

Original Article

Interacting Tsallis Holographic Dark Energy Models with Constant Deceleration Parameter

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Received: 13 June 2023

Revised: 28 July 2023

Accepted: 10 August 2023

Published: 25 August 2023

Abstract - In this work, we investigate an anisotropic and homogeneous Bianchi Type-I universe filled with Dark Matter (DM) and Dark Energy (DE) in the framework of Einstein's theory of Relativity. We choose an interaction between DM and DE with the interacting term, $Q = 3H\gamma\rho_m$, by considering Tsallis generalized entropy. The exact solutions of the field equations are obtained by using the laws of the Hubble parameter proposed by Berman [1]. The EoS parameter of Tsallis Holographic DE (THDE) is explaining the evolution of the universe as The EoS parameter is Phantom like, Quintessence like and approaches to Λ CDM model. The EoS parameter, Anisotropy Parameter, Deceleration parameter, Total Energy Density parameter are explained by graphical representation in both models. The results, we obtained are consistent with the observational data.

Keywords - Tsallis holographic dark energy, Bianchi-I Universe, Constant deceleration parameter, General relativity.

1. Introduction

One of the most prevalent problems in cosmology is to explain the accelerated expansion of the universe, until the late 1990s, it was believed that the expansion of the universe was decelerating. Observations of Type Ia Supernovae, Cosmic Microwave Background, Large Scale Structure [2-3] Suggested that the Universe is accelerating. Dark energy (DE) with high negative pressure is the primary component of the acceleration of the universe [3-5]. Dark matter (DM) and DE stand between the mysteries of the scientific arena today, DM represents approximately 25 % of the total energy density of the Universe. The cosmologist [6-7] suggest that DE occupies 73% of the Universe, DM occupies 23%, whereas 4% is baryonic matter.

In order to describe late time acceleration, two approaches were recommended: i) To construct various DE candidates and ii) to modify Einstein's theory of gravitation. Among the numerous alterations of theories Brans–Dicke (BD), Saez–Ballester (SB), scalar tensor theories play important role in study of DE, there are interesting reviews on modified theories and DE models such as quintessence, Phantom, Quintom, Tachon, K-Essence, Chaplygin gas and modified Chaplygin gas models [8-11].

Amongst various DE candidates, the Holographic (HDE) and new agegraphic DE, Tsallis Holographic DE (THDE) are studied, among the many different approaches to describe the dark cosmological sector, so called HDE, it is another possible candidate which emerges from the famous holographic principle proposed to explain the thermodynamics of black hole physics, many researchers actively research on HDE such as [12-16].

Later, several researchers proposed different IR cutoffs, which caused some new cosmological problems, Tsallis and Cirto proposed a new HDE model in 2013 called as Tsallis HDE (THDE), The properties of THDE were proposed using IR cutoffs such as the particle horizon, Ricci horizon, and Granda-Oliveros cutoffs, which were inspired by Tsallis generalized entropy formalism [17]. It is discovered that the THDE model with a particle horizon as an IR cutoff explains the universe's current accelerated expansion and predict the age of the universe, THDE is a particular example of generalized Nojiri–Odintsov HDE [18]. In the THDE model by using Tsallis generalized entropy, $s_\delta = \gamma A^\delta$ where γ is an unknown constant and δ is a non-additive parameter [19] thus, the energy density of the THDE can be obtained as $\rho_T = CL^{2\delta-4}$, where C is an unknown parameter, if the Hubble horizon is used as the IR cutoff $L = H^{-1}$ then the energy density of the THDE is obtained as $\rho_T = CH^{-2\delta+4}$.

Recently, Koussour and Bennai have studied “Interacting Tsallis holographic dark energy and tachyon scalar field dark energy model in Bianchi type-II Universe” [20], Santhi, M. *et al.*, have studied “Marder space-time with Tsallis holographic dark energy model” [21], Pandey, *et al.*, have studied “Phantom model for Tsallis holographic dark energy” [22].



Motivating to above investigations, in this work we have considered spatially homogeneous and anisotropic Bianchi Type-I Universe with a THDE model in the framework of Einstein's theory of Relativity. We choose an interaction between DM and DE, the interaction term can be taken as, $Q = 3\gamma H\rho_m$, where γ is coupling parameter. Solutions of the field equations are obtained by considering special laws of the Hubble parameter yields a constant value of the deceleration parameter proposed by [1]. The geometrical and physical aspects of the model are studied in details.

2. Metric and Field Equation

The Metric for axially symmetric Bianchi-I space time is described as

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \tag{2.1}$$

where A and B are the cosmic scale factor.

The Einstein's field equation in natural limit ($8\pi G = c = 1$) are given as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(T_{ij} + \bar{T}_{ij}) \tag{2.2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar and g_{ij} is the metric tensor. T_{ij} and \bar{T}_{ij} are the energy momentum tensors of DM and THDE, respectively.

The energy momentum tensors are given as

$$T_{ij} = \rho_m u_i u_j \tag{2.3}$$

and

$$\bar{T}_{ij} = (\rho_T + p_T)u_i u_j - g_{ij}p_T \tag{2.4}$$

where ρ_m, ρ_T and p_T are the energy density of DM, energy density of THDE and pressure of the THDE respectively.

The THDE density with the Hubble horizon as the IR cutoff is [46, 64,65].

$$\rho_T = CH^{-2\delta+4} \tag{2.5}$$

where C is an unknown parameter, H is the Hubble parameter and δ is a free parameter. If $\delta = 1$ THDE reduces to HDE and for $\delta = 2$, DE behave like Cosmological Constant.

In co-moving coordinate systems, the Einstein's field equations (2.2) for the metric (2.1) with the help of Eqs. (2.3) - (2.4) can be written as

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 = -p_T \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p_T \tag{2.7}$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = \rho_m + \rho_T \tag{2.8}$$

where an overhead dot ($\dot{}$) represent derivative with respect to time t .

The energy conservation law $T_{;j}^{ij} = 0$, gives the continuity equation as

$$\dot{\rho}_m + \dot{\rho}_T + 3H(\rho_m + \rho_T + p_T) = 0 \tag{2.9}$$

In the interacting model, we consider that DE interacts with DM with interaction term Q , then the continuity equation becomes

$$\dot{\rho}_m + 3H\rho_m = Q \tag{2.10}$$

$$\dot{\rho}_T + 3H(\rho_T + p_T) = -Q \tag{2.11}$$

Here, Q indicates the interaction term explaining energy flow between the DE and DM. There are usual options for the interaction term Q such as $3H(\gamma_1\rho_T + \gamma_2\rho_m)$, here γ_1, γ_2 are the coupling constant, a single coupling constant can be used as $Q_1 = 3H\gamma\rho_T, Q_2 = 3H\gamma\rho_m, Q_3 = 3H\gamma(\rho_T + \rho_m)$ [28, 66-67].

In this work, for linear interaction we choose $Q = 3\gamma H\rho_m$, where γ is the coupling parameter between DM and DE, if we take $\gamma = 0$ we obtain the non-interacting model.

Using Eq. (2.5) in (2.11), The EoS parameter of THDE is obtained as

$$\omega_T = \frac{p_T}{\rho_T} = -1 + \frac{(2\delta-4)\dot{H}}{3H^2} - \gamma \frac{\rho_m}{\rho_T} \tag{2.12}$$

The Directional Hubble Parameter in the direction of x, y and z respectively, and Average Hubble parameter is defined as

$$H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B} = H_3 \tag{2.13}$$

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} (H_1 + 2H_2) \tag{2.14}$$

The special Volume V and Average Scale factor a is defined as

$$V = a^3 = AB^2, a = (AB^2)^{\frac{1}{3}} \tag{2.15}$$

The deceleration parameter $q(t)$ is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{2.16}$$

The mean anisotropy parameter of expansion A_m , the expansion scalar θ and the shear scalar σ^2 are defined for the metric as

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \tag{2.17}$$

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = 3H \tag{2.18}$$

$$\sigma^2 = \frac{1}{2} \left[\left(\frac{\dot{A}}{A} \right)^2 + 2 \left(\frac{\dot{B}}{B} \right)^2 \right] - \frac{\theta^2}{6} \tag{2.19}$$

From the Eqs. (2.3) - (2.4), the cosmic scale factor can be written as

$$A = F_2^2 a \exp(2F_3 \int a^{-3} dt) \tag{2.20}$$

$$B = F_2^{-1} a \exp(-F_3 \int a^{-3} dt) \tag{2.21}$$

where F_1, F_2 and F_3 are constant such that $3F_3 = F_1$

3. Cosmological Solutions of the Model

In order to obtain the exact solutions of the field Eqs. (2.6) - (2.6) and cosmic scale factor A and B , we assume the special law of variation for the Hubble parameter which yields the constant value of the deceleration parameter [1]. According to this law the variation of the mean Hubble parameter is given by

$$H = k a^{-m} \tag{3.1}$$

where $k > 0$ and $m \geq 0$.

Here we obtain the cosmological models I) Model for $m = 0$ and II) Model for $m \neq 0$.

3.1. Model for $m = 0$ [Exponential Volumetric Expansion Model]

Form $m = 0$, Equ. (3.1) gives the volume scale factor as

$$V = c_1 e^{3kt} \tag{3.2}$$

where $c_1 > 0$ is a constant of integration.

Using Eq. (3.2) in Eqs. (2.20) -(2.21), we obtain the exact values of scale factors as

$$A=F_2^2 c_1^{1/3} \exp\left(kt - \frac{2F_3}{3kc_1} e^{-3kt}\right) \quad (3.3)$$

$$B=F_2^{-1} c_1^{1/3} \exp\left(kt + \frac{2F_3}{3kc_1} e^{-3kt}\right) \quad (3.4)$$

The mean Hubble parameter H , deceleration parameter q and mean anisotropy parameter of expansion A_m for the model are respectively given by

$$H = k \quad (3.5)$$

$$q = -1 \quad (3.6)$$

$$A_m = \frac{2F_3^2 e^{-6kt}}{k^2 c_1^2} \quad (3.7)$$

The expansion scalar and the shear scalar are obtained respectively as

$$\theta = 3H = 3k \quad (3.8)$$

$$\sigma^2 = \frac{3F_3^2 e^{-6kt}}{c_1^2} \quad (3.9)$$

Using Eq. (3.5) in Eq. (2.5), we get the energy density of THDE as

$$\rho_T = C(K)^{-2\delta+4} \quad (3.10)$$

Using Eq. (3.5) in Eq. (2.10), we get the energy density of DM as

$$\rho_m = \frac{c_2}{e^{3k(1-\gamma)t}} \quad (3.11)$$

where $c_2 > 0$ is constant of integration.

The matter density parameter (Ω_m) and THDE density parameter (Ω_T) is given as

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{c_2}{3k^2 e^{3k(1-\gamma)t}} \quad (3.12)$$

$$\Omega_T = \frac{\rho_T}{3H^2} = \frac{C(k)^{-2\delta+2}}{3} \quad (3.13)$$

The overall density parameter obtained by using Eqs. (3.12) and (3.13)

$$\Omega = \Omega_m + \Omega_T = \frac{c_2}{3k^2 e^{3k(1-\gamma)t}} + \frac{C(k)^{-2\delta+2}}{3} \quad (3.14)$$

Using Eqs. (3.5), (3.10) and (3.11) in Eq. (2.12), the EoS parameter of THDE is given by

$$\omega_T = -1 - \gamma \left[\frac{c_2(k)^{2\delta-4}}{C e^{3k(1-\gamma)t}} \right] \quad (3.13)$$

3.2. Model for $m \neq 0$ [Power-law Volumetric Expansion Model]

For $m \neq 0$, Eq. (3.1) gives the volume scale factor as

$$V = (mkt + c_3)^{3/m} \quad (3.14)$$

where $c_3 > 0$ is a constant of integration.

Using Eq. (3.14) in Eqs. (2.20) - (2.21), we obtain the exact values of scale factors as

$$A=F_2^2 (mkt + c_3)^{1/m} \exp\left(\frac{2F_3}{k(m-3)} (mkt + c_3)^{\frac{m-3}{m}}\right) \quad (3.15)$$

$$B=F_2^{-1} (mkt + c_3)^{1/m} \exp\left(\frac{-F_3}{k(m-3)} (mkt + c_3)^{\frac{m-3}{m}}\right) \quad (3.16)$$

The mean Hubble parameter H , deceleration parameter q and mean anisotropy parameter of expansion A_m for the model respectively given as

$$H = k(mkt + c_3)^{-1} \tag{3.17}$$

$$q = m - 1 \tag{3.18}$$

$$A_m = \frac{2F_3^2(mkt+c_3)^{\frac{2(m-3)}{m}}}{k^2} \tag{3.19}$$

The expansion scalar and the shear scalar are obtaining respectively as

$$\theta = 3H = 3k(mkt + c_3)^{-1} \tag{3.20}$$

$$\sigma^2 = 3F_3^2(mkt + c_3)^{-6/m} \tag{3.21}$$

Using Eq. (3.17) in Eq. (2.5), we obtain an energy density of THDE as

$$\rho_T = \frac{C(mkt+c_3)^{2\delta-4}}{k^{2\delta-4}} \tag{3.22}$$

Using Eq. (3.17) in Eq. (2.10), we obtain an energy density of DM as

$$\rho_m = \frac{c_4}{(mkt+c_3)^{\frac{3(1-\gamma)}{m}}} \tag{3.23}$$

where $c_4 > 0$ is constant of integration.

The matter density parameter (Ω_m) and THDE density parameter (Ω_T) is given as

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{c_4}{3k^2(mkt+c_3)^{\frac{3(1-\gamma)}{m}-2}} \tag{3.24}$$

$$\Omega_T = \frac{\rho_T}{3H^2} = \frac{C(mkt+c_3)^{2\delta-2}}{3k^{2\delta-2}} \tag{3.25}$$

The overall density parameter obtained by using Eqs. (3.24) and (3.25)

$$\Omega = \Omega_m + \Omega_T = \frac{c_4}{3k^2(mkt+c_3)^{\frac{3(1-\gamma)}{m}-2}} + \frac{C(mkt+c_3)^{2\delta-2}}{3k^{2\delta-2}} \tag{3.26}$$

Using Eqs. (3.17), (3.22) and (3.23) in Eq. (2.12), the EoS parameter of THDE is given by

$$\omega_T = -1 - \frac{(2\delta-4)m}{3} - \gamma \left[\frac{c_4(k)^{2\delta-4}(mkt+c_3)^{4-2\delta}}{C(mkt+c_3)^{3(1-\gamma)/m}} \right] \tag{3.27}$$

4. Discussion

The physical and geometrical behaviours of the cosmological model are as follow

4.1. The Deceleration Parameter (q)

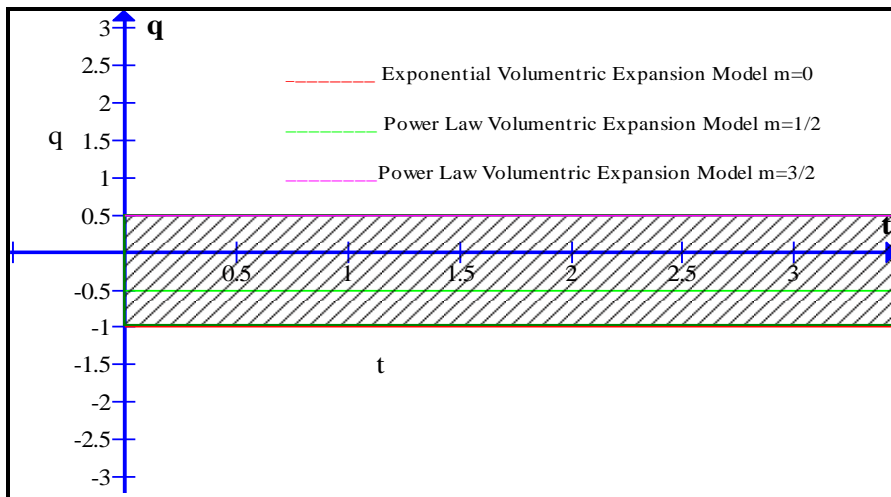


Fig. 1 The variation of q vs. time t

In the evolution of universe deceleration parameter play an important role, Astronomical observations of SNIa and CMB indicate that the current Universe is in a phase of accelerated expansion and the value of the deceleration parameter lies in between $-1 \leq q < 0$.

From the equation (3.18), it is observed that the deceleration parameter q is negative for $m = 0$ and for $0 < m < 1$. This indicates that the universe is accelerating as shown in fig. (1).

In other words, the sign of q indicates whether the model accelerates or not. The positive sign of q (for $m > 1$) corresponds to the deceleration of the universe, whereas the negative sign $-1 \leq q < 0$ for $0 \leq m < 1$ indicating inflation of the universe and for $m = 1$ gives $q = 0$ corresponds to expansion with constant velocity i.e. Acceleration and deceleration of the power law model may depend on the value of m .

The observation of our model consistent with the observations of [2-4] of accelerate expansion of the universe and may find applications in the analysis of late time evolution of the actual universe. In the exponential model, $q = -1$ and $\frac{dH}{dt} = 0$, implies that the greatest value of the Hubble parameter and the fastest rate expansion of the universe. Thus, this model may represent the inflationary era in the early universe and the very late times of the universe.

4.2. The Anisotropy Parameter of Expansion (A_m)

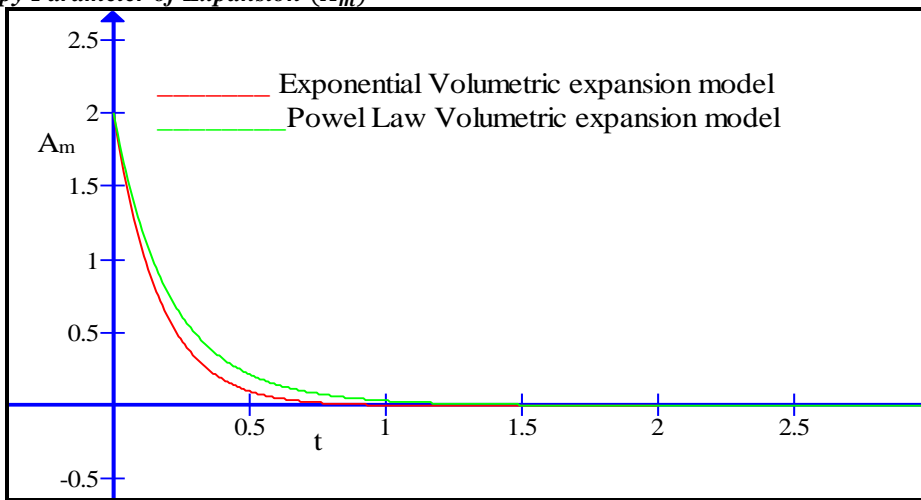


Fig. 2 The evolution of anisotropy parameter of expansion (A_m) vs. time t , for $F_3 = k = c_1 = m = 1$

In Fig. 2, we plotted an anisotropy parameter of expansion A_m for equations [(3.7) and (3.19)] against cosmic time t , from the fig. it is observed that the anisotropic parameter is positive and decreases with respect to time t and approaches to zero after some time t . Thus, the observed isotropy of the universe can be achieved in our derived model at the present epoch. The results show that the universe expands isotropically for large cosmic time t . Which match with the observational data that the universe is isotropic at large scale.

4.3. The Evolution of Total Energy Density Parameter $\Omega = \Omega_m + \Omega_T$

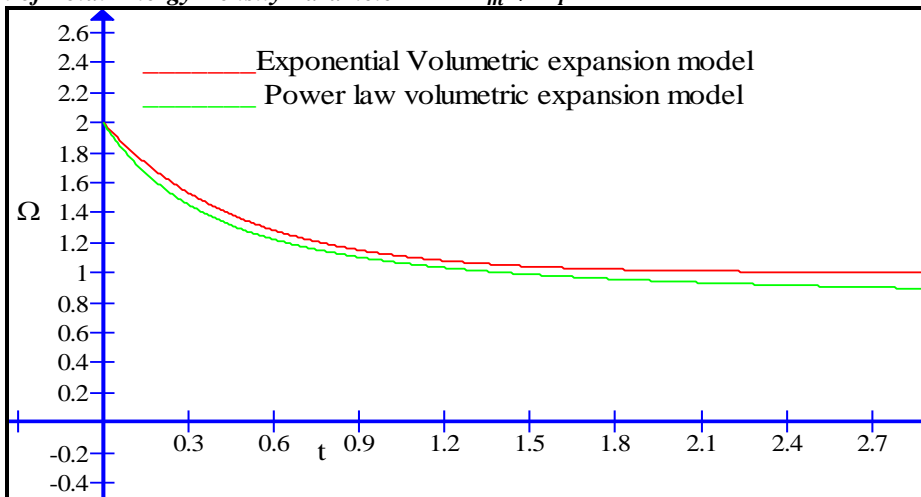


Fig. 3 The evolution of total energy density parameter $\Omega = \Omega_m + \Omega_T$ vs. time t , for $C = k = c_2 = m = 1, \gamma = 0.30, \delta = 1.4$

In fig. 3, we plotted the evolution of total energy density parameter $\Omega = \Omega_m + \Omega_T$ for the equations [(3.14), (3.26)] against cosmic time t . The density parameter Ω takes these values $\Omega > 1, \Omega = 1, \Omega < 1$ corresponds to the open, flat, and closed Universe respectively.

From the fig. 3, we observed that the total energy density parameter of the universe is positive i.e. $\Omega > 1$ and approaches to 1 (i.e. $\Omega \sim 1$) for sufficiently large cosmic time t in exponential and power law model, thus our model predicts a flat Universe at a later time. we can conclude that the Universe will proceed towards the isotropy in the future and the anisotropy of the Universe will damp out, which is supported by the current observational data.

4.4. The Evolution of EoS Parameter (ω_T)

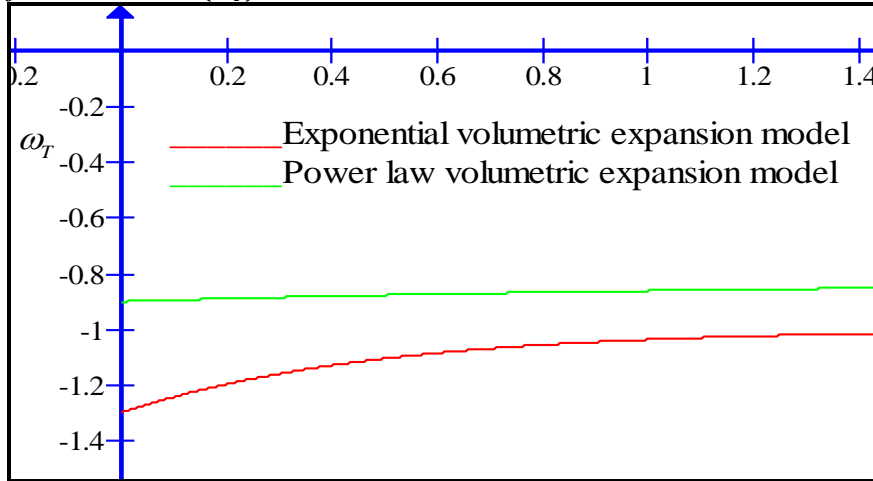


Fig. 4 The evolution of EoS parameter (ω_T) vs. time t , for $C = k = c_2 = c_4 = m = 1, \delta = 1.4, \gamma = 0.30$.

Figure 4, shows that the variation of the EoS parameter ω_T against cosmic time t in exponential and power law model. From the equation [3.13], for Exponential volumetric expansion model, we observed that the EoS parameter $\omega_T < -1$ i.e., start with phantom region and after some time t , the EoS parameter attains the value $\omega_T = -1$ i.e., the model approaches to Λ CDM model (cosmological constant dominated Universe). Mathematically, the cosmological constant is equivalent to the vacuum energy. The phantom, a hypothetical exotic matter, which has $EoS \omega_T \leq -1$, violates the strong energy condition and it is a leading candidate for DE, which can be responsible for accelerating expansion of the Universe. The cosmological observation shows that a Λ CDM model or a model reduces to Λ CDM is an excellent model to describe the cosmological evolution of the universe.

From the equation [3.27], for Power law volumetric expansion model we observed that the EoS parameter $-1 < \omega_T > -1/3$ i.e. The EoS parameter of THDE behaves like quintessence. From fig. 4, for the two cases, $m = 0$ and $m \neq 0$, we can conclude that ω_T is evolving with negative values, thus our model is realistic model.

Hence, from Fig. 4, we observed that THDE behave quintessence-like, phantom-like, or crosses the phantom-divide line (quintom). The results are consistent with the observations of [23, 24].

5. Concluding Remarks

In this work, we have investigated a spatially homogeneous and anisotropic Bianchi Type- I universe filled with DM and DE in the framework of Einstein’s Theory of Relativity. We state the interaction between DM and DE with the interaction term $Q = 3H\gamma\rho_m$ by considering the THDE density with the Hubble horizon as the IR cutoff. The exact solutions of the Einstein’s field equation are obtained by using the condition of Hubble parameter is reduced to a constant value of deceleration parameter.

We have summarized the conclusions as follows:

- The sign of q indicates whether the model accelerates or not. The positive sign of q ($form > 1$) corresponds to the deceleration of the universe, whereas the negative sign $-1 \leq q < 0$ for $0 \leq m < 1$ indicating inflation of the universe and for $m = 1$ gives $q = 0$ corresponds to expansion with constant velocity i.e. Acceleration and deceleration of the power law model may depend on the value of m .
- We observe that, in power law volumetric expansion model parameter such as $H, \vec{\epsilon} \leftrightarrow p_T, \rho_T, \theta$ and σ diverges and the special volume vanishes at point $t = -c_3/mk$, thus the model has Big bang singularity at, $t = -c_3/mk$, it can be shifted

to $t = 0$ by considering $c_3 = 0$. The singularity is point type as the scale factors $A(t)$ and $B(t)$ vanishes at initial [68-70].

- If $\delta = 1$ in Eq. (2.5), THDE reduces to HDE and for $\delta = 2$, DE behave like Cosmological Constant.
- For $Q = 0$, the non-interacting case the model reduces to [26] for special case.
- The THDE density parameter ρ_T and energy density of DM ρ_m in the power law model are the remains positive in throughout evolution and decreasing function with respect to time t , it shows that there is a volumetric expansion of the Universe at late time. For exponential volumetric expansion model THDE density parameter ρ_T is constant, and energy density of DM ρ_m decreases faster towards zero than THDE density parameter ρ_T , hence THDE is responsible for accelerated expansion of the present-day universe.
- In our model, we observe that EoS parameter of THDE can be quintessence-like, phantom-like, or crosses the phantom-divide line (quintom). It is concluded that THDE model coincide with the flat Λ CDM model for large cosmic time t , which matches with the current observations.
- From the equation [3.7 and 3.22], we conclude that Anisotropy of expansion dies out very quickly also the special volume V is finite at $t \rightarrow 0$ and diverges for $t \rightarrow \infty$ thus the universe approaches to isotropy for large cosmic time. Our model of THDE is suitable to describe the late time evolution of the universe.

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