a distributive q-lattice

Definition: 1.2.1

Definition: Ideal element of a distributive q-lattice :

Let A be a distributive q-lattice and 'b' be a special element of A satisfying property $b \vee b = b$, and for any element r of A, $r \wedge b = b$, then we call element 'b' as an ideal element of a distributive q-lattice A.

Definition: 1.2.2

Let A be distributive q-lattice and 'b' be an ideal element of A; then, for any element a of A, we define annihilator (a: b) of a relative to b as $(a: b) = \{x \in A / x \wedge a = b\}$

Definition: 1.2.3

If P be a nonempty set of a distributive q-lattice A, then we consider the annihilator (P: b) of P relative to an ideal element b as $t \in (P: b)$ then $(P: b) = \{x \in A / x \wedge a = b, \text{ for all } x \in P\}$

In the following theorem, we observed the annihilator (a_1: b) of $a_1 \in A$ relative to b is an ideal of A.

Throughout this paper, we consider 'b' to be an ideal element of a distributive q-lattice A.

Theorem: 1.2.4

If b be an ideal element of distributive q-lattice A, then the annihilator (a_1: b) of $a_1 \in A$ is an ideal of A.

Proof : (i) Let $x_1 \in (a_1: b)$ and $y_1 \in (a_1: b)$.

Implies $x_1 \wedge a_1 = b$ and $y_1 \wedge a_1 = b$.

Now to show $x_1 \vee y_1 \in (a_1: b)$.

Consider $(x_1 \vee y_1) \wedge a_1 = a_1 \wedge (x_1 \vee y_1)$ (∵ commutativity)

$= (a_1 \wedge x_1) \vee (a_1 \wedge y_1)$ (∵ A is a distributive q-lattice)

$= (x_1 \wedge a_1) \vee (y_1 \wedge a_1)$ (∵ commutativity)

$= b \vee b$

$= b$

implies $(x_1 \vee y_1) \wedge a_1 = b$, implies $x_1 \vee y_1 \in (a_1: b)$.

ii) Let $x_1 \in (a_1: b)$ and $r_1 \in A$.

Implies $x_1 \wedge a_1 = b$ and $r_1 \in A$.

To show $r_1 \wedge x_1 \in (a_1: b)$ means to show $(r_1 \wedge x_1) \wedge a_1 = b$.

As $(r_1 \wedge x_1) \wedge a_1 = r_1 \wedge (x_1 \wedge a_1)$.

Implies $r_1 \wedge (x_1 \wedge a_1) = r_1 \wedge b = b$

implies $r_1 \wedge x_1 \in (a_1: b)$

Implies (a_1: b) is an ideal of a distributive q-lattice A.

Theorem: 1.2.5

If b be an ideal element of distributive q-lattice A and P be a non-empty subset A. Then we have the following

$$\bigcap_{x \in P} (x: b) = (P: b)$$

Proof: Suppose $t \in (P: b)$

Then $t \wedge x = b$ for all $x \in P$, means $t \wedge x = b$ for all $x \in \{x\}$.

Hence $t \in (x: b)$ for all $x \in P$.

Therefore $t \in \bigcap_{x \in P} (x: b)$.

Therefore $(P: b) \subseteq \bigcap_{x \in P} (x: b)$.

And reversely, let $t \in \bigcap_{x \in P} (x: b)$

Implies $t \in (x: b)$ for all $x \in P$

implies $t \wedge x = b$ for all $x \in P$

implies $t \in (P: b)$.

Therefore $\bigcap_{x \in P} (x: b) \subseteq (P: b)$.

Hence $\bigcap_{x \in P} (x: b) = (P: b)$.

Theorem: 1.2.6

If (a_1: b) and (b_1: b) are ideals of a distributive q-lattice A, then $(a_1: b) \cap (b_1: b)$ is an ideal of A.

Proof : Let $x , y \in (a_1: b) \cap (b_1: b)$

Then $x \in (a_1: b)$, $x \in (b_1: b)$ and $y \in (a_1: b)$, $y \in (b_1: b)$

Implies $x \wedge a_1 = b$, $x \wedge b_1 = b$ and $y \wedge a_1 = b$, $y \wedge b_1 = b$

Now to show $x \vee y \in (a_1: b) \cap (b_1: b)$

Consider $(x \vee y) \wedge a_1 = a_1 \wedge (x \vee y)$ (∵ commutativity)

$= (a_1 \wedge x) \vee (a_1 \wedge y)$ (∵ A is a distributive q-lattice)

$= (x \wedge a_1) \vee (y \wedge a_1)$ (∵ commutativity)

$= b \vee b$

$= b$

therefore $(x \vee y) \in (a_1: b)$

similarly

Consider $(x \vee y) \wedge b_1 = b_1 \wedge (x \vee y)$ ($\because$ commutativity)

$= (b_1 \wedge x) \vee (b_1 \wedge y)$ ($\because$ A is a distributive q-lattice)

$= (x \wedge b_1) \vee (y \wedge b_1)$ ($\because$ commutativity)

$= b \vee b$

$= b$

therefore $(x \vee y) \in (b_1: b)$

Implies $x \vee y \in (a_1: b) \cap (b_1: b)$

Now let $x \in (a_1: b) \cap (b_1: b)$ and $r \in A$.

Then $x \in (a_1: b)$ and $x \in (b_1: b)$ since $(a_1: b)$ and $(b_1: b)$ are ideals

Then $r \wedge x \in (a_1: b)$ and $r \wedge x \in (b_1: b)$

Therefore $r \wedge x \in (a_1: b) \cap (b_1: b)$.

Hence $(a_1: b) \cap (b_1: b)$ is an ideal of a distributive q-lattice A.

Theorem: 1.2.7

If b be an ideal element of distributive q-lattice A and $a_1, b_1 \in A$, then

(i) If $a_1 \wedge b_1 = b_1$ then $(a_1: b) \subseteq (b_1: b)$

(ii) $(a_1 \wedge b_1: b) = (b_1 \wedge a_1: b)$

(iii) $(a_1 \vee b_1: b) = (b_1 \vee a_1: b)$

(iv) If property $(x \wedge a_1) \vee (x \wedge b_1) = b$ implies $x \wedge a_1 = b$ and $x \wedge b_1 = b$ then $(a_1 \vee b_1: b) = (a: J) \cap (b: J)$

(v) $(a_1: b) = A$ if and only if a_1 has property for every $x \in A$, $x \wedge a_1 = b$

Proof : (i) Suppose $a_1 \wedge b_1 = b_1$

Let $x \in (a_1: b)$ implies $x \wedge a_1 = b$

Consider $x \wedge b_1 = x \wedge (a_1 \wedge b_1)$

$= (x \wedge a_1) \wedge b_1$ $\because$ commutativity

$= b \wedge b_1$ $\because$ associativity

$= b$

Implies $x \in (b_1: b)$

Therefore $(a_1: b) \subseteq (b_1: b)$

(ii) Let $x \in (a_1 \wedge b_1 : b)$

$\Leftrightarrow x \wedge (a_1 \wedge b_1) = b$

$\Leftrightarrow x \wedge (b_1 \wedge a_1) = b$

$\Leftrightarrow x \in (b_1 \wedge a_1: b)$.

Therefore $(a_1 \wedge b_1: b) = (b_1 \wedge a_1: b)$

(iii) Let $x \in (a_1 \vee b_1 : b)$

$\Leftrightarrow x \wedge (a_1 \vee b_1) = b$

$\Leftrightarrow x \wedge (b_1 \vee a_1) = b$ $\because$ commutativity

$\Leftrightarrow x \in (b_1 \vee a_1: b)$.

Therefore $(a_1 \vee b_1: b) = (b_1 \vee a_1: b)$

(iv) Let $x \in (a_1 \vee b_1: b)$

Implies $x \wedge (a_1 \vee b_1) = b$

implies $(x \wedge a_1) \vee (x \wedge b_1) = b$, by given property

implies $x \wedge a_1 = b$ and $x \wedge b_1 = b$

implies $x \in (a_1: b)$ and $x \in (b_1: b)$

implies $x \in (a_1: b) \cap (b_1: b)$

implies $(a_1 \vee b_1: b) \subseteq (a_1: b) \cap (b_1: b)$

Conversely, let $x \in (a_1: b) \cap (b_1: b)$ then $x \in (a_1: b)$ and $x \in (b_1: b)$.

Implies $x \wedge a_1 = b$ and $x \wedge b_1 = b$

Now consider $x \wedge (a_1 \vee b_1) = (x \wedge a_1) \vee (x \wedge b_1)$

Since $x \wedge a_1 = b$, $x \wedge b_1 = b$

therefore $(x \wedge a_1) \vee (x \wedge b_1) = b \vee b = b$

Hence $x \wedge (a_1 \vee b_1) = b$

Therefore $x \in (a_1 \vee b_1: b)$

Thus $(a: J) \cap (b: J) \subseteq (a_1 \vee b_1: b)$.

Therefore $(a_1 \vee b_1: b) = (a: J) \cap (b: J)$

(v) Consider $a_1 \in A$, clearly $(a_1: b) \subseteq A$.

Now let $x \in A$, and as a_1 has property for every $x \in A$, $x \wedge a_1 = b$ implies

$x \wedge a_1 = b$
$x \in (a_1: b)$
therefore $A \subseteq (a_1: b)$
Hence $(a_1: b) = A$ if and only if a_1 has property for every $x \in A$, $x \wedge a_1 = b$

2. Conclusion

This study develops the concept of an annihilator (a: b) in a distributive q-lattice A called annihilator of 'a' relative to 'b' for a special element 'b'. We proved annihilator (a: b) is an ideal of A. From this study, we defined annihilator (P:b) for a non-empty set P of a distributive q-lattice A, and we can extend the study for properties of an annihilator (a: b) and an annihilator (P:b).

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