Extended Approach to Fuzzy Soft Topological Spaces

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Abstract-
In this research paper, we continue the study on fuzzy soft topological spaces and investigate the properties and theorems of fuzzy soft local base point, fuzzy soft contact point. Then discuss the relationship between soft derived sets and fuzzy soft closed sets and also define and discuss the proportions of interior points which are important for further research on soft topology. This research not only can form the theoretical basis for further application of topology on soft sets but also lead to the development of information system.

Keywords*:* soft sets, topological space, fuzzy soft local base point, fuzzy soft contact point, fuzzy soft closed sets, fuzzy soft interior point.

I. INTRODUCTION

The fundamental concept of fuzzy Soft set theory was firstly introduced by Zadeh [9] in 1965 as a general mathematical tool for dealing with uncertain fuzzy not clearly defined objects. This theory brought a paradigmatic change in mathematics. But there exists difficulty, how to set the membership function in each particular case. The theory of fuzzy soft sets is more generalized concept than the theory of fuzzy soft sets but this theory has same difficulties. In 1999, Molodtsov [3] initiated the concept of soft sets. Molodtsov [3], is free of the difficulties present in this theories. Pie and Mio [4] discussed the relationship between soft sets is a parameterized classifications of the objects of the universe. The topological structures of soft set theories dealing with uncertainties were first studied by Cheang [2] .He introduces the notion of fuzzy topology and also studied some basic properties. K. Borgohain [7, 8] 2014 studied fuzzy soft separation axioms, fuzzy soft compact spaces. Some other [6],[10],[11],[12] studied on the compact fuzzy soft topological space. My main aim in this research paper is to develop the basic properties of fuzzy soft local base points and establish several equivalent forms of fuzzy soft topological spaces.

II. PRELIMINARIES

Definition2.1 [13] Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$, a pair (F,A) is called a fuzzy soft set over U . Where F is a mapping given by $F: A \to I^U$ Where I^U denotes the collection of all fuzzy subset of U.

Definition 2.2[13]If T is a fuzzy soft topology on (U,E) , then (U,E,T) is said to be a fuzzy soft topological space . Also each member of T is called a fuzzy soft open set in (U,E,T)

Definition 2.3[13] The complement of a fuzzy soft set (F,A) is denoted by $(F,A)^C$ and is defined by

$(F, A)^C = (F^C, A)$. Where $F^C: A \to \tilde{P}$ (U) is mapping given by

$F^C(\alpha) = U—F(\alpha)=[F(\alpha)]^C \ \forall \propto \in A$

Definition 2.4[6]

Union of two fuzzy soft sets (F, A) and (G,B) over a common universe U is a fuzzy soft set (H,C) where C= AUB $\forall$ ε Є C

H (ε) = F(ε) if ε Є A-B, G (B) if ε Є B-A, F (ε) ∪ G (ε) if ε Є A∩ B

And is written as F (A) Û (G,B) = (H,C)

Definition 2.5[6]

Intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is a fuzzy soft set (H,C), Where C= A ∩ B and ε Є C, H(ε)= F(ε) ∩ G(ε) and is written as (F,A)∩ (G,B) = (H,C)

Definition2.6[13] Let (X,E,T) be fuzzy soft topological space. A fuzzy soft set is called fuzzy soft closed if its complement is a member of $\tilde{T}$.

Definition2.7 [13] if T is a fuzzy soft topology on (X,E), the triple (X,E,T) is said to be fuzzy soft topological space. Each member of T is called fuzzy soft open set in (X,E,T).

Definition2.8 [7] Let (X,E,T) be a fuzzy soft topological space. Let (F,A) be fuzzy soft set over (X,E).Then the fuzzy soft closure of (F,A) denoted by $\overline{(F,A)}$ is defined as the intersection of all fuzzy soft closed sets which contains (F,A).

Definition2.8[8] Let (X,E,T) be a fuzzy soft topological space. Let (F,A) be a fuzzy soft set over (X,E). Then the fuzzy soft interior of (F,A) denoted by $(F,A)^o$, is defined as union of all fuzzy soft open sets contained in (F,A). That is $(F,A)^o = \tilde{\cup}\{(G,B):(G,B)$ is fuzzy soft open and $(G,B) \tilde{\subseteq} (F,A)\}$

Definition 2.9[13] A fuzzy soft topology T on (U,E) is a family of fuzzy soft sets over (U,E) satisfying the following properties

(i)$\bar{\phi}, \bar{E} \in \tilde{T}$

(ii) If F_A, $G_B \in$ T then $F_A \tilde{\cap}$ $G_B \in \tilde{T}$

(iii) If $F^{\alpha} A_{\alpha} \in$ T for all $\alpha \in \Delta$,then $\cup F^{\alpha} A_{\alpha} \in \tilde{T}$

III. Main Results

Definition 3.1 Let (X, E, T) be fuzzy soft topological space and $x \in X$. A non empty collection (B_x, E) of fuzzy soft topological nbds over (X,E) is called a fuzzy soft based for topological nbds system of x if every nbd (N,E) of x there is a $(B,E) \in (B_x, E)$ such that $x \in (B,E) \tilde{\subset} (N,E)$. Then the sub family (B_x, E) of all topological nbds of x is called fuzzy soft local base.

Example3.1 Let (X,D,T) be a fuzzy soft discrete topological space and $x \in X$, there will be many topological neighborhood of point x but the collection $(B_x, E) = \{\{x\}\}$ consisting of a single member $\{x\}$ which is fuzzy soft open sub sets over (X,E) and nbds of x is called fuzzy soft local base at $x \in X$. Because if we consider any fuzzy soft discrete nbds (N, E) of x there member (B, E) in (B_x, E) i.e $\{x\}$ such that $x \in \{x\} \tilde{\subset} (N,E)$

Theorem3.1 Let (X, E, T) be fuzzy soft topological space and taking (B_x, E) be fuzzy soft local base point of X then

1) $(B_x, E) \neq \tilde{\phi}$,For every $x \in X$

2) if $(B,E) \in (B_x, E)$ then $x \in (B,E)$

3) if $(A,E) \in (B_x, E) and (B,E) \in (B_{x,} E)$ $(A,E) \in (B_x, E) and (B,E) \in (B_{x,} E)$ then there exist a fuzzy set $(C,E) \in (B_x, E)$ such that $(C,E) \tilde{\subset} (A,E) \tilde{\cap} (B,E)$

4) if $(A,E) \in (B_x, E)$ then there exists a fuzzy set (B,E)such that $x \in (B,E) \tilde{\subset} (A,E)$ and for every point $y \in (B,E)$ there exists a $(C,E) \in (B_{y,} E)$ such that $(C,E) \tilde{\subset} (B,E)$

Proof:1) We know that X is a fuzzy soft open set and every fuzzy soft open set is a nbds of each of its points and such X is a fuzzy topological nbds of $x \in X$,Taking (B_x, E) be fuzzy soft local base at x and hence each fuzzy soft topological nbds of x then there exists a $(B,E)\in(B_x,E)$ such that $x\in(B,E)\tilde{\subset}X$ .Thus $(B_x,E)\neq\tilde{\phi},\forall x\in X$

2) if $(B,E)\in(B_x,E)$.Then (B,E) is a fuzzy topological nbds of x and hence $x\in(B,E)$

3) if $(A,E)\in(B_x,E),(B,E)\in(B_x,E)\Rightarrow(A,E),(B,E)$ are fuzzy topological nbds of x $\Rightarrow(A,E)\tilde{\cap}(B,E)$ is also fuzzy topological nbds of x but (B_x,E) be fuzzy soft local base at x. So $(C,E)\tilde{\subset}(A,E)\tilde{\cap}(B,E)$.

4) If $(A,E)\in(B_x,E)\Rightarrow(A,E)$ is fuzzy topological nbds of x $\Rightarrow$ there exists a fuzzy topological open set (B,E) such that $x\in(B,E)\tilde{\subset}(A,E)$. Again (B,E) being fuzzy topological open set and every fuzzy topological open set is fuzzy topological nbds each of its points $y\in(B,E)$ but (B_y,E) is a fuzzy soft local base at y and (B,E) being a fuzzy topological nbds of y therefore $(C,E)\in(B,E)$ such that $(C,E)\tilde{\subset}(B,E)$.

Definition3.2
Let (X,E,T) be a fuzzy soft topological space and A family (B,E) be fuzzy subspace of X is said to form a fuzzy base for T iff

1) $(B,E)\in\tau,\forall(B,E)\in(B_x,E)i.e(B_x,E)\tilde{\subset}\tau$

2) For each $(G,E)\in\tau,\forall x\in(G,E)$ then there exist some $(B,E)\in(B_x,E)$ such that $x\in(B,E)\tilde{\subset}(G,E)$

Example3.2: Let X={a,b,c,d} and τ ={x, ϕ ,{a},{b},{a,b},{c,d},{a,b,c},{b,c,d}} be a fuzzy soft topology for X .
Then (B, E)={{a},{b},{c, d}} is a fuzzy soft base for τ .
Let fuzzy topological open set (G, E)={a,c,d}

$a\in(G,E)and(B,E)=\{a\}\in(B_x,E)s.t,x\in(B,E)\tilde{\subset}(G,E)$

$c\in(G,E)and(B,E)=\{c,d\}\in(B_x,E)s.t,x\in(B,E)\tilde{\subset}(G,E)$

$d\in(G,E)and(B,E)=\{c,d\}\in(B_x,E)s.t,x\in(B,E)\tilde{\subset}(G,E)$

Proposition 3.1 Let (X,E,T) be a fuzzy soft topological space and let (B_x,E) be a fuzzy soft base topology for τ then 1) $X=\tilde{\cup}\{(B,E):(B,E)\in(B_x,E)\}$

2) for each $(B_1,E),(B_2,E)\in(B_x,E)$ and every $x\in(B_1,E)\tilde{\cap}(B_2,E)$ then there exist $(B,E)\in(B_x,E)$ such that $x\in(B,E)\tilde{\subset}(B_1,E)\tilde{\cap}(B_2,E)$

Proof; 1) (B_x,E) be a fuzzy soft base topology for τ then there exists fuzzy soft open set (G, E) and $x\in(G,E)$ then there exists fuzzy soft open set $x\in(B,E)\tilde{\subset}(G,E)$.

Choose (G, E)=X fuzzy soft open set then $x\in(B,E)\tilde{\subset}X$.

There fore $X=\tilde{\cup}\{(B,E):(B,E)\in(B_x,E)\}$

2) if $(B_1,E),(B_2,E)\in(B_x,E)$ Then (B_1,E) and (B_2,E) are fuzzy soft open sets and so $(B_1,E)\tilde{\cap}(B_2,E)$ be fuzzy soft open set and (B_x,E) being fuzzy soft base for τ . Then we consider $(B,E)\in(B_x,E)$ such that $x\in(B,E)\tilde{\subset}(B_1,E)\tilde{\cap}(B_2,E)$.

Remarks3.1 Let (X,E,T) be a fuzzy soft topological space and X be a fuzzy soft non empty set and (B_x,E) be the collection of fuzzy soft subset of X then (B_x,E) be the fuzzy soft base topology on X .Then $X=\tilde{\cup}\{(B,E):(B,E)\in(B_x,E)\}$. For every $(B_1,E)\in(B_x,E)$ and $(B_2,E)\in(B_x,E)$ then there exists $(B,E)\in(B_x,E)$ such that $x\in(B,E)\tilde{\subset}(B_1,E)\tilde{\cap}(B_2,E)$.

Definition3.3: (Fuzzy soft Sub base) Let (X,E,T) be a fuzzy soft topological space and (B^*,E) be fuzzy soft subsets over (X,E) is called a fuzzy soft topology T iff finite intersection of members of (B^*,E) forms a fuzzy soft base for T.

Example3.3 Let X={a,b,c,d} and T={X, ϕ ,{a},{a,c},{a,d},{ac,d}} and (B^*,E)={{a,c},{a,d}} is a fuzzy soft sub-base for T because the fuzzy soft family (B,E) of finite intersection of (B^*,E) is given by (B^*,E)={{a},{a,c},{a,d},X}

Remarks3.2 let (X,E,T) be a fuzzy soft topological space and (A,E) be a fuzzy soft subsets over (X,E). Then a point fuzzy soft point $x \in X$ is called a fuzzy soft contact point of (A,E) if and only if every fuzzy soft open nbds over (X,E) contains at least one point of (A,E) i.e $(N,E)\tilde{\cap}(A,E) \neq \phi$. Where (N, E) is fuzzy soft nbd of $x \in X$.

Definition3.4 Let (X,E,T) be a fuzzy soft topological space and (A,E) be a fuzzy soft subspace over (X,E).Then a point $x \in X$ is called fuzzy soft accumulation point of (A,E) iff [(N,E)-{x}] $\tilde{\cap}$ (A,E) $\neq \tilde{\phi}$ for every nbd (N,E) of (X,E).

Definition3.5 Fuzzy Soft Derived Set: Let (X, E, T) be a fuzzy soft topological space and A be the all soft limits of fuzzy soft subset over (X, E) is called a fuzzy soft derived set and denoted by $\tilde{D}(A,E)$.

Definition3.6 Fuzzy Soft Isolated Point: Let (X, E, T) be a fuzzy soft topological space and a fuzzy soft point $x \in (A,E)$ .Where (A,E) is a fuzzy soft subsets over (X,E) is called fuzzy soft isolated point if $(A,E)\tilde{\cap}\{(N,E)-\{x\}\} = \tilde{\phi}$. Where (N, E) is any fuzzy soft nbd over (X, E).

Theorem3.2 Let (X, E, T) be a fuzzy soft topological space and (A, E), (B, E) be fuzzy soft subsets over (X, E) then the fuzzy soft derived sets $\tilde{D}(A,E)$ and $\tilde{D}(B,E)$ satisfies the following properties

(1) $\tilde{D}(\phi) = \tilde{\phi}$

(2) $(A,E)\tilde{\subseteq}(B,E) \Rightarrow x \in \tilde{D}[(A,E)-\{x\}]$

(3) $\tilde{D}[(A,E)\tilde{\cup}(B,E)] = \tilde{D}(A,E)\tilde{\cup}\tilde{D}(B,E)$

(4) $\tilde{D}[(A,E)\tilde{\cap}(B,E)] \tilde{\subseteq} \tilde{D}(A,E)\tilde{\cap}\tilde{D}(B,E)$

Proof :(1) let (X,E,T) be a fuzzy soft topological space and $x \in X$ be any fuzzy soft point and $x \in \tilde{D}(\phi)$ .Since ϕ contains no fuzzy soft point. Therefore no. x, $x \in X$ is a fuzzy soft limit point of ϕ i.e $x \in \tilde{D}(\phi)$ .Hence $\tilde{D}(\phi) = \tilde{\phi}$.

(2)Let $x \in \tilde{D}(A,E) \Rightarrow$ x is a limit point of (A,E). Every fuzzy nbd of x containing at least one point other than x over (A,E)-{x}. Hence $x \in \tilde{D}(A,E) \Rightarrow x \in D\{(A,E)-\{x\}\}$

(3)We know that $(A,E)\tilde{\subseteq}(A,E)\tilde{\cup}(B,E) \Rightarrow \tilde{D}(A,E)\tilde{\subseteq}\tilde{D}[(A,E)\tilde{\cup}(B,E)]$

$(B,E)\tilde{\subseteq}(A,E)\tilde{\cup}(B,E) \Rightarrow \tilde{D}(B,E)\tilde{\subseteq}\tilde{D}[(A,E)\tilde{\cup}(B,E)]$

Hence $\tilde{D}(A,E)\tilde{\cup}\tilde{D}(B,E)\tilde{\subseteq}\tilde{D}[(A,E)\tilde{\cup}(B,E)]............(a)$

Now, $\tilde{D}(A,E)\tilde{\cup}\tilde{D}(B,E)\tilde{\subseteq}\tilde{D}(A,E)\tilde{\cup}\tilde{D}(B,E)............(b)$

From (a) &(b) we get $\tilde{D}[(A,E)\tilde{\cup}(B,E)] = \tilde{D}(A,E)\tilde{\cup}\tilde{D}(B,E)$

(4) if $(A,E)\tilde{\cap}(B,E)\tilde{\subseteq}(A,E) \Rightarrow \tilde{D}[(A,E)\tilde{\cap}(B,E)]\tilde{\subseteq}\tilde{D}(B,E)$

Again if $(A,E)\tilde{\cap}(B,E)]\tilde{\subseteq}(B,E) \Rightarrow \tilde{D}(A,E)\tilde{\cap}(B,E)]\tilde{\subseteq}\tilde{D}(B,E)$

Hence, $\tilde{D}(A,E)\tilde{\cap}(B,E)]\tilde{\subseteq}[\tilde{D}(A,E)\tilde{\cap}\tilde{D}(B,E)]$

Definition3.7 Let (X,E,T) be a fuzzy soft topological space an $(A,E)\tilde{\subseteq}X$. The intersection of all fuzzy soft closed supersets of (A,E)is called the fuzzy soft closure of (A,E) and denoted by cl(A,E) or $\overline{(A,E)}$.

Theorem3.3 Let (X,E,T) be fuzzy soft topological space and $(A,E)\tilde{\subseteq}X$ .Then

(1) $\overline{(A,E)}$ is the smallest fuzzy soft closed set containing (A,E).

(2) (A,E) is fuzzy soft closed $\Leftrightarrow \overline{(A,E)} = (A,E)$

Proof: 1) Let $\{(A_\mu, E) : \mu \in \Lambda\}$ be the collection of all fuzzy soft closed super set of (A,E).Then $(A,E) = \tilde{\cap}\{(A_\mu, E) : \mu \in \Lambda\}$ is fuzzy soft closed . Now each (A_μ, E) is a fuzzy soft super set of (A,E) therefore $(A,E) \tilde{\subset} (F,E)$ .Again $(A,E) \tilde{\cap} (B,E) \tilde{\subset} (B,E) i.e (F,E) \tilde{\subset} (A_\mu, E)$ for each $x \in \Lambda$. Hence (F,E) is the fuzzy soft smallest closed set containing (A,E).

Therefore, cl(A,E) is the smallest fuzzy closed set containing (A,E).

2) Let (A,E) be fuzzy soft closed then $(A,E) \tilde{\subset} (A,E)$ and hence $\overline{(A,E)} = (A,E)$

Conversely, Let $\overline{(A,E)} = (A,E)$ then (A,E) is closed fuzzy soft set $\overline{(A,E)}$ is the smallest closed fuzzy soft set containing (A,E) and form $\overline{(A,E)} = (A,E)$ .Hence (A,E) is fuzzy soft closed set.

Proposition3.2 Let (X, E, T) be a fuzzy soft topological space .Let (A, E) and (B, E) be two fuzzy soft sets over (X, T) .Then

(i) $\overline{\tilde{\phi}} = \tilde{\phi}$

(ii) $(A,E) \tilde{\subseteq} \overline{(A,E)}$

(iii) $(A,E) \tilde{\subseteq} (B,E) \Rightarrow \overline{(A,E)} \tilde{\subseteq} \overline{(B,E)}$

(iv) $\overline{(A,E) \tilde{\cup} (B,E)} = \overline{(A,E)} \tilde{\cup} \overline{(B,E)}$

(v) $\overline{(A,E) \tilde{\cap} (B,E)} \tilde{\subseteq} \overline{(A,E)} \tilde{\cap} \overline{(B,E)}$

(vi) $\overline{\overline{(A,E)}} = \overline{(A,E)}$

Proof :(i) $\tilde{\phi}$ is a fuzzy soft closed set $\Rightarrow \overline{\tilde{\phi}} = \tilde{\phi}$

(ii) Since $\overline{(A,E)}$ is the smallest fuzzy soft closed set containing (A, E).Thus $(A,E) \tilde{\subseteq} \overline{(A,E)}$ or $\overline{(A,E)} \tilde{\supseteq} (A,E)$

(iii)Let $(A,E) \tilde{\subseteq} (B,E)$ .Also $(B,E) \tilde{\subseteq} \overline{(B,E)} \Rightarrow (A,E) \tilde{\subseteq} \overline{(B,E)}$ .So $\overline{(A,E)}$ is the smallest fuzzy soft closed set containing (A,E).Hence $\overline{(A,E)} \tilde{\subseteq} \overline{(B,E)}$

(iv) $(A,E) \tilde{\subseteq} (A,E) \tilde{\cup} (B,E)$ and $(B,E) \tilde{\subseteq} (A,E) \tilde{\cup} (B,E)$ Therefore,

$\overline{(A,E)} \tilde{\subseteq} \overline{(A,E) \tilde{\cup} (B,E)}$, $\overline{(B,E)} \tilde{\subseteq} \overline{(A,E) \tilde{\cup} (B,E)} \Rightarrow \overline{(A,E)} \tilde{\cup} \overline{(B,E)} \tilde{\subseteq} \overline{(A,E) \tilde{\cup} (B,E)}$

Also ,(A,E) $\tilde{\subseteq} \overline{(A,E)}$,(B,E) $\tilde{\subseteq} \overline{(B,E)} \Rightarrow \overline{(A,E)} \tilde{\cup} \overline{(B,E)}$

Now $\overline{(A,E)} \tilde{\cup} \overline{(B,E)}$ is a fuzzy soft closed set containing (A,E) $\tilde{\cup}$ (B,E).Also , $\overline{(A,E) \tilde{\cup} (B,E)}$ is smallest fuzzy soft closed set containing (A,E) $\tilde{\cup}$ (B,E).

(V) $(A,E) \tilde{\cap} (B,E) \tilde{\subseteq} (A,E), (A,E) \tilde{\cap} (B,E) \tilde{\subseteq} (B,E)$

$\Rightarrow \overline{(A,E) \tilde{\cap} (B,E)} \tilde{\subseteq} \overline{(A,E)}, \overline{(A,E) \tilde{\cap} (B,E)} \tilde{\subseteq} \overline{(B,E)}$

$\Rightarrow \overline{(A,E) \tilde{\cap} (B,E)} \tilde{\subseteq} \overline{(A,E)} \tilde{\cap} \overline{(B,E)}$

(VI)We know ,A be fuzzy soft closed then $\overline{(A,E)}$ =(A,E),Also $\overline{(A,E)}$ is fuzzy soft closed . so, we have $\overline{\overline{(A,E)}} = \overline{(A,E)}$

Proposition3.3Let (X, E, T) be a fuzzy soft topological space. Let (A, E) and (B, E) be two fuzzy soft subset over (X, T), Then

(i) $\tilde{X}^0 = \tilde{X}, \tilde{\phi}^0 = \tilde{\phi}$

(ii) $(A,E)^0 \tilde{\subseteq} (A,E)$

(iii) $(A,E) \tilde{\subseteq} (B,E) \Rightarrow (A,E)^0 \tilde{\subseteq} (B,E)^0$

(iv) $[(A,E) \tilde{\cap} (B,E)]^0 = (A,E)^0 \tilde{\cap} (B,E)^0$

(v) $(A,E)^0 \tilde{\cup} (B,E)^0 \tilde{\subseteq} [(A,E) \tilde{\cap} (B,E)]^0$

(vi) $(A,E)^{0^o} = (A,E)^0$

Proof :(i) X and $\tilde{\phi}$ are fuzzy soft open sets. So, $\tilde{X}^0 = \tilde{X}, \tilde{\phi}^0 = \tilde{\phi}$

(ii) $x \in (A,E)^0 \Rightarrow$ x is an fuzzy soft interior point of (A,E)

$\Rightarrow$(A,E)is a fuzzy soft nbd of x

$\Rightarrow x \in (A,E) \Rightarrow (A,E)^0 \tilde{\subseteq} (A,E)$

(iii)Let $x \in (A,E)^0$,x is any fuzzy soft interior point of (A,E) $\Rightarrow$(A,E)is a nbd of x, so $(B,E) \tilde{\supseteq} (A,E)$,(B,E) is also a fuzzy soft nbd of x $\Rightarrow x \in (B,E)^0 \Rightarrow (A,E)^0 \tilde{\subseteq} (B,E)^0$

(iv) $(A,E) \tilde{\cap} (B,E) \tilde{\subseteq}$(A,E), $(A,E) \tilde{\cap} (B,E) \tilde{\subseteq}$(B,E)

And $[(A,E) \tilde{\cap} (B,E)]^0 \tilde{\subseteq} (A,E)^0, [(A,E) \tilde{\cap} (B,E)]^0 \tilde{\subseteq} (B,E)^0$

$\Rightarrow [(A,E) \tilde{\cap} (B,E)]^0 \tilde{\subseteq} (A,E)^0 \tilde{\cap} (B,E)^0$

Also, $x \in (A,E)^0 \tilde{\cap} (B,E)^0$

$\Rightarrow x \in (A,E)^0, x \in (B,E)^0$

$\Rightarrow$x is a fuzzy soft interior point of each fuzzy soft sets (A,E)and (B,E)

$\Rightarrow x \in [(A,E) \tilde{\cap} (B,E)]^0$

$\Rightarrow (B,E)^0 \tilde{\cap} (B,E)^0 \tilde{\subseteq} [(A,E) \tilde{\cap} (B,E)]^0$, Therefore, $[(A,E) \tilde{\cap} (B,E)]^0 = (A,E)^0 \tilde{\cap} (B,E)^0$

(v)We have ,

$(A,E) \tilde{\subseteq} (A,E) \tilde{\cup} (B,E) \Rightarrow (A,E)^0 \tilde{\subseteq} [(A,E) \tilde{\cup} (B,E)]^0 \Rightarrow (A,E)^0 \tilde{\cup} (B,E)^0 \tilde{\subseteq} [(A,E) \tilde{\cup} (B,E)]^0$

(vi) $(A,E)^0$ is fuzzy soft open set .So, $(A,E)^{0^o} = (A,E)^0$

Example3.4 Consider the usual topology U of R and

Let (A,E)=[0,1[,(B,E)=]1,2[, $(A,E)^0$ =]0,1[, $(B,E)^0$ =]1,2[

$(A,E)^0 \tilde{\cup} (B,E)^0 =]0,1[\tilde{\cup}]1,2[=]0,2[- \{1\}$.

Also, $(A,E) \tilde{\cup} (B,E) = [0,2[$.

Hence, $[(A,E) \tilde{\cup} (B,E)]^0 =]0,2[$

V. Conclusion.

Fuzzy topological space are powerful mathematical tools for modeling ; uncertain systems in industry, nature and humanity and facilitators for common sense reasoning in decision making in the absence of complete and precise information. In this present work I have continued to study the properties of fuzzy soft topological spaces. Introduced Fuzzy soft local base point and base for Fuzzy soft Topological space and fuzzy soft subspace, fuzzy soft derived set and have established several interesting theorems, examples. I hope that the findings in this research paper will help researchers enhance and promote the father study on fuzzy soft topology to carry out a general framework for their applications in practical life.

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