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Original Article

Skolem Odd Vertex Graceful Signed Graphs for Star Graph

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Abstract *- In this article, skolem odd vertex graceful signed graphs on directed graphs have been introduced. A graph G(P, m, n) is a bijective function $f:V(G) \rightarrow \{1, 3, 5, 7, ..., 2p-1\}$ such that, when each edge $uv \in E(G)$ is assigned by $f(uv) = f(v) - f(u)$ the positive edges receive distinct labels from the set $\{1, 3, 5, ..., 2m-1\}$ and the negative edges receive distinct labels from the set $\{-1, -3, -5, -7, ..., -2n-1\}$, it is called as a skolem odd vertex graceful signed graphs. In this article, star graph is investigated under skolem odd vertex graceful labeling for signed graph.*

Keywords *- Graceful labeling, Signed graphs, Additive labeling, Skolem edge graceful labeling.*

1. Introduction

Graph labeling is an assignment of integer to its vertices or edges under certain condition. All Graphs in this paper are finite and directed. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of the edge set is called the size of G denoted by q edges is called a (p,q) graph. A graph labeling is an assignment of integers to the vertices or edges. A sigraph is an ordered pair S = (G, s), where G = (V, E) is a (p, q) graph called its underlying graph and s: $E \rightarrow \{+,-\}$ is a function from the set of edges to the set {+, -}, called a signing of G; hence an edge receiving '+' ('-') in the signing is said to be positive (negative). Let $E^+(S)$ and $E^-(S)$ denote respectively, the set of positive and negative edges of S. By a (p, m, n)-sigraph, it means a sigraph S having p vertices, m positive edges and n negative edges. This concept is extended from undirected graphs which is cited as [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. A sigraph is said to be homogeneous, if it is either all-positive sigraph(all-negative sigraph is defined similarly). A sigraph is said to be homogeneous, if it is either all-positive or all-negative and non-homogeneous otherwise. Graph labeling concept can be extended to automata theory which is cited as [10, 11, 12, 13, 14]. Graph labeling is also extended to domination [1, 5, 8, 9].

2. Main Result

Definition 2.1

A graph G(P, m, n) is a bijective function $f:V(G) \rightarrow \{1, 3, 5, 7, \dots, 2p-1\}$ such that, when each edge $uv \in E(G)$ is assigned by $f(uv) = f(v) - f(u)$ the positive edges receive distinct labels from the set $\{1, 3, 5, \dots, 2m-1\}$ and the negative edges receive distinct labels from the set $\{-1, -3, -5, -7, ..., -2n-1\}$, it is called as a skolem odd vertex graceful signed graphs.

Definition 2.2

A graph in which every edges associates, either positive or negative sign with distinct even numbers are called a skolem odd vertex graceful signed graphs.

Theorem 2.1

The star $K_{1,n}$ is a skolem odd vertex graceful signed graphs for $n \geq 2$.

Proof

Let G be a graph of star $K_{1,n.}$

Let $\{v_0, v_1, v_2, v_3, \dots, v_n\}$ be the vertices of $K_{1,n}$ and $\{e_1, e_2, e_3, \dots, e_n\}$ be the edges of $K_{1,n.}$

The star $K_{1,n}$ consists of n+1 vertices and n edges.

Case (i)

Edges receive positive signs with distinct even numbers.

If v_0 is the smallest number among $v_1, v_2, \ldots, v_n$ then, the edges receive positive signs.
Let us set an arbitrary labeling as follows:

The vertices of $K_{1,n}$ are labelled as given below.
Define $f: V(G) \rightarrow \{1, 3, 5, \ldots, 2n + 1\}$ as follows:

$f(v_0) = 1$

$f(v_i) = 2i + 1; 1 \leq i \leq n$

Then the edge labels are:

$f(e_i) = 2i ; 1 \leq i \leq n$

Clearly, the edge labels are distinct positive even numbers.

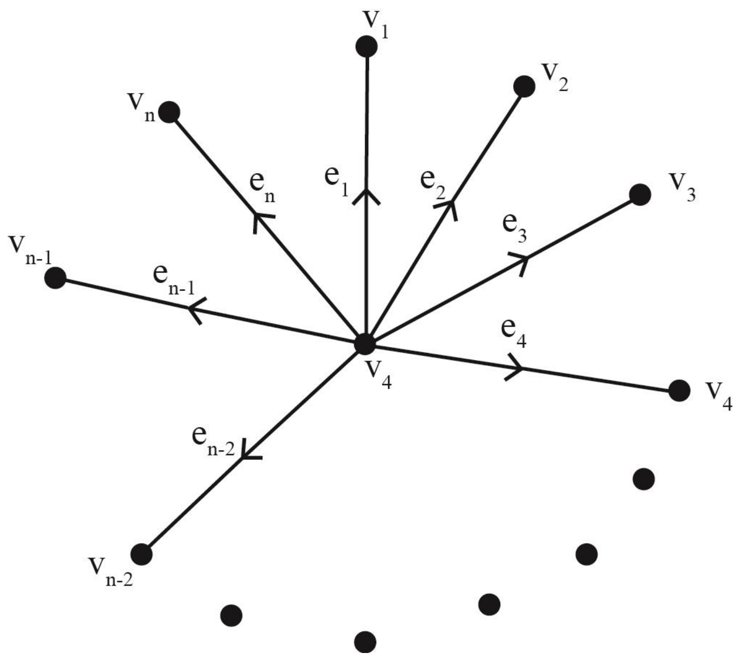

Fig. 2.1 Star $K_{1,n}$ with ordinary labeling

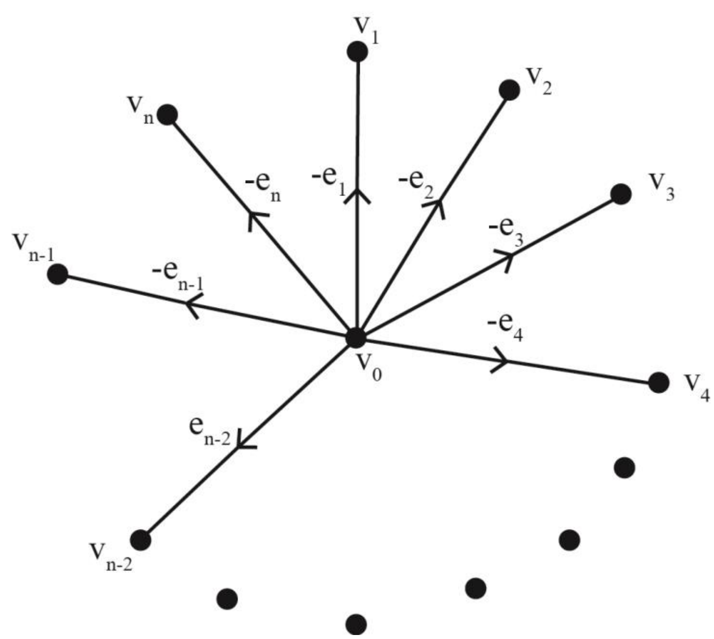

Fig. 2.2 Star $K_{1,n}$ with ordinary labelling

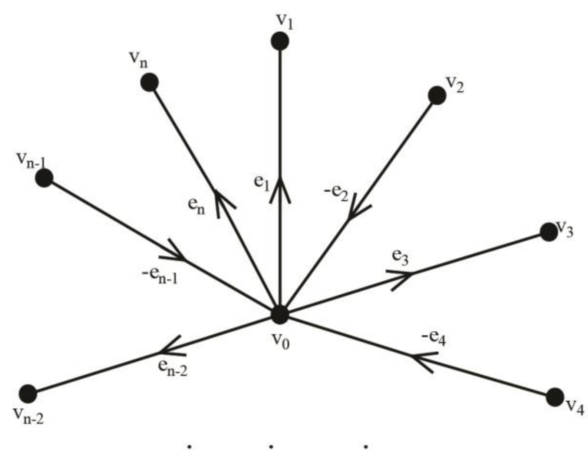

Fig. 2.3 Star $K_{1,n}$ with ordinary labeling when 'n' is odd

Case (ii)

Edges receive negative signs with distinct even numbers.

If v_0 is the largest number among $v_1, v_2, \ldots, v_n$ then, the edges receive negative signs.
The vertices of $K_{1,n}$ are labelled as given below.
Define f: V(G) $\rightarrow$ {1, 3, 5, …, 2n + 1} as follows:

$f(v_0) = 2n + 1$

$f(v_i) = 2i - 1 \; ; 1 \leq i \leq n$

Then the edge labels are:

$f(e_i) = -(2n + 2 - 2i) \; ; 1 \leq i \leq n$

Clearly, the edge labels are distinct negative even numbers.

Case (iii)

Edges receive both positive and negative signs with distinct even numbers.

Let us set an arbitrary labeling as follows:
The vertices of $K_{1,n}$ are labelled as given below.
Define f: V(G) $\rightarrow$ {1, 3, 5, …, 2n + 1} as follows:

$f(v_0) = 1$

$f(v_i) = 2i + 1; 1 \leq i \leq n$

Then the edge labels are:

$$f(e_i) = \begin{cases} 2i & \text{if i is odd} \\ -2i & \text{if i is even} \end{cases}$$

Clearly, the edge labels are positive and negative signs with distinct even numbers.

Hence, the star $K_{1,n} (n \geq 2)$ is a skolem odd vertex graceful signed graphs.

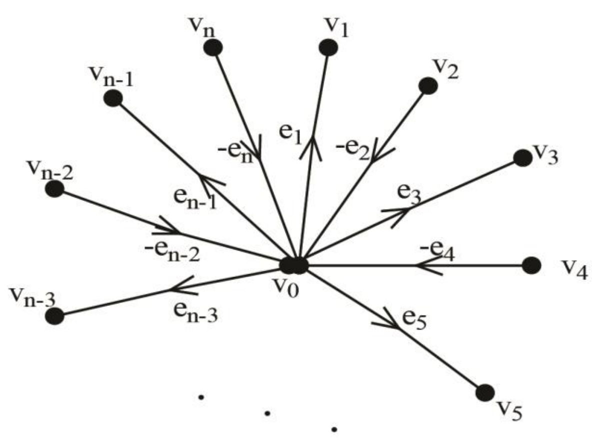

Fig. 2.4 Star $K_{1,n}$ with ordinary labeling when 'n' is even

Example: 2.1

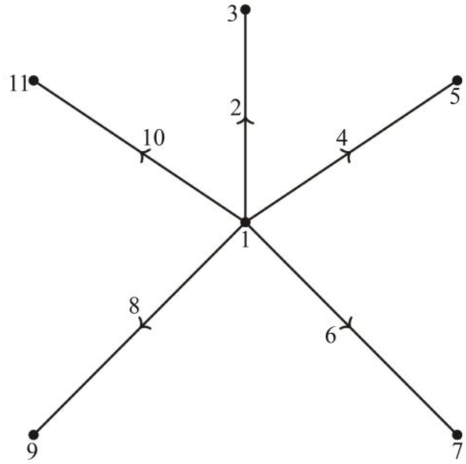

Fig. 2.5 Star $K_{1,5}$

Example: 2.2

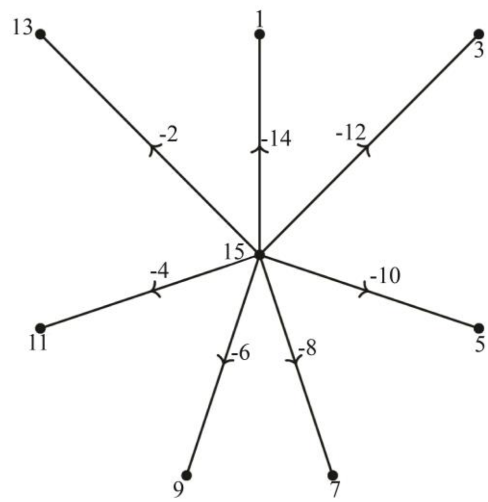

Fig. 2.6 Star $K_{1,7}$

Example: 2.3

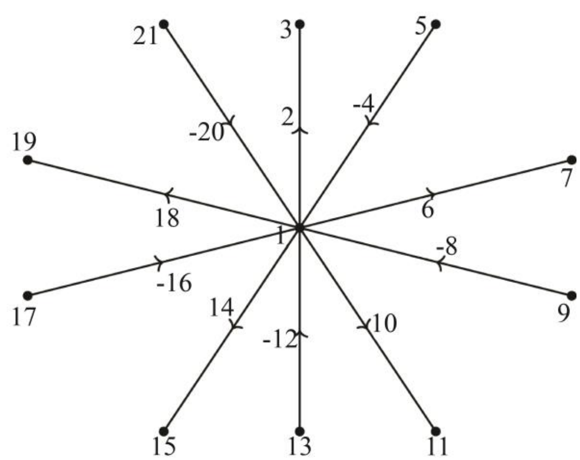

Fig. 2.7 Star $K_{1,10}$

3. Conclusion

In this article, Skolem odd vertex graceful signed has been discussed. Star graph has been proved under skolem odd vertex graceful labeling for directed graphs.

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