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Original Article

Rainfall Forecasts by Mathematical Model using Monthly Rainfall Data for Ranchi District of Jharkhand in India

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Abstract *- Climate change is becoming a threat for the whole world and for the livelihood of humans. Rainfall is one of the major climatic variables which heavily gets impacted by it. Rainfall variability is causing flash floods in some region and droughts in other region. The rainfall prediction has become very important so that the adverse effect of extreme events such as floods and droughts can be minimized. Mathematical model helps in future forecasting of rainfall. Holt-Winter exponential method is used in this paper to forecast rainfall from year 2023 up to 2030. The RMSE and normalized BIC is calculated for the selection of better model between AHW and MHW method. The results suggest that the MHW method yields better forecasts than the AHW method as the RMSE and normalized BIC for MHW method is low. The future forecasts emphasize that the rainfall will gradually increase in future without affecting the seasonal behavior of rainfall. The monsoon rainfall will continue to contribute maximum rainfall to annual rainfall and winter rainfall will contribute the least rainfall to the annual rainfall. The monsoon rainfall of 2030 will contribute 80.8% to the annual rainfall while winter rainfall will contribute the least 2.8% rainfall.*

Keywords - *Mathematical model, Holt-Winter exponential method, Rainfall forecasts, Climate change, Rainfall trend.*

1. Introduction

Climate change is becoming a threat for the whole world and for the livelihood of humans. Rainfall is one of the major climatic variables which gets impacted by climate change. The variability in rainfall affects the overall climate of a region. Climate change is causing variation in rainfall pattern of a region which results in flash floods [1] and drought. Researchers around the globe are getting attracted to the rainfall prediction and analysis of extreme rainfall events like floods and drought. Mathematical method helps in predicting rainfall by analyzing historical data of rainfall. There are several mathematical methods for the forecasts of rainfall like interpolation method [2], ARIMA method [3], SARIMA method [4], Holt-Winter Exponential smoothing method [5]. In this paper, Rainfall forecasts were done using Holt-Winter smoothing exponential method. Holt-Winters exponential smoothing methods are used in the many forecasting such as rainfall prediction [6], tourist forecasting [7], revenue forecasting [8], electricity forecasting [9], and in waste water treatment [10]. Holt-Winter method uses historical data of rainfall to forecast the future rainfall.[11] Holt-Winters method gives relatively more weightage to the recent data points than the older data points.[12] The Holt-Winter method estimates three smoothing parameters which are level, trend and seasonality.

The seasonality is of two different types, one is additive seasonality and the other is multiplicative. When the additive seasonality is considered then the method is known as Additive Holt-Winter (AHW) method and when the multiplicative seasonality is considered then it is known as Multiplicative Holt-Winter (MHW) method. There is a drawback in MHW method as it requires the data points to be non-zero however there is no such restriction in AHW method. In general, MHW method is preferred over AHW method as the forecasts by MHW method is more accurate. Karthika et. al. used the Holt-Winter method to forecast the rainfall of Tamil Nadu in India and concluded that the MHW method is more accurate than the AHW method.[13] However, the Holt-Winter method is applied on the rainfall data of Quetta region in Pakistan resulted that the AHW method is more accurate than the MHW method.[14] The type of past data set decides which method is better for forecasting. So, in this research paper, Holt-Winter method is used to forecast the future rainfall of Ranchi.

2. Study Area

Ranchi is the capitol of the state Jharkhand in India. It is situated in the Chota Nagpur plateau of eastern India. It lies between 22º52'-23º45' North latitude and 84º45'-85º50' East longitude. It is situated at average mean sea level of 629m with an area of 5097 Km^2. The Ranchi has a pleasant climate and it characterized as subtropical climate. The average annual rainfall of Ranchi is 1300 mm. Ranchi receives a good amount of monsoonal rainfall and it receives 81% of the total annual rainfall from south-west monsoon during June to September.[15] Ranchi also receives unique convectional rainfall during summer season.

3. Data and Methodology

The daily rainfall data of Ranchi is obtained from the website of National Centre For Environmental Information (https://www.ncdc.noaa.gov/cdo-web/) for the period 1973-2022. The monthly rainfall data were calculated from the daily rainfall data.

The monthly rainfall data for the period 1973-2021 is used to forecast the future rainfall from the year 2023 up to 2030. The monthly rainfall data of 2022 is used to verify the efficiency of model. For the sake of stability of model, the value 0 is replaced by 0.00001 in the data set. The method used for the forecasting is Holt-Winter's exponential smoothing method. The Root mean square error (RMSE) and normalized Bayesian Information Criterion (BIC) is calculated for the forecasted values to find out the error percentage in the rainfall forecasts. Holt-Winter method is based on four equations that compute Level which is average value of the sequence in the past, Trend which is trend of evaluation in the future, Seasonality which is a seasonal term and the final equation computes the weighted sum of the previous term as forecast. When additive seasonal term is considered in the equation then the method is known as Additive Holt-Winter (AHW) method and when multiplicative seasonal term is considered, it is known as Multiplicative Holt-Winter (MHW) method.

3.1. Additive Holt-Winter Method

The Level, Trend and Seasonality for the AHW method is computed by following formulas:

$$L_t = \alpha(Y_t - S_{t-m}) + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-m}$$

$$\hat{Y}_t = L_t + kT_t + S_{t-m+k}$$

Where L_t, T_t, S_t and $\hat{Y}_t$ represents Level, Trend, Seasonality and Forecasts respectively and k represents the length of future forecast from the end of past data.

Also, m=4 for quarterly observation and m=12 for monthly observation.

Here, α, β, γ are the smoothing parameters and which lies in the interval [0,1].

The initial value of Level, Trend and seasonality can be calculated as follows:

$$S_p = Y_p - {\sum_{i=1}^{m} Y_i}/{m}$$

$$L_p = Y_p - S_p$$

$$T_p = {L_p}/{S_p} - {Y_{p-1}}/{S_{p-1}}$$

3.2. Multiplicative Holt-Winter Method

The Level, Trend and Seasonality for the AHW method is computed by following formulas:

$$L_t = \alpha(Y_t/S_{t-m}) + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma(Y_t/L_t) + (1-\gamma)S_{t-m}$$

$$\hat{Y}_t = (L_t + kT_t)S_{t-m+k}$$

The initial value of Level, Trend and seasonality can be calculated as follows:

$$S_p = \frac{Y_t}{{\sum_{i=1}^{m} Y_i}/{m}}$$

$$L_p = Y_p/S_p$$

$$T_p = {}^{L_p}\!/_{S_p} - {}^{Y_{p-1}}\!/_{S_{p-1}}$$

3.3. Root Mean Square Error

RMSE is used to check the accuracy of different models of forecast for a particular data set. RMSE is always non-negative. Lower the value of RMSE, better the model. The value 0 represents the best fit which is practically impossible. It is the square root of the average of squared errors.

Let x_t and y_t be the actual and the forecast value at any time t. The RMSE is calculated as follows:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(y_t - x_t)^2}{n}}$$

Where n is the length of the prediction.

3.4. Normalized BIC

It is used for the selection of model. It is possible to increase the maximum likelihood by adding parameters when fitting a model, but this can result in overfitting. BIC tries to resolve this problem by introducing a penalty term for the number of parameters in the model.

The BIC is defined as,

$$BIC = k\ln(n) - 2\ln(\hat{L})$$

Where $\hat{L} = p(x\,|\hat{\theta}, M)$ is the maximize value of the model M and $\hat{\theta}$ is the parameter value that maximize the likelihood function

And x is the observed value and n is the sample size of the data set, k is the number of parameters estimated by the model.

4. Results

AHW and MHW methods were applied on the monthly rainfall data of period 1973-2021. Firstly, the rainfall for the year 2022 is forecasted. The rainfall forecasts for the year 2022 is then compared with the actual rainfall of the year 2022 and error of forecasting by both the methods is calculated for all months [Table 2]. The Root Mean Square Error (RMSE) and normalized Bayesian Information Criterion (BIC) are then calculated. The lower value of RMSE and normalized BIC for a method emphasizes that the method is more accurate than the other method. The coefficient of determination i.e., R^2 is also calculated to find that which method forecasts more accurately. A higher value of R^2 emphasize more accurate method. The RMSE value for the AHW and MHW method are respectively 73.326 and 72.168.

Also, the normalized BIC for AHW and MHW are respectively 8.622 and 8.591. The lower value RMSE and normalized BIC of MHW method emphasize that MHW method is better for forecasting than AHW method for this particular data set Table 1.

The value of R^2 is also high for MHW method. The forecasts done by MHW method is more accurate than the AHW method for all months except January, June and July for the year 2022 [Fig. 2]. The rainfall forecasts were plotted together with the actual rainfall for the year 2022 [Fig. 1].

The future rainfall forecasts are then done by both the methods. The forecasts by both the methods show an increasing trend in rainfall pattern. AHW method also shows an upward trend in the winter seasonal rainfall however there is no such trend is observed by MHW method. The rainfall forecasts by both the method are plotted in Fig. 3 and Fig. 4. This result also emphasizes that the MHW method is more accurate than AHW method as the seasonal trend of rainfall is maintained by the MHW method.

The future rainfall forecasted value by MHW method from the year 2023 up to 2030 is mentioned in the Table 3. The annual rainfall of the year 2030 will be 1627.9 mm out of which 80.8% rainfall will be received in the monsoon season. The summer, post-monsoon and winter will receive respectively 8%, 8.3% and 2.8% of the total annual rainfall of 2030.

Table 1. Model statistical parameter of AHW & MHW method

Method	R squared	RMSE	Normalized BIC
MHW	0.540	72.168	8.591
AHW	0.525	73.326	8.622

Table 2. Rainfall forecasts by AHW & MHW method for the year 2022

Year 2022	Actual Rainfall	AHW Forecast	AHW Error	MHW Forecast	MHW Error
January	36.068	35.20	0.868	19.78	16.288
February	18.034	36.10	-18.066	19.67	-1.636
March	0.254	47.16	-46.906	27.74	-27.486
April	6.096	34.74	-28.644	21.99	-15.894
May	42.164	85.75	-43.586	62.54	-20.376
June	96.52	177.50	-80.98	205.80	-109.28
July	144.526	317.50	-172.974	361.38	-216.854
August	556.26	266.64	289.62	319.34	236.92
September	255.016	216.78	38.236	248.23	6.786
October	29.21	88.59	-59.38	87.18	-57.97
November	0.00001	33.38	-33.38	17.14	-17.14
December	0.00001	33.59	-33.59	13.52	-13.52

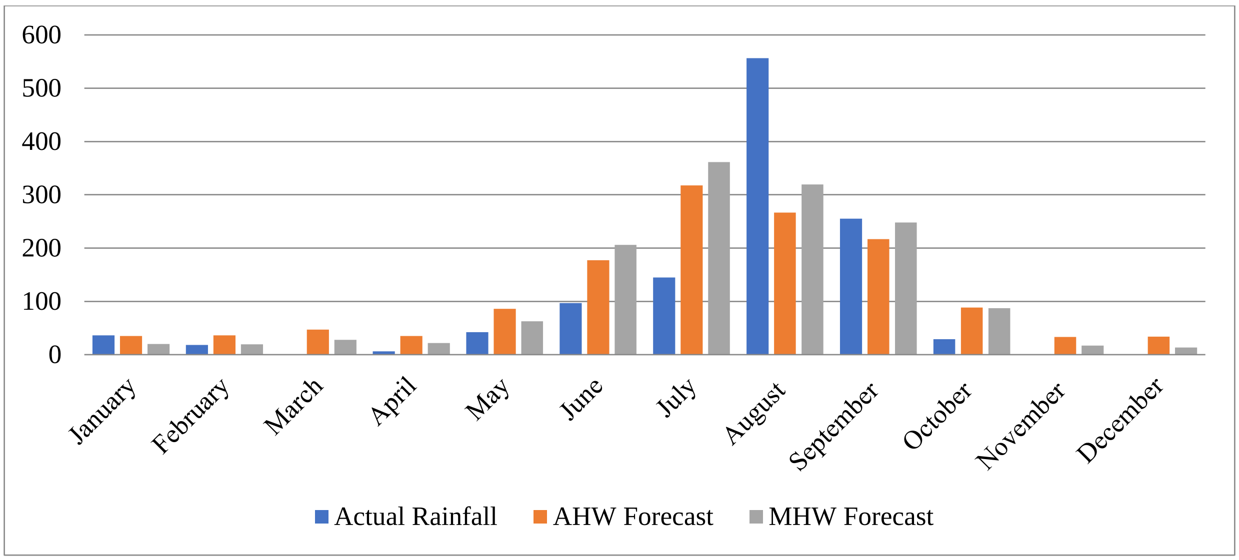

Fig. 1 Forecasts rainfall and actual rainfall for the year 2022

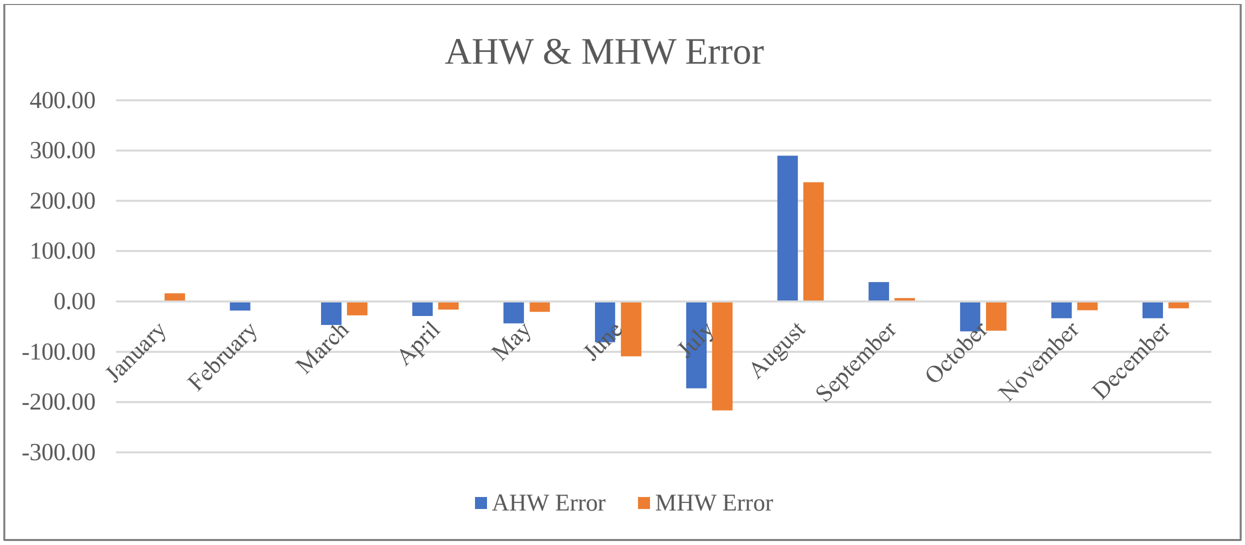

Fig. 2 Error of forecasting for the year 2022

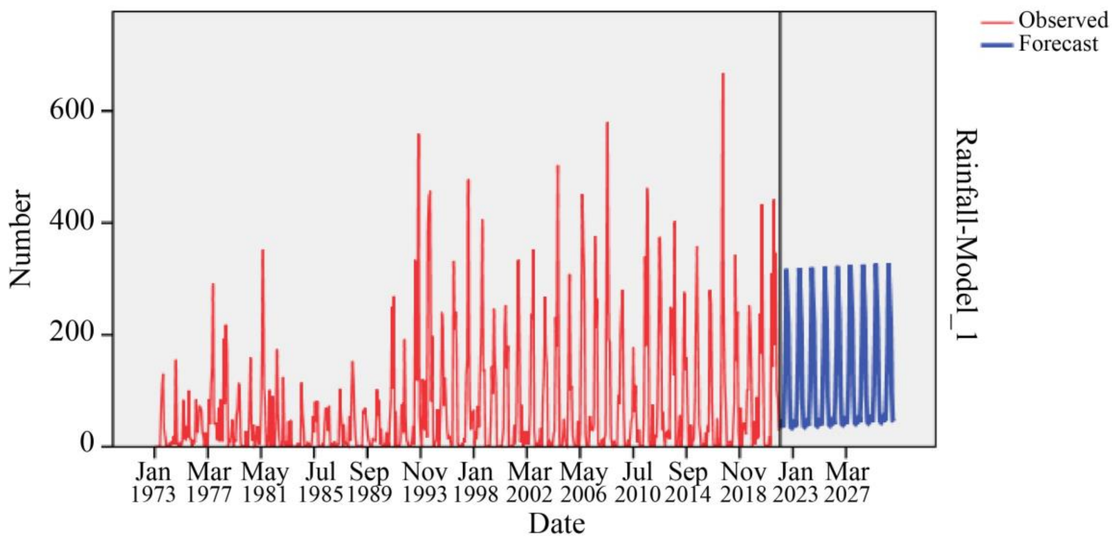

Fig. 3 Rainfall forecasts by AHW method

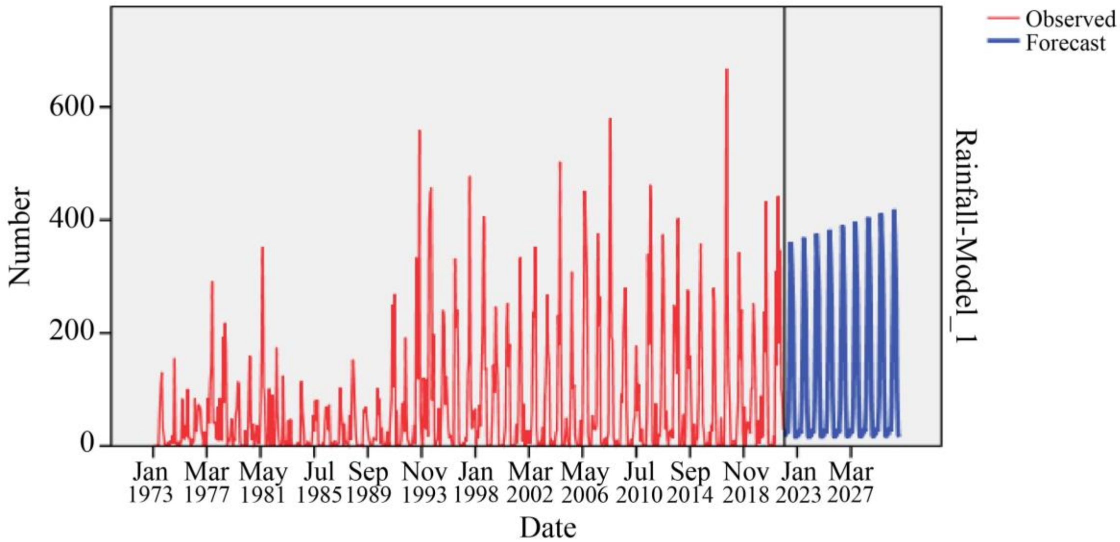

Fig. 4 Rainfall forecasts by MHW method

Table 3. Rainfall forecasting for upcoming years up to 2030

Months\Year	2023	2024	2025	2026	2027	2028	2029	2030
January	20.18	20.58	20.97	21.37	21.77	22.17	22.57	22.96
February	20.07	20.46	20.86	21.25	21.65	22.04	22.44	22.83
March	28.3	28.86	29.41	29.97	30.52	31.08	31.64	32.19
April	22.43	22.87	23.31	23.75	24.19	24.63	25.07	25.51
May	63.79	65.04	66.29	67.54	68.79	70.04	71.29	72.54
June	209.91	214.01	218.12	222.22	226.33	230.43	234.54	238.64
July	368.57	375.77	382.97	390.16	397.36	404.55	411.75	418.95
August	325.69	332.04	338.39	344.74	351.09	357.44	363.78	370.13
September	253.16	258.09	263.01	267.94	272.87	277.79	282.72	287.65
October	88.91	90.63	92.36	94.09	95.82	97.54	99.27	101
November	17.47	17.81	18.15	18.49	18.83	19.17	19.51	19.85
December	13.79	14.05	14.32	14.59	14.85	15.12	15.39	15.65

5. Conclusion

This paper presents analysis of mathematical model for the forecasting of rainfall in Ranchi from the period 2023 up to 2030. The mathematical model used in this paper is Holt-Winter exponential method. The historical rainfall data of Ranchi from the period 1973 to 2021 is used for forecasting. The rainfall data of 2022 is used to verify the accuracy of the mathematical model. The results suggest that the MHW method is more accurate than AHW method for the rainfall forecasting in Ranchi. The future forecasts by MHW method of rainfall from 2023 to 2030 clearly shows that the rainfall will gradually increase without affecting the seasonal behavior of rainfall. The monsoon rainfall will continue to contribute maximum rainfall to annual rainfall and winter rainfall will contribute the least rainfall to the annual rainfall. The annual rainfall of the year 2030 will be 1627.9 mm out of which 80.8% rainfall will be received in the monsoon season. The winter rainfall will contribute only 2.8% rainfall to the annual rainfall.

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