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Original Article

Computational Investigation of the Impact of Availability and Efficacy of Control on the Transmission Dynamics of Schistosomiasis

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Abstract *- In this paper, we carried out an elaborate quantitative analysis of the Reproduction Number of the availability and efficacy of control Schistosomiasis model by varying key sensitive parameters. Numerical simulation was also carried using parameter values from literature and the numerical study showed that both high levels of availability and efficacy of controls measures are needed for effective disease control. In particular, it was shown that with a* 100% *efficacious control measure that prevents transmission from snails to humans* (ϕ) *as well as from humans to snails* (π) *Schistosomiasis may not be controlled with a* 50% *availability of these control measures. Thus, efficacy as well as availabililty or coverage of the controls should be emphasised. It was also shown that it is more advantageous to have a control that prevents transmission from snails to humans as this does not just result in a reduction of human Schistosomiasis cases but also a reduction in snail Schistosomiasis cases.*

Keywords *- Schistosomiasis, Availability, Efficacy, Control, Stability, Numerical simulation.*

1. Introduction

Schistosomiasis also known as Snail fever or Katayama fever remains one of the most challenging public health issues in Africa, South America, the Middle East, Asia, and the Caribbean [13], [15]. The pathogenesis of Schistosomiasis is well documented in literature [1], [5], [31]. Schistosomiasis is only second to Malaria in terms of prevalence and impact as the most brutal parasitic disease [5], [31].

Mathematical Models for Schistosomiasis transmission dynamics have been formulated since 1973 as seen in [2]-[4],[6]-[12], [17]-[19], [24], [26]-[28], [32], [33]-[37]. These formulated models have indeed brought great insight into dynamics of Schistosomiasis but none of these models have quantitatively investigated the influence of the availability and efficacy of control on Schistosomiasis dynamics to the best of our knowledge.

In this paper, we analyzed the Reproduction number and also numerically simulated a non-linear coupled deterministic mathematical model which described the disease dynamics for Schistosomiasis in a population with control measures in place. These Control parameters built to the force of infections (which seeks to prevent the interactions between humans and Cercariae; between Snail and Miracidia on the other hand) are varied within a defined range and its impact on the Schistosomiasis model is reported in both the Analysis of the reproduction number and Numerical simulation. The cumulative incidence over a period of time is obtained by simulating the forces of infections of the model (for both human and snail populations in the patches) using parameter values from literature.

2. Mathematical Model

The Mathematical model to be analyzed in this section is seen Olowu and Nwankwo (2023), which is a working paper as at the time of writing this article.

The availability and efficacy control Schistosomiasis model we seek to analyze is given by:

$$N_h(t) = S_h(t) + E_h(t) + I_h(t) \text{ and } N_n(t) = M(t) + U(t) + L(t) + I_s(t) + C(t).$$

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \lambda_h S_h + \gamma_h I_h - \mu_h S_h \\
\frac{dE_h}{dt} &= \lambda_h S_h - (\kappa_h + \mu_h) E_h \\
\frac{dI_h}{dt} &= \kappa_h E_h - (\gamma_h + \delta_h + \mu_h) I_h \\
\frac{dM}{dt} &= \theta_M I_h - \mu_M M \\
\frac{dU}{dt} &= \Lambda_s - \lambda_s U - \mu_s U \\
\frac{dL}{dt} &= \lambda_s U - (\kappa_s + \mu_s) L \\
\frac{dI_s}{dt} &= \kappa_s L - (\delta_s + \mu_s) I_s \\
\frac{dC}{dt} &= \theta_C I_s - \mu_C C.
\end{aligned}$$

Where

$$\lambda_h = \beta_h \frac{(1-\phi\xi)C}{C_0 + \varepsilon C} \text{ and } \lambda_s = \beta_s \frac{(1-\pi\nu)M}{M_0 + \varepsilon M}$$

are the forces of infection for human and snail subpopulations.

The State variables and Parameters of the model are defined in Tables 1 and 2.

Table 1. State variables of the model

Variable	Description
$S_h(t)$	Susceptible individuals
$E_h(t)$	Latently tainted individuals
$I_h(t)$	Infected individuals
$M(t)$	Miracidia concentration
$U(t)$	Uninfected snails
$L(t)$	Latently-infected snails
$I_s(t)$	Tainted snails not yet releasing cercariae
$C(t)$	Free swimming Cercariae ready to enter human skin

Table 2. Parameters of the Model

Parameter	Description
μ_k (k = h, s)	Natural death rate for the kth sub population.
Λ_k (k=h, s)	Recruitment rate for the kth sub population.
β_k (k=h, s)	Cercariae and Miracidia infectious rate respectively for the kth sub population.
C_0	Saturation constant for Cercariae.
M_0	Saturation constant for Miracidia.
ε	Limitation of growth velocity of Cercariae and Miracidia.
K_k ($k = h, s$)	progression rate from latent class to infectious classes in the kth sub population.
δ_k ($k = h, s$)	Disease and parasite induced death respectively for humans and snails the kth sub population.
γ_h	Recovery rate for humans.
θ_M	Rate at which egg produced by adult Schistosome hatch and develop to free swimming Miracidia.
θ_C	Rate at which patent infected snails release cercariae.
ϕ	Efficacy of control in the human population.
ξ	Availability of control in the human population.
π	Efficacy of control in the aquatic (Snail) environment.
ν	Availability of control in the aquatic environment
μ_M	Natural death rate for miracidia
μ_C	Natural death rate for cercariae

3. Results established in Olowu and Nwankwo (2023)

(a) It was established that the orbits $X(t)$, where: $X(t) = \big(S_h(t), E_h(t), I_h(t), M(t), U(t), L(t), I_s(t), C(t)\big)$ of the Schistosomiasis model with non-negative initial conditions will always be non-negative for all time $t > 0$.

(b) It was also reported in Olowu and Nwankwo (2023) that the trajectories of the model, $S_h(t), E_h(t), I_h(t), M(t), U(t), L(t), I_s(t), C(t)$ with initial conditions and the biological feasible region is given by the set $\mathcal{D} = \mathcal{D}_h \times \mathcal{D}_M \times \mathcal{D}_S \times \mathcal{D}_C \subset \mathbb{R}_+^3 \times \mathbb{R}_+^1 \times \mathbb{R}_+^3 \times \mathbb{R}_+^1 \subset \mathbb{R}_+^8$ where:

$$\mathcal{D}_h = \left\{(S_h, E_h, I_h) \in \mathbb{R}_+^3 : N_h \leq \frac{\Lambda_h}{\mu_h}\right\}$$
$$\mathcal{D}_M = \left\{(M) \in \mathbb{R}_+^1 : M \leq \frac{\theta_M \Lambda_h}{\mu_M \mu_h}\right\}$$
$$\mathcal{D}_S = \left\{(U, L, I_s) \in \mathbb{R}_+^3 : N_S \leq \frac{\Lambda_s}{\mu_s}\right\}$$
$$\mathcal{D}_C = \left\{(C) \in \mathbb{R}_+^1 : C \leq \frac{\theta_C \Lambda_s}{\mu_C \mu_s}\right\}$$

And that these domain are positively-invariant and attracts all the non-negative trajectories of the model.

(c) Using the method of next generation matrix operator proposed by [28] and using notations similar to the ones used in [28], Olowu and Nwankwo (2023) obtained the Reproduction Number as follows:

The reproduction number $\mathcal{R}_c = \rho(FV^{-1})$, with $\rho(\cdot)$ being the largest eigenvalue associated with matrix FV^{-1}, is given by

$$\mathcal{R}_c = \sqrt{\frac{A_1 A_2 \beta_h \beta_s \Lambda_h \Lambda_s \kappa_h \kappa_s \theta_M \theta_C}{T_1 T_2 T_3 T_4 C_0 M_0 \mu_h \mu_M \mu_s \mu_C}},$$

where: $A_1 = (1 - \pi\nu), A_2 = (1 - \phi\xi), T_1 = (\kappa_h + \mu_h), T_2 = (\gamma_h + \delta_h + \mu_h), T_3 = (\kappa_s + \mu_s), T_4 = (\delta_s + \mu_s)$.

(d) The Endemic Equilibrium Point (EEP) of the model was given as

$\mathcal{E}_1^p = (S_h^{**}, E_h^{**}, I_h^{**}, M^{**}, U^{**}, L^{**}, I_s^{**}, C^{**})$, where

$$S_h^{**} = \frac{\Lambda_h T_1 T_2}{T_5 \lambda_C^{**} + \mu_h T_1 T_2},\ E_h^{**} = \frac{\Lambda_h \lambda_C^{**} T_2}{T_5 \lambda_C^{**} + \mu_h T_1 T_2},\ I_h^{**} = \frac{\Lambda_h \lambda_C^{**} \kappa_h}{T_5 \lambda_C^{**} + \mu_h T_1 T_2}$$
$$M^{**} = \frac{\theta_M \kappa_h + \lambda_C^{**} \Lambda_h}{\mu_M (T_5 \lambda_C^{**} + \mu_h T_1 T_2)},\ U^{**} = \frac{\Lambda_s}{\lambda_s^{**} + \mu_s},\ L^{**} = \frac{\Lambda_s \lambda_s}{T_3 (\lambda_s^{**} + \mu_s)}$$
$$I_s^{**} = \frac{\Lambda_s \lambda_s \kappa_s}{T_3 T_4 (\lambda_s^{**} + \mu_s)},\ C^{**} = \frac{\Lambda_s \lambda_s \kappa_s \theta_C}{T_3 T_4 \mu_s (\lambda_s^{**} + \mu_s)}$$

where:

$T_1 = (\kappa_h + \mu_h), T_2 = (\gamma_h + \delta_h + \mu_h), T_3 = (\kappa_s + \mu_s), T_4 = (\delta_s + \mu_s), \quad T_5 = T_1 T_2 - \gamma_h \kappa_h > 0, T_6 = C_0 \mu_C T_3 T_4 + \varepsilon \Lambda_s \kappa_s \theta_s, T_7 = M_0 \mu_M T_5 + \varepsilon \theta_M \kappa_h \Lambda_h$.

On Substituting the EEP into the force of infection and after several algebraic simplifications, the EEP of the model was shown to satisfies the following polynomial at the steady state:

$$\lambda_s^{**} K_1 + K_0 = 0$$

where

$$K_1 = A_1\beta_h\Lambda_s\kappa_s\theta_C T_7 + M_0\mu_M\mu_h T_1 T_2 T_{6,}$$
$$K_0 = C_0 M_0\mu_M\mu_C\mu_s\mu_h T_1 T_2 T_3 T_4(1 - \mathcal{R}_c^2)$$

Which of course showed the the model has a unique EEP and the condition for disease eradication of having $\mathcal{R}_c \leq 1$ is both a necessary and sufficient condition for effective Schistosomiasis control, irrespective of the starting sizes of the sub-populations.

(e) The Disease Free Equilibrium (DFE) of the model was shown to be Globally Asymptotically Stable (GAS) whenever $\mathcal{R}_c \leq 1$ since a unique EEP exist for all time. Hence, Schistosomiasis will be eliminated from the population whenever $\mathcal{R}_c \leq 1$.

(f) The endemic steady state of the model was shown to be GAS, whenever treatment/recovery is negligible ($\gamma_h = 0$) whenever $\mathcal{R}_c > 1$. Hence, Schistosomiasis will persist in the population regardless of the initial sizes of the subpopulation whenever $\mathcal{R}_c > 1$.

4. Numerical Simulations

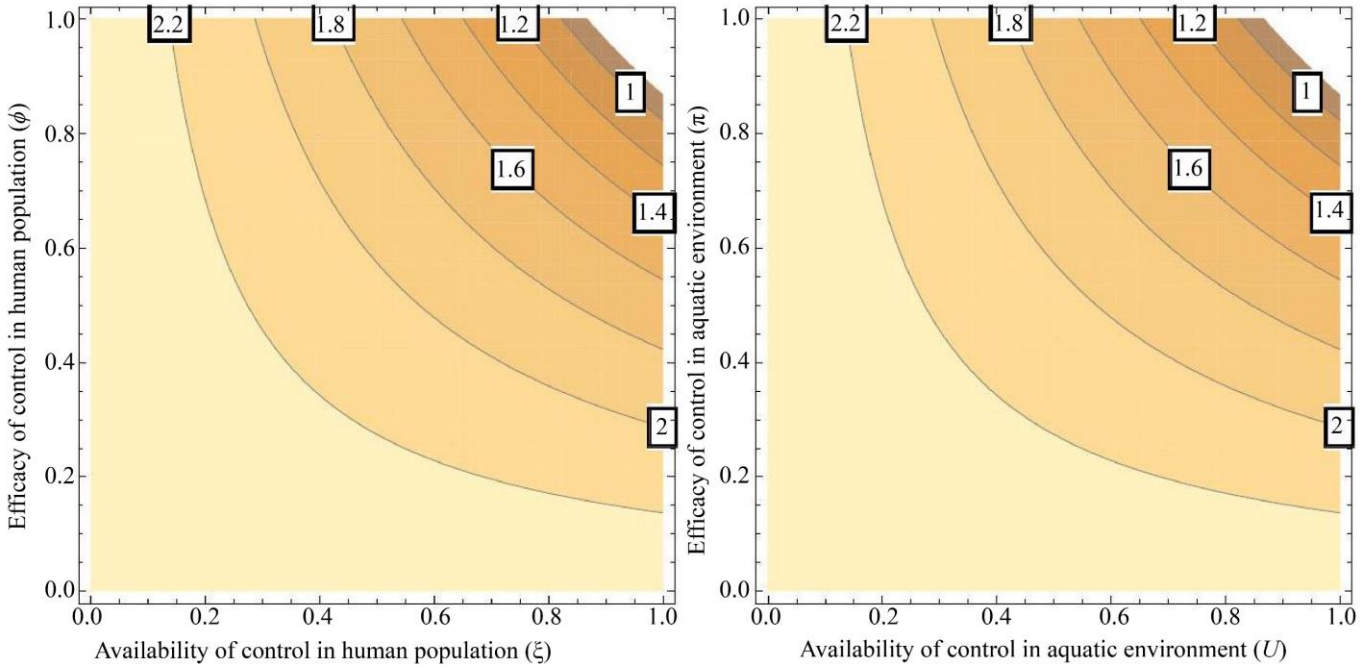
In this Section, the model is numerically simulated so as to investigate the impact of changing certain important parameters. The parameter values of the model in Table 3 are used for the simulation. Some demographic and epidemiological parameters peculiar to Nigeria are used. As at 2015, the population of Nigeria was estimated to be 213,401,323 (Countrymeter, 2021), hence $\Lambda_h/\mu_h = 213{,}401{,}323$ at the DFE. It is assumed that Schistosomiasis has a prevalence of about 50,000. 4.9704e-05

Table 3. Baseline values for the parameters of the schistosomiasis model

Parameter	Baseline values	References
μ_h	$0.000049704\text{day}^{-1}$	Calculated
μ_m	2.526dav^{-1}	Mangal et al. (2008)
μ_s	0.003dav^{-1}	Mangal et al. (2008)
μ_c	1.000dav^{-1}	Mangal et al. (2008)
Λ_h	$1_{10606\text{day}^{-1}}$	Calculated
Λ_s	15000day^{-1}	Mangal et al. (2008)
β_h	0.000013 day^{-1}	Assumed
β_s	0.000021 day^{-1}	Assumed
C_0	90,000	Assumed
M_0	100,000	Assumed
ε	0.2	Chiyaka and Garira (2009)
κ_h	0.017857 day^{-1}	Ngarakana-Gwasira et al. (2015)
κ_s	0.036 day^{-1}	Mangal et al. (2008)
δ_h	0.0039day^{-1}	Ngarakana-Gwasira et al. (2015)
δ_s	0.0004012 day^{-1}	Chiyaka and Garira (2009)
γ_h	0.8day^{-1}	Ngarakana-Gwasira et al. (2015)
θ_M	500 dav^{-1}	Mangal et al. (2008)
θ_C	2.6 day^{-1}	Chiyaka and Garira (2009)
ϕ	0.8	Variable
ξ	0.5	Variable
π	0.8	Variable
ν	0.5	Variable

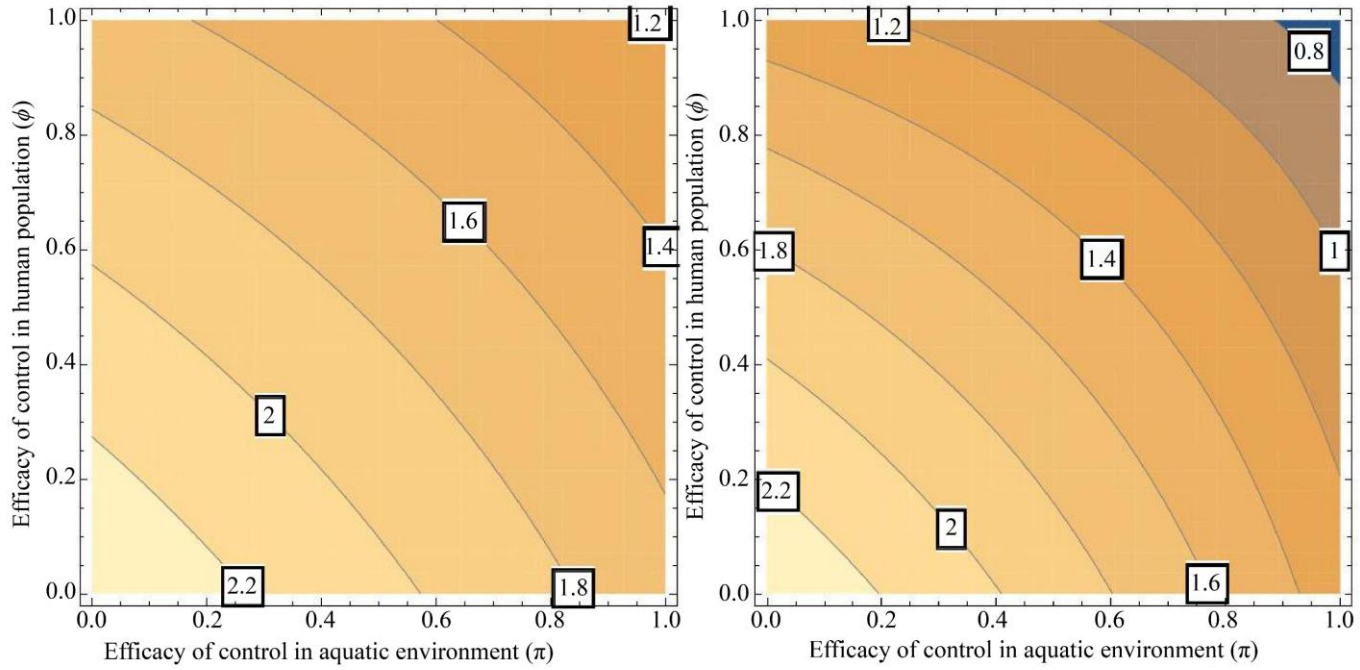
We shall numerically, test the sensitivity of the threshold quantity, $\mathcal{R}_c$ to efficacy (ϕ) and availability (ξ) of control measures (sanitation, public health education, provision of safe water, preventive chemotherapy) in the human population, as well as the efficacy (π) and availability (ν) of control measures (snail control) in aquatic environment.

(a) Using contour plots and parameter values as given in Table 3, we have the following



(a) Impact of ϕ and ξ **(b) Impact of π and ν**

Fig.1 Contour plot showing the impact of efficacy and availability of controls on $\mathcal{R}_c$

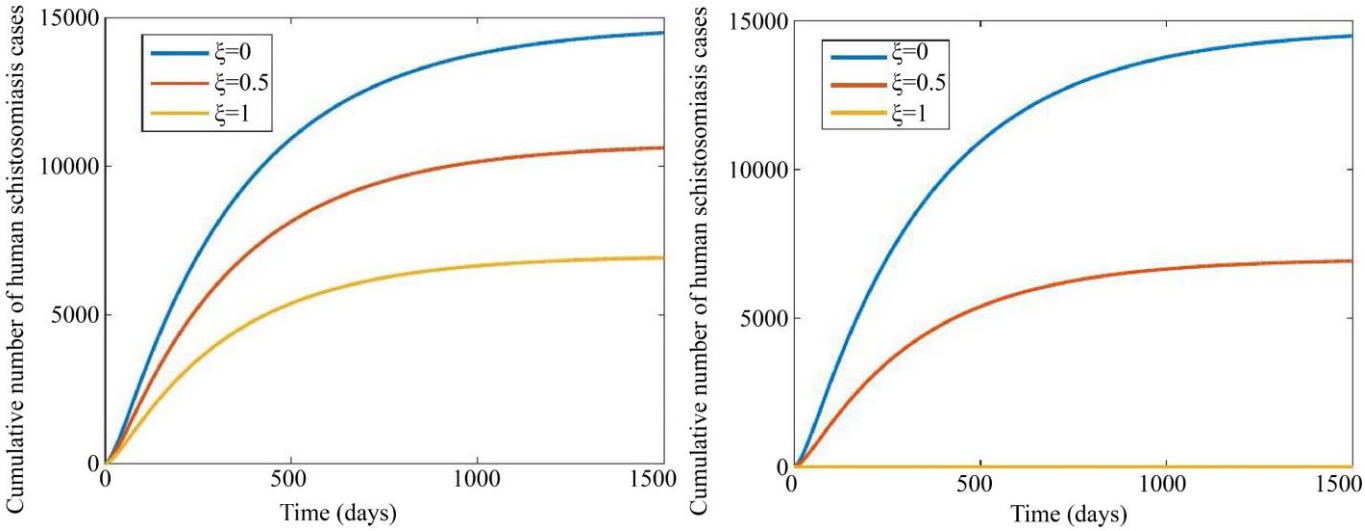


(a) Impact of ϕ and π with 50% availability **(b) Impact of ϕ and π with 80% availability**

Fig. 2 Contour plot showing the impact of efficacy and availability of controls on $\mathcal{R}_c$

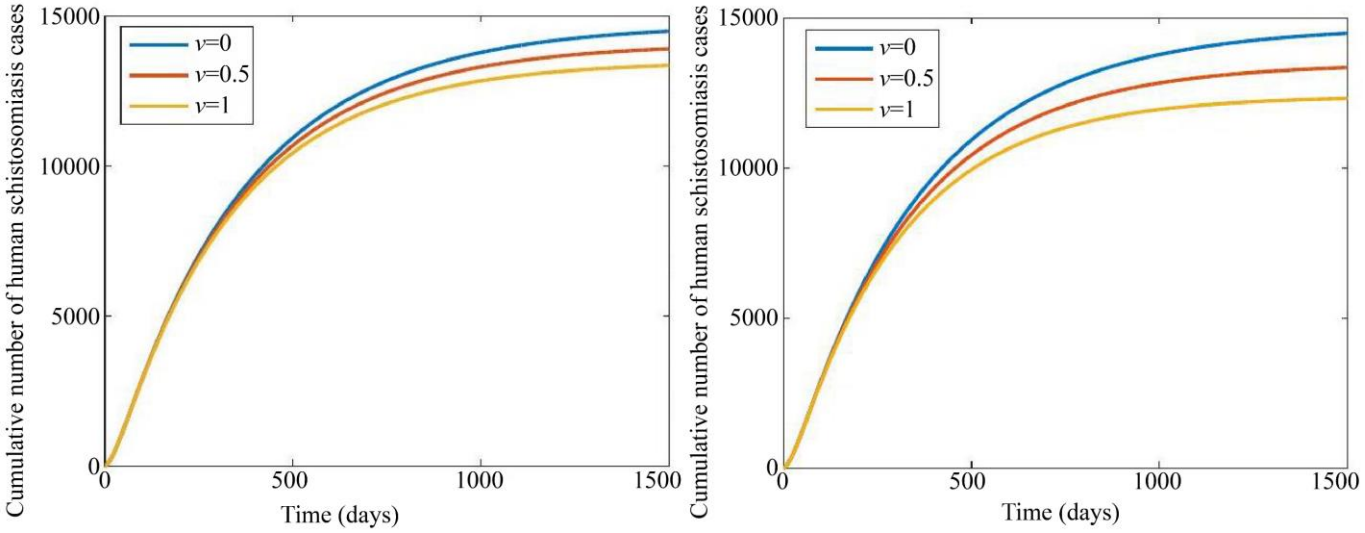
Figure 1(a) shows that in the absence of snail control (that is if $= \nu = 0$), the efficacy of the control deployed in the human environment must be over 85% with an availability of over 95% for effective disease control. Also, Figure 1(b) shows that when $\xi = \phi = 0$ the efficacy deployed in the aquatic environment must be over 85% with an availability of over 95% for effective disease control. However, Figure 2(a) shows that even with 100% efficacious control measures that prevents transmission from snails to humans (ϕ) as well as from humans to snails (π) Schistosomiasis cannot be controlled if availability of the controls is less than or equal to 50% (that is, $\xi = 50\%$ and $\nu = 50\%$). With an increase to an availability of 70% (that is, $\xi = 70\%$ and $= 70\%$), we see from Figure 2(b) that an efficacy of about 80% and 85% for π (control in the aquatic environment) and ϕ (control in the human environment) respectively, will help drive down the burden of Schistosomiasis. Thus, emphasis should not just rest on the efficacy of the controls deployed but also on the coverage of such controls.

(b) The impact of the availability and efficacy of controls on the model is again assessed via numerical simulations.



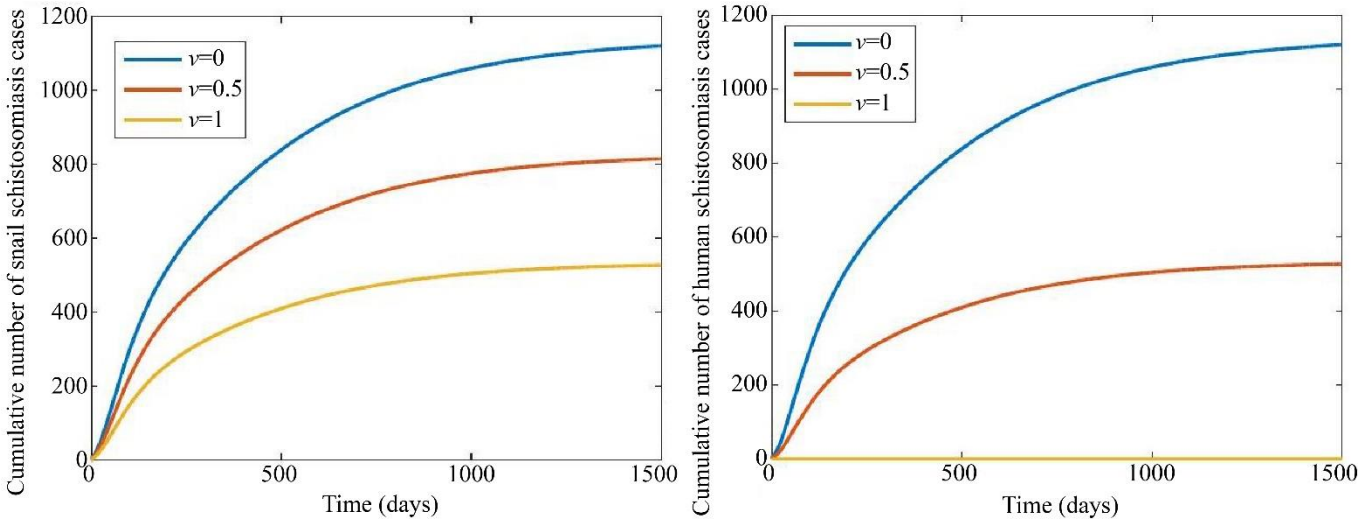
(a) $\phi = 0.5$ **(b) $\phi = 1$**

Fig. 3 Plot showing the cumulative incidence of Schistosomiasis in humans with $\pi = 0$ and $\nu = 0$



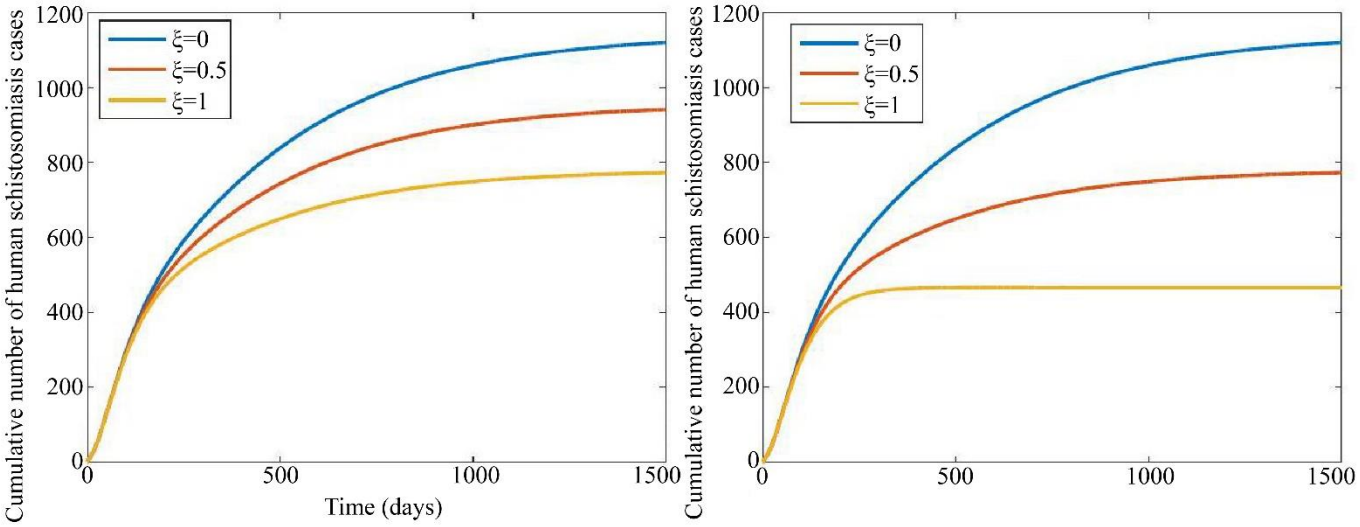
(a) $\pi = 0.5$ **(b) $\pi = 1$**

Fig. 4 Plot showing the cumulative incidence of Schistosomiasis in humans with $\phi = 0$ and $\xi = 0$



(a) $\pi = 0.5$ **(b) $\pi = 1$**

Fig. 5 Plot showing the cumulative incidence of Schistosomiasis in snails with $\phi = 0$ and $\xi = 0$



(a) $\phi = 0.5$ **(b) $\phi = 1$**

Fig. 6 Plot showing the cumulative incidence of Schistosomiasis in snails with $\pi = 0$ and $\nu = 0$

We observe from Figure 3 that in the absence of any control that prevents transmission from humans to snails, the availability of the control measure that prevents transmission from snails to humans has significant impact on the cumulative incidence of human Schistosomiasis. We also observe by comparing Figures 3(a) and 3(b) with an increase in the efficacy of the control measure that prevents transmission from snails to humans the more significant the impact of the availability of this control measure in reducing the cumulative cases of human schistosomias is in the population. However, in the absensce of any control that prevents transmission from snails to humans, Figure 4 depicts that the availability of a control that prevents transmission to snails has minimal impact on the cumulative cases of human Schistosomiasis irrespective of the efficacy of such control.

Furthermore, Figure 6 shows that in the absence of any control that prevents transmission from snails to humans, the availability of the control measure that prevents transmission to snails has significant impact on the cumulative incidence of snail Schistosomiasis. We also observe by comparing Figures $6(a)$ and $6(b)$ with an increase in the efficacy of the control measure that prevents transmission to snails, the more significant the impact of the availability of this control measure in reducing the cumulative cases of snail Schistosomiasis. In the same vein, in the absence of any control that prevents transmission from humans to snails, Figure 5 depicts that the availability of a control that

prevents transmission to humans has significant impact on the cumulative cases of snail Schistosomiasis. This impact is even more significant with an increase in the efficacy of such control. Thus, we can conclude from the above results that it is more advantageous to have a control that prevents transmission from snails to humans as this does not just result in a reduction of human Schistosomiasis cases but also a reduction in snail Schistosomiasis cases.

5. Discussions and Conclusion

Numerical study of the model shows that with about 70% availability of controls that prevents transmission from snails to humans (ξ) as well as from humans to snails (ν) Schistosomiasis can be kept in check in as much as the efficacy of both controls is as high as 85%. However, even with a 100% efficacious control measure that prevents transmission from snails to humans (ϕ) as well as from humans to snails (π) Schistosomiasis cannot be controlled as much as the availability (coverage) of the controls is less than or equal to 50% (that is, $\xi = 50\%$ and $\nu = 50\%$). Thus, efficacy as well as coverage should be emphasised. Furthermore, it was shown that an increase in the efficacy of the control measures whether in the human or aquatic environment increases the impact of the availability of such control on the dynamics of the model. Implying that mass administration of highly efficacious control measures which ranges from prophylaxis, vaccine, protective wears to limiting the activities of individuals around water bodies suspected to be highly infected with Cercariae by way of legislation can help eliminate Schistosomiasis from the population.

Again, it was shown that in the absence of control measures in the human environment, deploying control in the aquatic environment has minimal impact on the number of human Schistosomiasis but significant impact on the number of snail Schistosomiasis cases. However, in the absence of any control that prevents transmission from humans to snails, the availability of the control measures in the human environment has significant impact on the cumulative incidence of both human and snail Schistosomiasis.

Thus, this work has shown that it is more advantageous to have a control that prevents transmission from snails to humans than having a control that prevents transmission from humans to snails as the former does not just result in a reduction of human Schistosomiasis cases but also a reduction in snail Schistosomiasis cases.

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