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Original Article

Minimum Equitable Eccentric Dominating Energy of Graphs

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Abstract *- For a graph $G = (V, E)$, the minimum equitable eccentric dominating energy $\mathbb{M}_{eqed}(G)$ is the sum of the eigen values obtained from the minimum equitable eccentric dominating $n \times n$ matrix $\mathbb{M}_{eqed}(G) = (m_{ij})$. In this paper, $\mathbb{M}_{eqed}(G)$ of standard graphs are computed. Properties, upper and lower bounds for $\mathbb{M}_{eqed}(G)$ are established.*

Keywords *- Equitable eccentric dominating eigen values, Minimum equitable eccentric dominating set, Minimum equitable eccentric dominating energy.*

1. Introduction

In 1978 I. Gutman[1] introduced energy of a graph. Inspired by Gutman many authors have explored different types of energy in graph theory. M. R. Rajesh Kanna et al[2] found the minimum dominating energy of a graph. For a graph $G = (V, E)$, let $A = (a_{ij})$ be the minimum dominating matrix defined by

$$(a_{ij}) = \begin{cases} 1, if\ v_i v_j \in E, \\ 1, if\ i = j\ and\ v_i \in D, \\ 0, otherwise \end{cases}$$

and $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigen values of A. The minimum dominating energy $E_D(G) = \sum_{i=1}^{n} |\lambda_i|$. Eccentric domination was introduced by T. N. Janakiraman et al[3] in 2010. For a graph $G = (V, E)$, a set $S \subseteq V$ is said to be a dominating set, if every vertex in $V - S$ is adjacent to some vertex in S. The eccentricity $e(v)$ of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v): u \in V\}$. For a vertex v, each vertex at a distance $e(v)$ from v is an eccentric vertex. Eccentric set of a vertex v is defined as $E(v) = \{u \in V / d(u, v) = e(v)\}$. A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric vertex of v in D. An eccentric dominating set with minimum cardinality is called a minimum eccentric dominating set. The eccentric domination number $\gamma_{ed}(G)$ of a graph G equals the minimum cardinality of an eccentric dominating set. Swaminathan et al[4] introduced equitable domination number in graphs. Riyaz Ur Rehman A and A Mohamed Ismayil[5] introduced equitable eccentric domination in graphs. An eccentric dominating set $S \subseteq V(G)$ is called an equitable eccentric dominating set(EQED-set) if for every $v \in V - S$ there exist at least one vertex $u \in S$ such that $vu \in E(G)$ and $|d(v) - d(u)| \leq 1$.

An equitable eccentric dominating set S is called a minimal equitable eccentric dominating set if no proper subset of S is equitable eccentric dominating set. The equitable eccentric domination number $\gamma_{eqed}(G)$ of a graph G is the minimum cardinality among the minimal equitable eccentric dominating sets of G. Inspired by Kanna et al we introduce minimum equitable eccentric dominating energy $\mathbb{E}_{eqed}(G)$ of graphs. In this paper we find $\mathbb{E}_{eqed}(G)$ of standard graphs.

2. The Minimum Equitable Eccentric Dominating Energy-$\mathbb{E}_{eqed}(G)$

In this section, the minimum equitable eccentric dominating matrix and its energy are defined. Minimum equitable eccentric dominating energy of some standard graphs are obtained.

Definition 2.1: Let $= (V, E)$ be a simple graph where $V(G) = \{v_1, v_2, \ldots v_n / n \in \mathbb{N}\}$ is the set of vertices and E is the set of edges. Let D be a minimum equitable eccentric dominating set of G then the minimum equitable eccentric dominating matrix of G is a $n \times n$ defined by $\mathbb{M}_{eqed}(G) = (m_{ij})$, where

$$(m_{ij}) = \begin{cases} 1, if\ |\deg(v_i) - \deg(v_j)| \leq 1, (v_i, v_j) \in E(G)\ and\ v_i \in E(v_j)\ or\ v_j \in E(v_i), \\ 1, if\ i = j\ and\ v_i \in D, \\ 0, otherwise \end{cases}$$

Definition 2.2: The characteristic polynomial of $\mathbb{M}_{eqed}(G)$ is defined by $\mathcal{G}_n(G, \psi) = \det\,(\mathbb{M}_{eqed}(G) - \psi I)$.

Definition 2.3: The minimum equitable eccentric dominating eigen values of G are the eigen values of $\mathbb{M}_{eqed}(G)$. Since $\mathbb{M}_{eqed}(G)$ is symmetric and real, the eigen values are real. We label the eigen values in non-increasing order $\psi_1 \geq \psi_2 \geq \cdots \geq \psi_n$.

Definition 2.4: The minimum equitable eccentric dominating energy of G is defined by $\mathbb{E}_{eqed}(G) = \sum_{i=1}^{n} |\psi_i|$.

Remark 2.1: The trace of $\mathbb{M}_{eqed}(G)$=Equitable eccentric domination number.

Example 2.1:

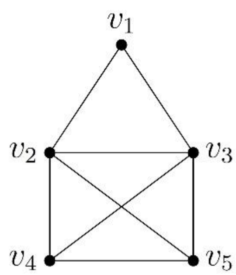


Figure 2.1. House X graph

Vertex	Eccentricity $e(v)$	Eccentric vertex $E(v)$
v_1	2	v_4, v_5
v_2	1	v_1, v_3, v_4, v_5
v_3	1	v_1, v_2, v_4, v_5
v_4	2	v_1
v_5	2	v_1

The minimum equitable eccentric dominating sets of an 'House X graph' are $D_1 = \{v_1, v_2\}$, $D_2 = \{v_1, v_3\}$, $D_3 = \{v_1, v_4\}$ and $D_4 = \{v_1, v_5\}$.

1. $D_1 = \{v_1, v_2\}$,

$$\mathbb{M}_{eqed}(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G, \psi) = -\psi^5 + 2\psi^4 + 4\psi^3 - 3\psi^2 - 2\psi$.
Minimum equitable eccentric dominating eigen values are $\psi_1 \approx 2.9354$, $\psi_2 \approx 1$, $\psi_3 \approx 0$, $\psi_4 \approx -0.4626$, $\psi_5 \approx -1.4728$.
Minimum equitable eccentric dominating energy $\mathbb{E}_{eqed}(G) \approx 5.8708$.

2. $D_2 = \{v_1, v_3\}$,

$$\mathbb{M}_{eqed}(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G,\psi) = -\psi^5 + 2\psi^4 + 4\psi^3 - 3\psi^2 - 2\psi$.
Minimum equitable eccentric dominating eigen values are $\psi_1 \approx 2.9354$, $\psi_2 \approx 1$, $\psi_3 \approx 0$, $\psi_4 \approx$ -0.4626, $\psi_5 \approx$-1.4728.
Minimum equitable eccentric dominating energy $\mathbb{E}_{eqed}(G) \approx 5.8708$.
3. $D_3 = \{v_1, v_4\}$,

$$\mathbb{M}_{eqed}(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G,\psi) = -\psi^5 + 2\psi^4 + 4\psi^3 - 4\psi^2 - 3\psi + 2$.
Minimum equitable eccentric dominating eigen values are $\psi_1 \approx 2.8136$, $\psi_2 \approx 0.5293$, $\psi_3 \approx 1$, $\psi_4 \approx$ -1, $\psi_5 \approx$-1.3429.
Minimum equitable eccentric dominating energy $\mathbb{E}_{eqed}(G) \approx 6.6858$.
4. $D_4 = \{v_1, v_5\}$,

$$\mathbb{M}_{eqed}(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{G}_n(G,\psi) = -\psi^5 + 2\psi^4 + 4\psi^3 - 4\psi^2 - 3\psi + 2$.
Minimum equitable eccentric dominating eigen values are $\psi_1 \approx 2.8136$, $\psi_2 \approx 0.5293$, $\psi_3 \approx 1$, $\psi_4 \approx$ -1, $\psi_5 \approx$-1.3429.
Minimum equitable eccentric dominating energy $\mathbb{E}_{eqed}(G) \approx 6.6858$.

Observation 2.1: The energy of 'House X' graph G varies for different minimum equitable eccentric dominating sets.
For the set D_1, D_2, $\mathbb{E}_{eqed}(G) \approx 5.8708$,
For the set D_3, D_4, $\mathbb{E}_{eqed}(G) \approx 6.6858$.

Remark 2.2: The minimum equitable eccentric dominating energy depends on the minimum equitable eccentric dominating set.

Theorem 2.1: For a complete graph K_n where $n > 2$ the minimum equitable eccentric dominating energy of a complete graph $\mathbb{E}_{eqed}(K_n) = (n-2) + \left|\frac{(n-1)+\sqrt{(n-1)^2+4}}{2}\right| + \left|\frac{(n-1)-\sqrt{(n-1)^2+4}}{2}\right|$.
Proof: Let K_n be a complete graph with the vertex set $V = \{v_1, v_2, \ldots v_n\}$. The minimum equitable eccentric dominating set is $D = \{v_1\}$ then

$$\mathbb{M}_{eqed}(K_n) = \begin{pmatrix} 1 & 1 & 1 & & 1 & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 0 & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 0 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 & 1 \\ 1 & 1 & 1 & & 1 & 1 & 0 \end{pmatrix}_{n\times n}$$

The characteristic equation is $\mathcal{G}_n(K_n,\psi) = \det(\mathbb{M}_{eqed}(K_n) - \psi I)$.

$$= \begin{vmatrix} 1-\psi & 1 & 1 & & 1 & 1 & 1 \\ 1 & -\psi & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & -\psi & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & -\psi & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & -\psi & 1 \\ 1 & 1 & 1 & & 1 & 1 & -\psi \end{vmatrix}$$

The characteristic equation is $\mathcal{G}_n(G,\psi) = (-1)^{n-2}(\psi+1)^{n-2}(\psi^2 - (n-1)\psi - 1)$.

The minimum equitable eccentric dominating eigen values are

$$\psi = -1\ (n-2\ times),$$
$$\psi = \frac{(n-1)+\sqrt{(n-1)^2+4}}{2} \text{ and}$$
$$\psi = \frac{(n-1)-\sqrt{(n-1)^2+4}}{2}.$$

The minimum equitable eccentric dominating energy of the complete graph K_n is given by

$$\mathbb{E}_{eqed}(K_n) = |(-1)|(n-2) + \left|\frac{(n-1)+\sqrt{(n-1)^2+4}}{2}\right| + \left|\frac{(n-1)-\sqrt{(n-1)^2+4}}{2}\right|.$$

$$\mathbb{E}_{eqed}(K_n) = (n-2) + \left|\frac{(n-1)+\sqrt{(n-1)^2+4}}{2}\right| + \left|\frac{(n-1)-\sqrt{(n-1)^2+4}}{2}\right|.$$

Theorem 2.2: For cocktail party where $n \geq 4$ and crown graph G where $n \geq 6$, the minimum equitable eccentric dominating energy $\mathbb{E}_{eqed}(G) = \frac{n}{2}$.

Proof: Let G be a graph with the vertex set $V = \{v_1, v_2. \ldots v_n\}$. The minimum equitable eccentric dominating set is $D = \{v_1, v_2, \ldots v_{\frac{n}{2}}\}$, $|D| = \frac{n}{2}$ then

$$\mathbb{M}_{eqed}(G) = \begin{pmatrix} 1 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & & 0 & 0 & 0 \end{pmatrix}_{n\times n}$$

The characteristic equation is $\mathcal{G}_n(K_n, \psi) = \det(\mathbb{M}_{eqed}(G) - \psi I)$.

$$= \begin{vmatrix} 1-\psi & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1-\psi & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1-\psi & & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & & -\psi & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -\psi & 0 \\ 0 & 0 & 0 & & 0 & 0 & -\psi \end{vmatrix}$$

The characteristic equation is $\mathcal{G}_n(K_n, \psi) = (\psi^2 - \psi - 1)^{\frac{n}{2}}$.

The minimum equitable eccentric dominating eigen values are

$$\psi = 0 \text{ and}$$
$$\psi = 1\ \left(\frac{n}{2}\ times\right).$$

The minimum equitable eccentric dominating energy of the cocktail party and crown graph G is given by

$\mathbb{E}_{eqed}(G) = |1|\frac{n}{2} = \frac{n}{2}$.

3. Properties of Minimum Equitable Eccentric Dominating Eigen Values

In this section we discuss the properties of eigen values of $\mathbb{M}_{eqed}(G)$ for complete, crown and cocktail party graphs. Bounds for minimum equitable eccentric dominating energy of some standard graphs are obtained.

Theorem 3.1: Let D be a minimum equitable eccentric dominating set and $\psi_1, \psi_2, \ldots \psi_n$ are the eigen values of minimum equitable eccentric dominating matrix $\mathbb{M}_{eqed}(G)$ then

1. For any graph G, $\sum_{i=1}^{n} \psi_i = |D|$,
2. For a complete graph K_n where $n > 2$, $\sum_{i=1}^{n} \psi_i^2 = |D| + (n)(n-1)$,
3. For a crown and cocktail party graph G, $\sum_{i=1}^{n} \psi_i^2 = |D|$.

Proof:

1. We know that the sum of eigen values of $\mathbb{M}_{eqed}(G)$ is the trace of $\mathbb{M}_{eqed}(G)$.

$\sum_{i=1}^{n} \psi_i = \sum_{i=1}^{n} m_{ii} = |D|$.

2. Similarly, for a complete graph K_n sum of square of eigen values of $\mathbb{M}_{eqed}(K_n)$ is trace of $\left[\mathbb{M}_{eqed}(K_n)\right]^2$

$$\text{Now } \sum_{i=1}^{n}\psi_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}m_{ij}$$
$$\sum_{i=1}^{n}\psi_i^2 = \sum_{i=1}^{n}(m_{ii})^2 + \sum_{i\neq j} m_{ij}m_{ij}$$
$$\sum_{i=1}^{n}\psi_i^2 = \sum_{i=1}^{n}(m_{ii})^2 + 2\sum_{i<j}\left(m_{ij}\right)^2$$
$$\sum_{i=1}^{n}\psi_i^2 = |D| + (n)(n-1)$$

[since for a complete graph K_n, $2\sum_{i<j}\left(m_{ij}\right)^2 = (n)(n-1)$]

3. Similarly, for a crown and cocktail party graph G sum of square of eigen values of $\mathbb{M}_{eqed}(G)$ is trace of $\left[\mathbb{M}_{eqed}(G)\right]^2$

$$\text{Now } \sum_{i=1}^{n}\psi_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}m_{ij}$$
$$\sum_{i=1}^{n}\psi_i^2 = \sum_{i=1}^{n}(m_{ii})^2 + \sum_{i\neq j} m_{ij}m_{ij}$$
$$\sum_{i=1}^{n}\psi_i^2 = \sum_{i=1}^{n}(m_{ii})^2 + 2\sum_{i<j}\left(m_{ij}\right)^2$$
$$\sum_{i=1}^{n}\psi_i^2 = |D| + 0$$

[since for a crown and cocktail party graph G, $2\sum_{i<j}\left(m_{ij}\right)^2 = 0$]

Theorem 3.2: For a complete graph K_n where $n > 1$, if D be the minimum equitable eccentric dominating set and $W = |\det\det \mathbb{M}_{eqed}(K_n)|$ then $\sqrt{|D| + n(n-1) + n(n-1)W^{2/n}} \leq \mathbb{E}_{eqed}(K_n) \leq \sqrt{n(n(n-1)+|D|)}$.

Proof: By Cauchy schwarz inequality $(\sum_{i=1}^{n} g_i h_i)^2 \leq (\sum_{i=1}^{n} g_i^2)(\sum_{i=1}^{n} h_i^2)$. If $g_i = 1$ and $h_i = \psi_i$ then

$$(\sum_{i=1}^{n}|\psi_i|)^2 \leq (\sum_{i=1}^{n} 1)(\sum_{i=1}^{n}\psi_i^2)$$
$$\left(\mathbb{E}_{eqed}(G)\right)^2 \leq n(|D| + n(n-1))$$
$$\Rightarrow \mathbb{E}_{eqed}(G) \leq \sqrt{n(|D| + n(n-1))}$$

Since the arithmetic mean is not smaller than geometric mean we have

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\psi_i|\left|\psi_j\right| \geq \left[\prod_{i\neq j}|\psi_i|\left|\psi_j\right|\right]^{\frac{1}{n(n-1)}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j}|\psi_i|\left|\psi_j\right| = \left[\prod_{i=1}^{n}|\psi_i|^{2(n-1)}\right]^{\frac{1}{n(n-1)}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j}|\psi_i|\left|\psi_j\right| = \left[\prod_{i=1}^{n}|\psi_i|\right]^{\frac{2}{n}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j}|\psi_i|\left|\psi_j\right| = \left[\prod_{i=1}^{n}\psi_i\right]^{\frac{2}{n}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j}|\psi_i|\left|\psi_j\right| = \left|\det \mathbb{M}_{eqed}(G)\right|^{\frac{2}{n}} = W^{\frac{2}{n}}$$
$$\sum_{i\neq j}|\psi_i|\left|\psi_j\right| \geq n(n-1)W^{\frac{2}{n}}$$

Now consider

$$\left(\mathbb{E}_{eqed}(K_n)\right)^2 = (\sum_{i=1}^{n}|\psi_i|)^2$$
$$\left(\mathbb{E}_{eqed}(K_n)\right)^2 = (\sum_{i=1}^{n}|\psi_i|)^2 + \sum_{i\neq j}|\psi_i|\left|\psi_j\right|$$
$$\left(\mathbb{E}_{eqed}(K_n)\right)^2 = (|D| + n(n+1)) + n(n-1)W^{\frac{2}{n}}$$
$$\mathbb{E}_{eqed}(K_n) \geq \sqrt{(|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}}}$$

Theorem 3.3: For a crown graph G where $n \geq 6$ and cocktail party graph G where $n \geq 4$, if D be the minimum equitable eccentric dominating set and $W = |\det \mathbb{M}_{eqed}(G)|$ then $\sqrt{|D| + n(n-1)W^{2/n}} \leq \mathbb{E}_{eqed}(G) \leq \sqrt{n(|D|)}$.

Proof: The proof follows on the similar lines of theorem-3.2.

Theorem 3.4: If $\psi_1(G)$ is the largest minimum equitable eccentric dominating eigen value of $\mathbb{M}_{eqed}(G)$ then

1. For a complete graph K_n, $\psi_1(K_n) \geq \frac{|D|+n(n-1)}{n}$,
2. For a crown and cocktail party G, $\psi_1(G) \geq \frac{|D|}{n}$.

Proof:

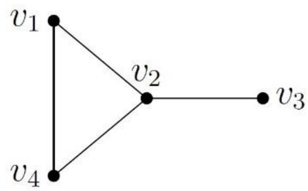
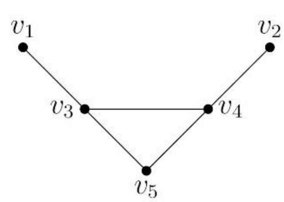
1. Let Y be a non-zero vector, then by ref.[7], we have $\psi_1\left(\mathbb{M}_{eqed}(G)\right) = \max_{Y\neq 0} \frac{Y^T\mathbb{M}_{eqed}(G)Y}{Y^TY}$.

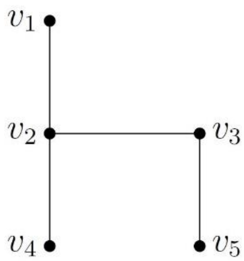
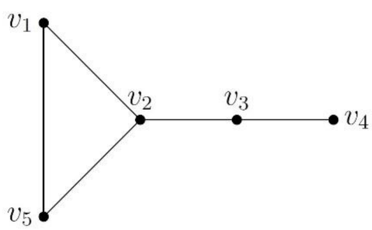
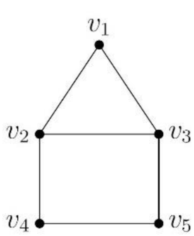
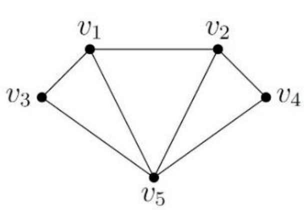
$\psi_1\left(\mathbb{M}_{eqed}(G)\right) \geq \frac{U^T\mathbb{M}_{eqed}(G)U}{U^TU} = \frac{|D|+n(n-1)}{n}$ where U is the unit matrix.

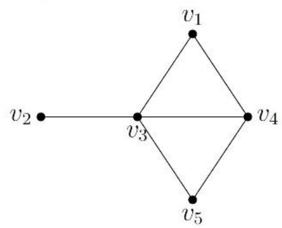
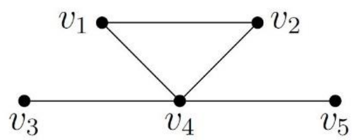
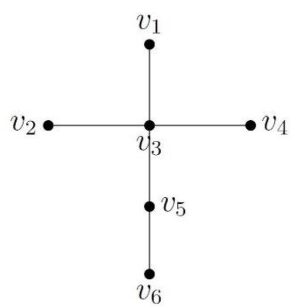
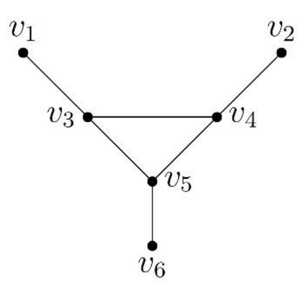
2. Let Y be a non-zero vector, then by ref.[7], we have $\psi_1\left(\mathbb{M}_{eqed}(G)\right) = \max_{Y\neq 0} \frac{Y^T\mathbb{M}_{eqed}(G)Y}{Y^TY}$.

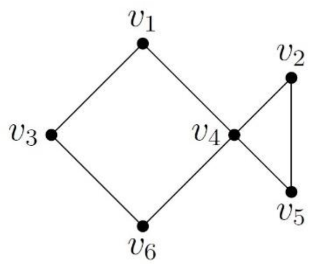
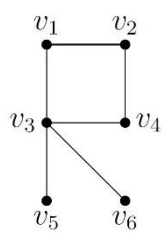
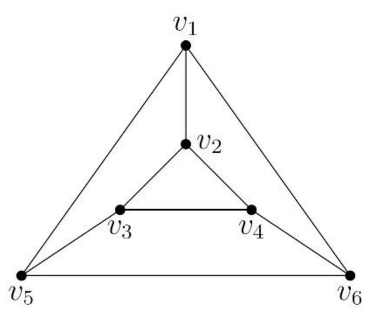
$\psi_1\left(\mathbb{M}_{eqed}(G)\right) \geq \frac{U^T\mathbb{M}_{eqed}(G)U}{U^TU} = \frac{|D|}{n}$ where U is the unit matrix.

Characteristic equation $\mathcal{G}_n(G,\psi)$, Roots $\psi(G)$ and Energy $\mathbb{E}_{eqed}(G)$ of Minimum EQED sets of various standard graphs are tabulated.

Graph	Figure	Minimum EQED set	Characteristic equation $\mathcal{G}_n(G,\psi)$	Roots $\psi(G)$	Energy $\mathbb{E}_{eqed}(G)$
Paw graph		$\{v_1, v_3\}$,	$\psi^4 - 2\psi^3 - \psi^2 + 3\psi - 1$.	$\psi_1 = 1.8019$, $\psi_2 = 1$, $\psi_3 = 0.445$, $\psi_4 = -1.247$.	4.4939
		$\{v_2, v_3\}$,	$\psi^4 - 2\psi^3 - \psi^2 + 2\psi$.	$\psi_1 = 2$, $\psi_2 = 1$, $\psi_3 = 0$, $\psi_4 = -1$.	4
		$\{v_3, v_4\}$.	$\psi^4 - 2\psi^3 - \psi^2 + 3\psi - 1$.	$\psi_1 = 1.8019$, $\psi_2 = 1$, $\psi_3 = 0.445$, $\psi_4 = -1.247$.	4.4939
Bull graph		$\{v_1, v_2, v_3\}$,	$-\psi^5 + 3\psi^4 - 3\psi^3 + \psi^2$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 0$.	3
		$\{v_1, v_2, v_4\}$,	$-\psi^5 + 3\psi^4 - 3\psi^3 + \psi^2$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 0$.	3
		$\{v_1, v_2, v_3\}$.	$-\psi^5 + 3\psi^4 - 3\psi^3 + \psi^2$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 0$.	3

Graph	Figure	Minimum EQED set	Characteristic equation $\mathcal{G}_n(G,\psi)$	Roots $\psi(G)$	Energy $\mathbb{E}_{eqed}(G)$
Fork graph		$\{v_1, v_2, v_3, v_4\}$,	$-\psi^5 + 4\psi^4 - 6\psi^3 + 4\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
		$\{v_1, v_2, v_4, v_5\}$,	$-\psi^5 + 4\psi^4 - 6\psi^3 + 4\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
		$\{v_1, v_3, v_4, v_5\}$.	$-\psi^5 + 4\psi^4 - 6\psi^3 + 4\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
(3,2)-Tadpole graph		$\{v_1, v_4\}$,	$-\psi^5 + 2\psi^4 - \psi^3$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$.	2
		$\{v_4, v_5\}$.	$-\psi^5 + 2\psi^4 - \psi^3$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$.	2
House graph		$\{v_2, v_4\}$,	$-\psi^5 + 2\psi^4 - \psi^3$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$.	2
		$\{v_3, v_5\}$.	$-\psi^5 + 2\psi^4 - \psi^3$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$.	2
Gem graph		$\{v_1, v_2\}$.	$-\psi^5 + 2\psi^4 - \psi^3 - 2\psi^2$.	$\psi_1 = 2$, $\psi_2 = 1$, $\psi_3 = 0$, $\psi_4 = -1$.	4

Graph	**Figure**	**Minimum EQED set**	**Characteristic equation** $\mathcal{G}_n(G,\psi)$	**Roots** $\psi(G)$	**Energy** $\mathbb{E}_{eqed}(G)$
Dart graph		$\{v_2, v_4\}$.	$-\psi^5 + 2\psi^4 - \psi^2$.	$\psi_1 = 1.618$, $\psi_2 = 1$, $\psi_3 = 0$, $\psi_4 = -0.618$.	3.236
Cricket graph		$\{v_1, v_3, v_4, v_5\}$,	$-\psi^5 + 4\psi^4 - 6\psi^3 + 4\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
		$\{v_2, v_3, v_4, v_5\}$.	$-\psi^5 + 4\psi^4 - 6\psi^3 + 4\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
Cross		$\{v_1, v_2, v_3, v_4, v_5\}$,	$\psi^6 - 5\psi^5 + 10\psi^4 - 10\psi^3 + 5\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 1$, $\psi_6 = 0$.	5
		$\{v_1, v_2, v_3, v_4, v_6\}$.	$\psi^6 - 5\psi^5 + 10\psi^4 - 10\psi^3 + 5\psi^2 - \psi$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 1$, $\psi_6 = 0$.	5
Net graph		$\{v_1, v_2, v_3, v_6\}$,	$\psi^6 - 4\psi^5 + 6\psi^4 - 4\psi^3 + \psi^2$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
		$\{v_1, v_2, v_4, v_6\}$.	$\psi^6 - 4\psi^5 + 6\psi^4 - 4\psi^3 + \psi^2$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4

Graph	Figure	Minimum EQED set	Characteristic equation $\mathcal{G}_n(G,\psi)$	Roots $\psi(G)$	Energy $\mathbb{E}_{eqed}(G)$
Fish graph		$\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$.	$\psi^6 - 3\psi^5 + 3\psi^4 - \psi^3$. $\psi^6 - 3\psi^5 + 3\psi^4 - \psi^3$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 0$. $\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 0$.	3 3
R graph		$\{v_2, v_3, v_5, v_6\}$.	$\psi^6 - 4\psi^5 + 6\psi^4 - 4\psi^3 + \psi^2$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$, $\psi_4 = 1$, $\psi_5 = 0$.	4
3-prism graph		$\{v_1, v_2\}$, $\{v_3, v_5\}$, $\{v_4, v_6\}$.	$\psi^6 - 2\psi^5 + \psi^4$. $\psi^6 - 2\psi^5 + \psi^4$. $\psi^6 - 2\psi^5 + \psi^4$.	$\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$. $\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$. $\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 0$.	2 2 2

4. Conclusion

In this paper we define the minimum equitable eccentric dominating energy of the graph and their properties are discussed. Minimum equitable eccentric dominating energy for family of graphs are determined and their bounds are calculated.

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