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Original Article

Percentiles of Exponentiated Generalized Inverse Rayleigh Distribution in Double Sampling Plan

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Abstract *- Acceptance sampling is one of the prominent techniques in quality control to reduce producer and consumer risk. If the 'lifetime' of a product is the main characteristic of interest, then sampling plans designed for testing the acceptability of a product are called reliability sampling plans. In this paper, double sampling plan based on percentile for Exponentiated Generalized Inverse Rayleigh distribution is proposed. The operating characteristic values as well as the minimum number of samples that guaranty the consumer's risk are computed. An illustrative example is given to show the strength of our proposed plan in the manufacturing industry.*

Keywords *- Double sampling plan, Exponentiated generalized inverse rayleigh distribution, Operating characteristic value, Percentiles, Producer's risk, Truncated life tests.*

1. Introduction

Quality control is the regulatory process through which the measurements of actual quality performance, comparison with other standards is made. Quality control is usually concern with what is called acceptance sampling (Harrison et. al.2004). Acceptance sampling plan is an important vital element in the control of quality of a product. It is a system that was developed to protect the consumer from getting unacceptably defective product. A good sampling plan will also protect the producer in the sense that lots produced at permissible levels of quality will have a good chance to be accepted by the plan (Schilling & Neubauer, 2008). Acceptance sampling is therefore an inspection procedure used to determine whether to accept or reject a specific quantity of product produced.

Reliability sampling plan is a statistical tool used in manufacturing industries for making decision about the disposition of lots based on the information obtained from a life test. If lifetime is a quality characteristic for some products, then sampling inspection for such products can be carried out by conducting suitable life test. An acceptance sampling plan under which sampling inspection is performed by conducting life test upon the sampled products may be termed as reliability sampling plan. Most of the acceptance sampling plans for a truncated life test has considered the determination of sample size as major issue this is due to the use of certain life time distribution.

Many authors studied truncated life test based on different distributions. Epstein (1954) in exponential case, Goode and Kao (1961) based on Weibull distribution, Gupta and Groll (1961) using Gamma distribution, Gupta (1962) considering normal and Log normal distribution, Fertig and Mann (1980) based on Two Parameter Weibull populations, Kantam and Rosaiah (1998) using Half Logistic distribution, Kantam et al. (2001) based on Log-Logistic model, Baklizi (2003) considering Pareto distribution of the second kind, Wu and Tsai (2005) using Birnbaum-Saunders distribution, Rosaiah and Kantam (2005) based on inverse Rayleigh distribution, Balakrishnan et al. (2007) using Birnbaum-Saunders distribution, Rao et al. (2008) considering Marshall–Olkin extended Lomax distribution etc. All these authors designed acceptance sampling plans based on the mean life time under a truncated life test.

The design of acceptance sampling plans based on the population mean under a truncated life test may not satisfy the requirement of engineering on the specific percentile of strength or breaking stress. When the quality of a specified low percentile is taken into account, the acceptance sampling plans based on the population mean could pass a lot which has the low percentile below the required standard of consumers. Furthermore, a small decrease in the mean with a simultaneous small increase in the variance can result in a downward shift in small percentiles of interest. This means that a lot of products could be accepted due to a small decrease in the mean life after inspection. But the material strengths of products are deteriorated significantly and may not meet the consumer's expectation. Therefore, engineers gave more attention to the percentiles of lifetimes than the mean life in life testing applications.

Lio et al. (2010) considered acceptance sampling plans for percentiles using truncated life tests and assuming Birnbaum-Saunders distribution. Srinivasa Rao and Kantam (2010) developed acceptance sampling plans for the percentiles of log-logistic distribution. Rao et al. (2013) studied acceptance sampling plans for percentiles assuming linear failure rate distribution. Rao (2013) considered acceptance sampling plans for percentiles based on the Marshall–Olkin extended Lomax distribution. Kaviyarasu and Fawaz (2017), developed acceptance sampling plans for percentiles based on the modified Weibull distribution. Jayalakshmi and Neena Krishna (2022) designed double sampling plan for truncated life tests based on percentiles using Kumaraswamy Exponentiated Rayleigh distribution.

2. Proposed Acceptance Sampling Plan

The Exponentiated Generalized Inverse Rayleigh distribution was developed by Fatima et al. in 2018. Assume that the lifetime of a product follows the Exponentiated Generalized Inverse Rayleigh distribution which has the following cumulative distribution function (c.d.f.) and probability density function (p.d.f.) respectively;

$$F(t) = [1 - (1 - e^{-\sigma^2/t^2})^{\alpha}]^{\gamma} \quad (1)$$

$$f(t;\alpha,\beta,\gamma) = \frac{2\alpha\gamma\sigma^2}{t^3} e^{-\sigma^2/t^2}(1 - e^{-\sigma^2/t^2})^{\alpha-1}[1 - (1 - e^{-\sigma^2/t^2})^{\alpha}]^{\gamma-1} \quad ; t > 0, \alpha,\sigma,\gamma > 0 \quad (2)$$

For given 0 < q < 1 the 100 qth actual percentile of the Exponentiated Generalized Inverse Rayleigh distribution can be given by

$$t_q = \frac{1}{\sigma}\left[-\ln(1 - (1 - q^{1/\gamma})^{1/\alpha})\right]^{-1/2} \quad (3)$$

The t_q increase as q increases

$$\text{Let } \eta = \left[-\ln(1 - (1 - q^{1/\gamma})^{1/\alpha})\right]^{-1/2} \quad (4)$$

Then from (3), $\lambda = \frac{\eta}{t_q}$

By letting $\delta = \frac{t}{t_q}$, $F(t)$ becomes

$$F(t) = [1 - (1 - e^{-(\delta\eta)^{-2}})^{\alpha}]^{\gamma} \quad (5)$$

Equation (5) gives the modified cdf and by partially differentiating the equation (5) w.r.t δ we will get the modified pdf for percentiles of Exponentiated Generalized Inverse Rayleigh distribution where t_q is the 10^{th} percentile of the given distribution.

A common practice in life testing is to terminate the life test by a pre-determined time t, the probability of rejecting a bad lot be at least P^*. The double sampling plan for percentiles under a truncated life test is to set up the minimum sample sizes n_1 and n_2 for the given acceptance numbers c_1 and c_2 such that the consumer's risk, the probability of accepting a bad lot, does not exceed $1 - P^*$. A bad lot means that the true $100q^{th}$ percentile, t_q, is below the specified percentile, t_q^0. Thus, the probability P^* is a confidence level in the sense that the chance of rejecting a bad lot with $t_q < t_q^0$ is at least equal to P^*. Therefore, for a given P^*, the proposed acceptance sampling plan can be represented as $(n_1, n_2, c_1, c_2, t/t_q^0)$.

3. Operating Procedure

- Draw a random sample of size n_1 from the lot and put on the test. The lot is accepted, if the number of failures d_1 less than c_1 that occurred before a pre-fixed experiment time t. Otherwise, the lot is rejected. If the failures d_1 are greater than c_2, the experiment will truncate before the time t.
- Draw the second sample of size n_2 and put them on the test during time t_q and count the number of defectives d_2 if the number of failures d_1 by t is between c_1 +1 and c_2. The lot is accepted if at most c_2 failures are observed from the two samples. Otherwise, the lot is rejected.

4. Minimum Sample Size

We have to obtain minimum sample size required to ensure a percentile life when the life test is terminated at a pre-specified time t_q^0 and when the observed number of failures does not exceed a given acceptance number c. For a fixed P^* our sampling plan is characterized by $\left(n_1, n_2, c_1, c_2, t/t_q^0\right)$. Here we consider sufficiently large sized lots so that the binomial distribution can be applied.

For given values of P^* ($0 < P^* < 1$), t_q^0 and c, the smallest positive integer, n required to assert that $t_q > t_q^0$ must satisfy the relation

$$L(p) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} + \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \sum_{d_2=0}^{c_2-d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \leq 1 - P^* \tag{6}$$

where $p = F(t, \delta_q)$, it is the probability of failure time during time t given a specified percentile of a lifetime t_q^0 and it depends on the $\delta\eta = \frac{t\eta}{t_q^0}$ since t_q^0 increases as q increases.

Accordingly, we have

$$F(t, \delta) < F(t, \delta_0) \Leftrightarrow \delta \leq \delta_0$$

$$F\left(t;\ t_q\right) < F\left(t;\ t_q^0\right) \Leftrightarrow t_q \geq t_q^0$$

The smallest sample size n satisfying (6) can be obtained for any given sampling plan $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$ is given in Table 1.

5. Operating Characteristic Function of the Proposed Sampling Plan

The OC function $L(p)$ of the acceptance sampling plan $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$ is the probability of accepting a lot as a function of $p = F\left(t, \delta_q\right)$ with $\delta_q = \frac{t}{t_q}$ is based on the number of failures from a sample of n items under a truncated life test at the time schedule t_q is given by

$$L(p) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} + \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \sum_{d_1=0}^{c_2-d_2} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \tag{7}$$

Therefore, we have

$p = F(t, \delta) = F\left({t}/{t_q^0}, {1}/{d_q}\right)$ where, $d_q = {t_q}/{t_q^0}$

Using eq. (7) the OC values can be obtained for any sampling plan $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$. The OC values for the proposed sampling plan is presented in Table 2.

6. Producer's Risk (λ)

The probability of rejecting a lot when $t_q > t_q^0$ is known as the producer's risk. For a given value of the producer's risk (λ), we are interested in knowing the value of d_q to ensure the producer's risk is less than or equal to λ if a sampling plan $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$ is developed at a specified confidence level P^*. Thus, one needs to find the smallest value d_q according to eq. (7)

$$L(p) \geq 1 - \lambda$$

Based on the sampling plans $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$ given in Table 1 the minimum ratios of $d_{0.10}$ at the producer's risk of λ = 0.05 are presented in Table 3.

7. Illustrative Example

Assume that the life distribution is Exponentiated Generalized Inverse Rayleigh distribution, and the experimenter is interested in showing that the true unknown 10th percentile life $t_{0.10}$ is at least 1000 hrs. Let $\alpha = 2, \gamma = 1$ and $q = 0.10$. It is desire to stop the experiment at time t=1000 hrs. For the Double Sampling plan $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$ = (34,35,0,2,1.0) and P^*= 0.95, the OC values obtained from the Table 2 are given below. The optimum sample sizes needed for the given requirement is found to be as $n_1 = 34, n_2 = 35$.

The respective OC values for the proposed ASP $\left(n_1, n_2, c_1, c_2, {t}/{t_q^0}\right)$ with P^*=0.95 for Exponentiated Generalized Inverse Rayleigh distribution from the Table 2 are given below.

$\boldsymbol{t_\theta}/\boldsymbol{t_\theta^0}$	0.75	1	1.25	1.5	1.75	2	2.25	2.5
$\boldsymbol{L(p)}$	0	0.0512	0.8678	0.9994	1	1	1	1

This shows that if the actual 10th percentile is equal to the required 10th percentile (${t_{0.10}}/{t_{0.10}^0} = 1$), the producer's risk is approximately 0.9488 (1–0.0512). The producer's risk is almost equal to zero when the actual 10th percentile is greater than or equal to 1.75 times the specified 10th percentile.

Table 3 gives the $d_{0.10}$ values for $c_1 = 0, c_2 = 2$ and different values of ${t}/{t_{0.10}^0}$ to assure that the producer's risk is less than or equal to 0.05. In this example, the value of $d_{0.10}$ is 1.3061 for $c_1 = 0, c_2 = 2, {t}/{t_{0.10}^0} = 1.0$ and λ =0.05. This means the product can have a 10th percentile life of 1.3061 times the required 10th percentile lifetime. That is under the above Double Sampling plan the product is accepted with probability of at least 0.95.

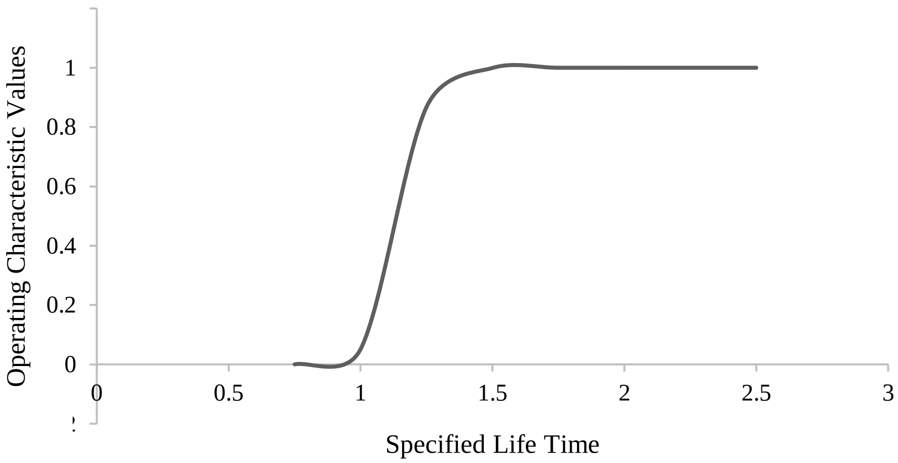

Fig. 1 OC curve for the sampling plan $\left(\boldsymbol{n_1 = 34, n_2 = 35., c_1 = 0, c_2 = 2,, {t}/{t_{0.10}^0} = 1.0}\right)$

8. Construction of the Table

Step 1: Find the value of η for the fixed values of $\alpha = 2, \gamma = 1$ *and* $q = 0.10$

Step 2: Set the value of ${t}/{t_q^0}$ =0.825, 0.85, 0.875, 0.9, 0.925, 0.95, 0.975, 1

Step 3: Find the sample size n by satisfying $L(p) \leq 1 - P^*$ when P^*=0.99, 0.95, 0.90 and 0.75

Here P^* is the probability of rejecting a bad lot and $L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$

Step 4: for the n value obtained find the $d_{0.10}$ value such that $L(p) \geq 1 - \lambda$ where $\lambda = 0.05$ and $p = F({t}/{t_q^0}, {1}/{d_q})$;$d_q = {t_q}/{t_q^0}$

9. Conclusion

In this paper, the reliability double sampling plan based on percentiles when the lifetime of a product follows Exponentiated Generalized Inverse Rayleigh distribution is developed. The procedure for construction of the proposed sampling plan for the 10th percentile is presented. The operating characteristic function values, the minimum number of samples required to ensure the specified life percentile and the corresponding producer's risk are obtained. Tables are provided to establish the proposed acceptance sampling plan. This plan is helpful for the industrial practitioner and the experimenter to save time of the experiment.

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Appendix

Table 1. Minimum Sample Size values necessary to assure 10th percentile for EGIR distribution

p^*	t/t_q^0							
	0.825	0.85	0.875	0.9	0.925	0.95	0.975	1
0.75	70,140	54,116	43,91	35,65	29,49	24,46	22,24	19,20
0.90	110,194	86,132	68,96	55,74	45,64	38,46	32,39	27,32
0.95	137,165	106,137	84,115	68,85	56,71	47,49	39,41	34,35
0.99	192,250	148,190	119,125	96,100	80,92	66,73	56,65	49,59

Table 2. Operating characteristic values of the sampling plan $\left(n_1, n_2, c_1, c_2, t/t_q^0\right)$ for a given P* under EGIR Distribution

p^*	n_1	n_2	t/t_q^0	$\frac{t_q}{t_q^0}$							
				0.75	1	1.25	1.5	1.75	2	2.25	2.5
0.75	70	140	0.825	0	0.2123	0.9918	1	1	1	1	1
	54	116	0.85	0	0.2056	0.9871	1	1	1	1	1
	43	91	0.875	0	0.2051	0.9821	1	1	1	1	1
	35	65	0.9	0.0001	0.2192	0.9793	1	1	1	1	1
	29	49	0.925	0.0001	0.2301	0.9753	1	1	1	1	1
	24	46	0.95	0.0003	0.2135	0.9637	0.9999	1	1	1	1
	22	24	0.975	0.0002	0.2785	0.9716	0.9999	1	1	1	1
	19	20	1	0.0004	0.2815	0.9656	0.9999	1	1	1	1
0.90	110	194	0.825	0	0.0684	0.9774	1	1	1	1	1
	86	132	0.85	0	0.0728	0.9723	1	1	1	1	1
	68	96	0.875	0	0.0784	0.9656	1	1	1	1	1
	55	74	0.9	0	0.0812	0.9564	1	1	1	1	1
	45	64	0.925	0	0.0781	0.9406	0.9999	1	1	1	1
	38	46	0.95	0	0.087	0.9351	0.9999	1	1	1	1
	32	39	0.975	0	0.0873	0.9201	0.9998	1	1	1	1
	27	32	1	0	0.0941	0.909	0.9996	1	1	1	1
0.95	137	165	0.825	0	0.0407	0.975	1	1	1	1	1
	106	137	0.85	0	0.0385	0.9618	1	1	1	1	1
	84	115	0.875	0	0.0366	0.9445	1	1	1	1	1
	68	85	0.9	0	0.0391	0.9332	0.9999	1	1	1	1
	56	71	0.925	0	0.038	0.9127	0.9999	1	1	1	1
	47	49	0.95	0	0.0452	0.9088	0.9998	1	1	1	1
	39	41	0.975	0	0.0482	0.892	0.9996	1	1	1	1
	34	35	1	0	0.0512	0.8678	0.9994	1	1	1	1
0.99	192	250	0.825	0	0.0079	0.9377	1	1	1	1	1
	148	190	0.85	0	0.0082	0.9164	1	1	1	1	1
	119	125	0.875	0	0.0086	0.9054	0.9999	1	1	1	1
	96	100	0.9	0	0.0087	0.8788	0.9999	1	1	1	1
	80	92	0.925	0	0.0073	0.8294	0.9997	1	1	1	1
	66	73	0.95	0	0.008	0.8034	0.9994	1	1	1	1
	56	65	0.975	0	0.0074	0.7554	0.9988	1	1	1	1
	49	59	1	0	0.0064	0.6965	0.9978	1	1	1	1

Table 3. Minimum ratio of true $d_{0.10}$ for the acceptability of a lot for the EGIR Distribution and producer's risk of α =0.05

p^*	n_1	n_2	t/t_q^0	$\frac{t_q}{t_q^0}$							
0.75	70	140	0.825	1.1828	1.1822	1.1831	1.183	1.1836	1.1832	1.1835	1.1827
	54	116	0.85	1.1960	1.1962	1.1954	1.1954	1.1959	1.1964	1.9656	1.1959
	43	91	0.875	1.2063	1.2064	1.2064	1.2058	1.2065	1.2062	1.2064	1.2072
	35	65	0.9	1.2105	1.2105	1.2109	1.2101	1.2111	1.2105	1.2115	1.2113
	29	49	0.925	1.2169	1.2165	1.2166	1.2161	1.2166	1.2169	1.2163	1.2168
	24	46	0.95	1.2344	1.2337	1.2332	1.2346	1.2339	1.2339	1.2340	1.2338
	22	24	0.975	1.2206	1.2203	1.2211	1.2215	1.2207	1.2206	1.2201	1.2209
	19	20	1	1.2301	1.2293	1.2295	1.2298	1.2297	1.2301	1.2294	1.2301
0.90	110	194	0.825	1.2201	1.2195	1.2203	1.2201	1.2208	1.2203	1.2205	1.2199
	86	132	0.85	1.2267	1.2264	1.2268	1.2262	1.2271	1.2267	1.2259	1.2271
	68	96	0.875	1.2341	1.234	1.2335	1.2339	1.2345	1.2333	1.2341	1.2341
	55	74	0.9	1.2441	1.2437	1.2441	1.2436	1.2432	1.2441	1.2432	1.244
	45	64	0.925	1.2576	1.2577	1.2579	1.2587	1.2587	1.2583	1.2586	1.2589
	38	46	0.95	1.2629	1.2633	1.2625	1.2635	1.2641	1.2643	1.2638	1.2642
	32	39	0.975	1.2761	1.2745	1.2751	1.2760	1.2762	1.2764	1.2758	1.2763
	27	32	1	1.2841	1.2831	1.2834	1.2843	1.2844	1.285	1.2844	1.2844
0.95	137	165	0.825	1.2246	1.2235	1.2248	1.2239	1.2246	1.2245	1.2241	1.2239
	106	137	0.85	1.2398	1.239	1.2394	1.239	1.2392	1.2395	1.2397	1.2393
	84	115	0.875	1.2546	1.2544	1.2549	1.2545	1.2552	1.2544	1.2551	1.2545
	68	85	0.9	1.2630	1.2631	1.2625	1.2636	1.2637	1.2631	1.2638	1.2633
	56	71	0.925	1.2774	1.2777	1.2769	1.2781	1.2773	1.2772	1.2772	1.2775
	47	49	0.95	1.2810	1.2805	1.2804	1.2815	1.2811	1.2809	1.2820	1.2806
	39	41	0.975	1.2916	1.2918	1.2920	1.2921	1.2924	1.2927	1.2926	1.2928
	34	35	1	1.3061	1.3066	1.3059	1.3054	1.3066	1.3064	1.3066	1.3061
0.99	192	250	0.825	1.2581	1.2588	1.2588	1.2583	1.2586	1.258	1.2586	1.2585
	148	190	0.85	1.2708	1.2719	1.2711	1.2715	1.2716	1.2709	1.2719	1.2717
	119	125	0.875	1.2779	1.2783	1.2775	1.2781	1.278	1.2778	1.2782	1.2782
	96	100	0.9	1.2912	1.2911	1.2915	1.2919	1.2911	1.2924	1.2918	1.292
	80	92	0.925	1.3118	1.312	1.3122	1.3114	1.3123	1.313	1.313	1.3134
	66	73	0.95	1.3229	1.3237	1.3237	1.3245	1.3248	1.3241	1.3242	1.3246
	56	65	0.975	1.3415	1.3409	1.3412	1.3423	1.3417	1.3421	1.3419	1.342
	49	59	1	1.3613	1.3613	1.3607	1.3617	1.362	1.362	1.3614	1.3618