(Sp)* Closed Sets in Topological Spaces

L. Elvina Mary,

Assistant Professor, Nirmala College for Women, Coimbatore.

R.Chitra,

PG student, Nirmala College for Women, Coimbatore.

Abstract:

In this paper we introduce a new class of sets namely, $(sp)^*$-closed sets and properties of this set are investigated. We introduce $(sp)^*$-continuous maps and $(sp)^*$-irresolute maps.

Keywords: $(sp)^*$-closed sets, $(sp)^*$-continuous and $(sp)^*$-irresolute.

1. INTRODUCTION:

Levine [10], Mashhour et. al. [14], Njastad [16] and Abd El-Monsef et. al. [1] introduced semi-open sets, preopen sets, α -sets and semi-pre-open sets respectively. Levine [9] introduced generalized closed (briefly g-closed) sets in 1970. Maki et. al.[12] and Bhattacharya and Lahiri [5] introduced and studied $g\alpha$ –closed sets and sg-closed sets respectively. Maki et. al. [11] introduced αg -closed sets. S.P.Arya and T.Nour [3] defined gs-closed sets in 1994. Dontchev [7] introduced gsp-closed sets by generalizing semi-pre-open sets. In this paper we introduce a new class of sets namely $(sp)^*$-closed sets. Further we introduce $(sp)^*$-continuous maps and $(sp)^*$-irresolute maps.

2. PRELIMINARIES:

Throughout this paper (X,τ) represents a non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X,τ), cl(A) and int(A) and α Cl(A)denote the closure, interior and α closure of the subset A.

Definition:2.1

A subset A of a topological space (X,τ) is said to be a

1. pre-closed[14] if cl(int(A)) $\subseteq$ A.

2. semi-closed[10] if int(cl(A)) $\subseteq$ A.

3. semi-pre-closed[1] if int(cl(Int(A))) $\subseteq$ A.

4. α -closed[16] if cl(Int(cl(A))) $\subseteq$ A.

5. g-closed[9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

6. gsp-closed[7] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

7. αg -closed[11] if $\alpha cl(A) \subseteq U$ whenever A $\subseteq$ U and U is open in X.

8. $g\alpha$ -closed[12] if $\alpha cl(A) \subseteq U$ whenever A $\subseteq$ U and U is α -open in X.

9. sg-closed[5] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.

10. gp-closed[13] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

11. α^* -closed[18] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α -open in X.

12. gs-closed[3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

13. ωg -closed[15] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

14. $\hat{g}$ -closed[17] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.

Definition:2.2

A function f : (X, τ) $\rightarrow$ (Y, σ) is called a

1. α -continuous[16] if f^{-1}(V) is α -closed in (X, τ) for every closed set V of (Y, σ).

2. g-continuous[4] if f^{-1}(V) is g-closed in (X, τ) for every closed set V of (Y, σ).

3. sg-continuous[5] if f^{-1}(V) is sg-closed in (X, τ) for every closed set V of (Y, σ).

4. gs-continuous[6] if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V of (Y, σ).

5. αg -continuous[8] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V of (Y, σ).

6. $g\alpha$ -continuous[12] if $f^{-1}(V)$ is $g\alpha$ -closed in (X, τ) for every closed set V of (Y, σ).

7. gsp-continuous[7] if $f^{-1}(V)$ is gsp- closed in (X, τ) for every closed set V of (Y, σ).

8. gp-continuous[2] if $f^{-1}(V)$ is gp-closed in (X, τ) for every closed set V of (Y, σ).

9. ωg -continuous[15] if $f^{-1}(V)$ is ωg -closed in (X, τ) for every closed set V of (Y, σ).

10. α^* -continuous[18] if $f^{-1}(V)$ is α^* -closed in (X, τ) for every closed set V of (Y, σ).

11. $\hat{g}$ -continuous[17] if $f^{-1}(V)$ is $\hat{g}$ -closed in (X, τ) for every closed set V of (Y, σ).

3. Basic Properties of $(sp)^*$-Closed Sets:

We introduce the following definition.

Definition 3.01: A subset A of a topological space (X,τ) is said to be $(sp)^*$-closed if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-pre-open in X.

Theorem 3.02: Every closed set is $(sp)^*$-closed.

Proof follows from the definition.

Theorem 3.03: Every $(sp)^*$-closed set is gsp-closed.

Proof: Let A be $(sp)^*$-closed. Let A $\subseteq$ U and U be open. Then A $\subseteq$ U and U is semi-pre-open and cl(A) $\subseteq$ U, since A is $(sp)^*$-closed. Then spcl(A) $\subseteq$ cl(A) $\subseteq$ U. Therefore A is gsp-closed.

The converse of the above theorem is not true as seen in the following example.

Example 3.04: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ A={a,b} is gsp-closed but not $(sp)^*$-closed in (X,τ)

Theorem 3.05: Every $(sp)^*$-closed set is g-closed.

Proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 3.06: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={a,c} is g-closed but not (sp)*-closed in (X,τ)

Theorem 3.07: Every (sp)*-closed set is gs-closed.

Proof: Let A be (sp)*-closed. Let A$\subseteq$U and U be open. Then A$\subseteq$U and U is semi-pre-open and cl(A)$\subseteq$U, since A is (sp)*-closed. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is (sp)*-closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.08: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={c} is gs-closed but not a (sp)*-closed set in (X,τ)

Theorem 3.09: Every (sp)*-closed set is gp-closed.

Proof: Let A be (sp)*-closed. Let A$\subseteq$U and U be open. Then A$\subseteq$U and U is semi-pre-open and cl(A)$\subseteq$U, since A is (sp)*-closed. Then $pcl(A) \subseteq cl(A) \subseteq U$. Hence A is gp-closed.

The converse of the above Theorem is not true always as seen in the following example.

Example 3.10: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={a,c} is gp-closed but not (sp)*-closed in (X,τ) .

Theorem 3.11: Every (sp)*-closed set is sg-closed.

Proof: Let A be (sp)*-closed. Let A$\subseteq$U and U be semi-pre-open. Then A$\subseteq$U and U is semi-pre-open and cl(A)$\subseteq$U since A is (sp)*-closed. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is sg-closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.12: Let X={a,b,c}, $\tau = \{\phi, \{a\}, X\}$. A={c} is sg-closed but not (sp)*-closed in (X,τ)

Theorem 3.13: Every (sp)*-closed set is $\hat{g}$-closed.

Proof follows from the definition.

The converse of the above theorem need not be true in general as it can be seen from the following example.

Example 3.14: Let X={a,b,c} $\tau = \{\phi, \{b,c\}, X\}$. A={a,c} is $\hat{g}$-closed but not $(sp)^*$-closed in (X,τ)

Theorem 3.15: Every $(sp)^*$-closed set is αg-closed.

Proof: Let A be $(sp)^*$-closed. Let A $\subseteq$ U and U be open. Then A $\subseteq$ U and U is semi-pre-open and cl(A) $\subseteq$ U, since A is $(sp)^*$-closed. Then α cl(A) $\subseteq$ cl(A) $\subseteq$ U. Hence A is αg-closed.

The following example supports that the converse of the above theorem is not true.

Example 3.16: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={b} is αg-closed but not $(sp)^*$-closed in (X,τ).

Theorem 3.17: Every $(sp)^*$-closed set is $g\alpha$-closed.

Proof: Let A be $(sp)^*$-closed. Let A $\subseteq$ U and U be α-open. Then A $\subseteq$ U and U is semi-pre-open and cl(A) $\subseteq$ U, since A is $(sp)^*$-closed. Then α cl(A) $\subseteq$ cl(A) $\subseteq$ U. Hence A is $g\alpha$-closed.

The converse of the above theorem is not true always as seen in the following example.

Example 3.18: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={c} is $g\alpha$-closed but not $(sp)^*$-closed in (X,τ)

Theorem 3.19: Every $(sp)^*$-closed set is ωg-closed.

Proof: Let A be $(sp)^*$-closed. Let A $\subseteq$ U and U be open. Then A $\subseteq$ U and U is semi-pre-open and cl(A) $\subseteq$ U, since A is $(sp)^*$-closed. Then $cl(\text{int}(A)) \subseteq cl(A) \subseteq U$ .Hence A is ωg-closed.

The converse of the above theorem is not true always as seen in the following example.

Example 3.20: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={b} is ωg -closed but not $(sp)^*$-closed in (X,τ) .

Theorem 3.21: Every $(sp)^*$-closed set is α^*–closed.

Proof follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 3.22: Let X={a,b,c}, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$. A={c} is α^*-closed but not $(sp)^*$-closed in (X,τ).

Theorem 3.23: If A and B are $(sp)^*$-closed, then $A \cup B$ is also $(sp)^*$-closed.

Proof: Let A and B are $(sp)^*$-closed sets. Let $A \cup B$ where U is semi-pre-open. $cl(A \cup B) = cl(A) \cup cl(B) \subseteq U$. Hence $A \cup B$ is $(sp)^*$-closed.

Theorem 3.24: If A is $(sp)^*$-closed set $\ni A \subseteq B \subseteq cl(A)$ then, B is also a $(sp)^*$-closed set.

Proof: Let A be $(sp)^*$-closed set and $A \subseteq B \subseteq cl(A)$. Let B $\subseteq$ U where U is semi-pre-open.

$B \subseteq cl(A), cl(B) \subseteq cl(A) \subseteq U$. Hence B is $(sp)^*$-closed.

Theorem 3.25: A is a $(sp)^*$-closed set of (X, τ) if and only if cl(A)\A does not contain any non-empty semi-pre-closed set.

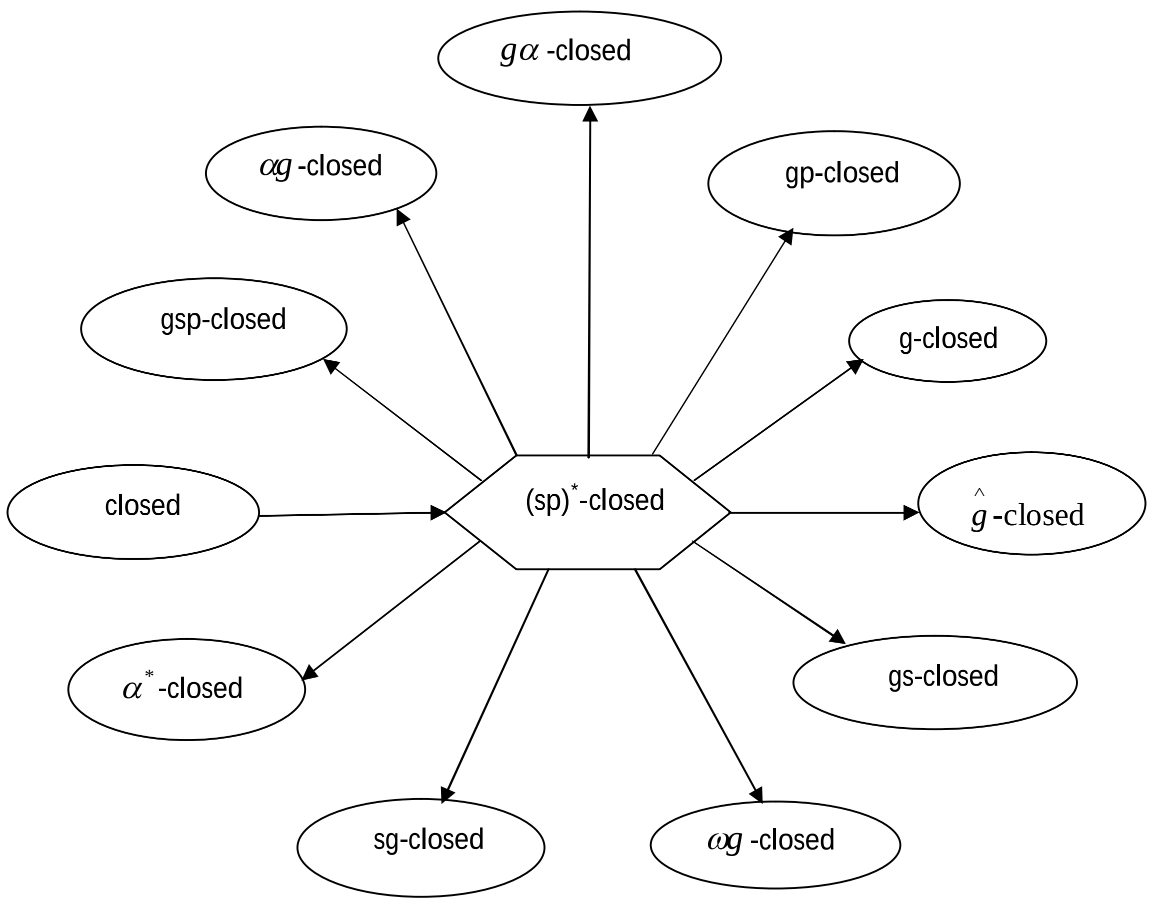
Proof: Necessity: Let F be a semi-pre-closed set of (X, τ) such that F $\subseteq$ cl(A)\A. Then A $\subseteq$ X\F. A is (sp)-closed and X\F is semi-pre-open, cl(A) $\subseteq$ X\F. Since F $\subseteq$ X\Cl(A).
So, F $\subseteq$ ((X\Cl(A)) $\cap$ ((Cl(A)\A)= ϕ , Therefore F= ϕ .

Sufficiency: Let A be a subset of (X, τ) such that Cl(A)\A does not contain any non-empty semi-pre-closed set. Let U be a semi-pre-open set of (X, τ) such that A $\subseteq$ U. If Cl(A) $\not\subseteq$ U, then Cl(A) $\cap U^c \neq \phi$ and Cl(A) $\cap U^c$ is semi-pre-closed. Therefore $\phi \neq$ Cl(A) $\cap U^c \subseteq$ Cl(A)\A. Therefore cl(A)\A contains a non-empty semi-pre-closed set, which is a contradiction. Therefore cl(A) $\subseteq$ U. Therefore A is a $(sp)^*$-closed set.

Theorem 3.26: If A is both semi-pre-open and $(sp)^*$-closed, then A is closed.

Proof: Let A be both semi-pre-open and $(sp)^*$-closed. Let $A \subseteq A$, where A is semi-pre-open. Then $cl(A) \subseteq A$, since A is $(sp)^*$-closed. Therefore A is closed.

The above results can be represented as the following diagram.



where $A \to B$ represents A implies B, but not B implies A.

4.$(sp)^*$-continuous And $(sp)^*$-irresolute Maps

We introduce the following definition.

Definition 4.01: A function $f:(X,\tau) \to (Y,\sigma)$ is called $(sp)^*$-continuous if $f^{-1}(V)$ is a $(sp)^*$-closed set of (X,τ) for every closed set V of (Y,σ).

Theorem 4.02: Every continuous map is $(sp)^*$-continuous.

Theorem 4.03: Every $(sp)^*$-contiuous map is gsp-continuous.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ).. Then f^{-1}(V) is a $(sp)^*$-closed, since f is $(sp)^*$-continuous and hence by theorem 3.03, it is gsp-closed in (X, τ). Therefore f is gsp-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.04: Let X=Y={a,b,c} τ ={X, ϕ ,{a}, {b,c}} σ ={Y, ϕ ,{b}}. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by an identity mapping. f^{-1}{a,c}={a,c} is gsp-closed but not $(sp)^*$-closed in (X,τ).

Theorem 4.05: Every $(sp)^*$-continuous map is g-continuous.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then f^{-1}(V) is a $(sp)^*$-closed set of(X τ), since f is $(sp)^*$-continuous and hence by theorem-3.5, f^{-1}(V) is g-closed in (X,τ). Therefore f is g-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.06: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},\{b,c\},X\}$, $\sigma=\{\phi,\{c\},Y\}$.

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by an identity mapping. f^{-1}{a,c}={a,c} is g–closed but not $(sp)^*$-closed.

Theorem 4.07: Every $(sp)^*$-continuous map is gs-continuous.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ).. Then f^{-1}(V) is a $(sp)^*$-closed set of X, since f is $(sp)^*$-continuous and hence by theorem-3.7, f^{-1}(V) is gs-closed in (X, τ). Therefore f is gs-continuous.

The converse of the above theorem is not true in general as it can be seen in the following example.

Example 4.08: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},\{b,c\},X\}$, $\sigma=\{\phi,\{b\},Y\}$. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by an identity mapping. f^{-1}{a,c}={a,c} is gs–closed but not $(sp)^*$-closed.

Theorem 4.09: Every $(sp)^*$-continuous map is gp-continuous.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ), since f is $(sp)^*$-continuous and hence by theorem-3.9, $f^{-1}(V)$ is gp-closed in (X, τ). Therefore f is gp-continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.10: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},\{b,c\},X\}$, $\sigma=\{\phi,\{c\},Y\}$.

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by an identity mapping. f^{-1}\{a,b\}=\{a,b\} is gp-closed but not $(sp)^*$-closed.

Theorem 4.11: Every $(sp)^*$-continuous map is sg-continuous.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ).. Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ), since f is $(sp)^*$-continuous, and hence by theorem-3.11, $f^{-1}(V)$ is sg-closed in (X, τ). Therefore f is sg-continuous.

The converse of the above theorem is not true always as seen in the following example.

Example 4.12: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},X\}$, $\sigma=\{\phi,\{a,c\},Y\}$. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by an identity mapping. f^{-1}\{b\}=\{b\} is sg–closed but not $(sp)^*$-closed.

Theorem 4.13: Every $(sp)^*$-continuous map is $\hat{g}$-continuous.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ), since f is $(sp)^*$-continuous and hence by theorem-3.13, $f^{-1}(V)$ is $\hat{g}$-closed in (X, τ). Therefore f is $\hat{g}$-continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.14: Let X={a,b,c}=Y, $\tau=\{\phi,\{b,c\},X\}$, $\sigma=\{\phi,\{a\},Y\}$. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined by an identity mapping. f^{-1}\{b,c\}=\{b,c\} is $\hat{g}$–closed but not $(sp)^*$-closed.

Theorem 4.15: Every $(sp)^*$-continuous map is αg-continuous.

Proof: Let f: $(X,\tau)\to(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ), since $(sp)^*$-continuous and hence by theorem-3.15, $f^{-1}(V)$ is αg-closed in (X, τ). Therefore f is αg-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.16: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},\{b,c\},X\}$, $\sigma=\{\phi,\{c\},Y\}$.

Let f: $(X,\tau)\to(Y,\sigma)$ be defined by an identity mapping. f^{-1}\{a,b\}=\{a,b\} is αg –closed but not $(sp)^*$-closed.

Theorem 4.17: Every $(sp)^*$-continuous map is $g\alpha$-continuous.

Proof: Let f: $(X,\tau)\to(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ), since $(sp)^*$-continuous and hence by theorem-3.17, $f^{-1}(V)$ is $g\alpha$-closed in (X, τ). Therefore f is $g\alpha$-continuous.

The converse of the above theorem is not true in general it can be seen from the following example.

Example 4.18: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},\{b,c\},X\}$, $\sigma=\{\phi,\{c\},Y\}$.

Let f: $(X,\tau)\to(Y,\sigma)$ be defined by an identity mapping. f^{-1}\{a,b\}=\{a,b\} is $g\alpha$ –closed but not $(sp)^*$-closed.

Theorem 4.19: Every $(sp)^*$-continuous map is ωg-continuous.

Proof: Let f: $(X,\tau)\to(Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ), since f is $(sp)^*$-continuous and hence by theorem-3.19, $f^{-1}(V)$ is ωg-closed in (X, τ). Therefore f is ωg-continuous.

The converse of the above theorem is not true always as seen in the following example.

Example 4.20: Let X={a,b,c}=Y, $\tau=\{\phi,\{a\},\{b,c\},X\}$, $\sigma=\{\phi,\{c\},Y\}$.

Let f: $(X,\tau)\to(Y,\sigma)$ be defined by an identity mapping. f^{-1}\{a,b\}=\{a,b\} is ωg –closed but not $(sp)^*$-closed.

Theorem 4.21: Every $(sp)^*$-continuous map is α^*-continuous.

Proof: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be $(sp)^*$-continuous. Let V be closed set of (Y, σ). Then $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ) , since $(sp)^*$-continuous and hence by theorem-3.21, $f^{-1}(V)$ is α^*-closed in (X, τ). Therefore f is α^*-continuous.

The converse of the above theorem is not true in general it can be seen from the following example.

Example 4.22: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$, $\sigma = \{\phi, \{c\}, Y\}$.

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be defined by an identity mapping. $f^{-1}\{a,b\}=\{a,b\}$ is α^* –closed but not $(sp)^*$-closed.

Definition 4.23: A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called $(sp)^*$-irresolute if $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ) for every $(sp)^*$-closed set V of (Y, σ).

Theorem 4.24: Every $(sp)^*$-irresolute function is $(sp)^*$-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.25: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$, $\sigma = \{\phi, \{a,c\}, Y\}$.

Define f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=c, f(b)=a and f(c)=b. $f^{-1}\{b\}=\{a\}$ is (sp)-closed in (X, τ). Therefore f is $(sp)^*$-continuous. {b,c} is $(sp)^*$-closed in Y. $f^{-1}\{b,c\}=\{a,b\}$ is not $(sp)^*$-closed in (X, τ). Therefore f is not $(sp)^*$-irresolute.

Theorem 4.26: Every $(sp)^*$-irresolute function is gsp-continuous.

The converse of the above Theorem is not true as seen in the following example.

Example 4.27: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$, $\sigma = \{\phi, \{a\}, Y\}$.

Define f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=c, f(b)=b and f(c)=a. $f^{-1}\{b,c\}=\{b,a\}=\{a,b\}$ is gsp-closed in (X, τ). Therefore f is gsp-continuous. {b,c} is $(sp)^*$-closed in Y. $f^{-1}\{b,c\}=\{a,b\}$ is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.28: Every $(sp)^*$-irresolute function is g-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.29: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$ $\sigma = \{\phi, \{a\}, Y\}$.

Define f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=c, f(b)=b and f(c)=a. f^{-1}\{b,c\}=\{c,a\}=\{a,c\} is g-closed in (X, τ). Therefore f is g-continuous. {b,c} is $(sp)^*$-closed set in y. f^{-1}\{b,c\}=\{a,b\} is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.30: Every $(sp)^*$-irresolute function is gs-continuous.

The following example supports that the converse of the above theorem is not true always.

Example 4.31: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$ $\sigma = \{\phi, \{a\}, Y\}$.

Define f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=c, f(b)=a and f(c)=b. f^{-1}\{b,c\}=\{a,b\} is gs-closed in (X, τ). Therefore f is gs-continuous. {b,c} is $(sp)^*$-closed set in Y. f^{-1}\{b,c\}=\{a,b\} is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.32: Every $(sp)^*$-irresolute function is gp-continuous.

The converse of the above Theorem is not true always as seen in the following example.

Example 4.33: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b,c\}, X\}$ $\sigma = \{\phi, \{a\}, Y\}$.

Define f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=b, f(b)=a and f(c)=c. f^{-1}\{b,c\}=\{a,c\} is gp-closed in (X, τ). Therefore f is gp-continuous. {b,c} is $(sp)^*$-closed set in Y. f^{-1}\{b,c\}=\{a,c\} is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.34: Every $(sp)^*$-irresolute function is sg-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.35: Let X={a,b,c}=y, $\tau = \{\phi, \{a\}, X\}$ $\sigma = \{\phi, \{b,c\}, Y\}$.

Define f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=b, f(b)=c and f(c)=a. f^{-1}\{a\}=\{b\} is sg-closed in (X, τ). Therefore f is sg-continuous. {a} is $(sp)^*$-closed set in Y. f^{-1}\{a\}=\{b\} is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.36: Every $(sp)^*$-irresolute function is $\hat{g}$ -continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.37: Let X={a,b,c}=Y, $\tau=\{\phi, \{b,c\}, X\}$ $\sigma=\{\phi,\{a\},\{b,c\},Y\}$.

Define f: $(X,\tau)\to(Y,\sigma)$ by f(a)=a, f(b)=c and f(c)=b. $f^{-1}\{a\}=\{a\}$ is $\hat{g}$-closed in (X, τ). Therefore f is g-continuous. {b,c} is $(sp)^*$-closed sets in Y. $f^{-1}\{b,c\}=\{c,b\}=\{b,c\}$ is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.38: Every $(sp)^*$-irresolute function is αg-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.39: Let X={a,b,c}=Y, $\tau=\{\phi, \{a\},\{b,c\}, X\}$ $\sigma=\{\phi,\{a\},Y\}$.

Define f: $(X,\tau)\to(Y,\sigma)$ by f(a)=c, f(b)=a and f(c)=b. $f^{-1}\{b,c\}=\{a,b\}$ is αg-closed in (X, τ). Therefore f is αg-continuous.{b,c} is $(sp)^*$-closed sets in Y. $f^{-1}\{b,c\}=\{a,b\}$ is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.40: Every $(sp)^*$-irresolute function is $g\alpha$-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.41: Let X={a,b,c}=Y, $\tau=\{\phi, \{a\},\{b,c\}, X\}$ $\sigma=\{\phi,\{a\},Y\}$.

Define f: $(X,\tau)\to(Y,\sigma)$ by f(a)=c, f(b)=b and f(c)=a.. $f^{-1}\{b,c\}=\{c,a\}=\{a,c\}$ is $g\alpha$-closed in (X, τ). Therefore f is $g\alpha$-continuous. {b,c} is $(sp)^*$-closed set in Y. $f^{1}\{b,c\}=\{a,b\}$ is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.42: Every $(sp)^*$-irresolute function is ωg-continuous.

The converse of the above theorem is not true as seen in the following example.

Example 4.43: Let X={a,b,c}=Y, $\tau=\{\phi, \{a\},\{b,c\}, X\}$ $\sigma=\{\phi,\{b,c\},Y\}$.

Define f: $(X,\tau)\to(Y,\sigma)$ by f(a)=b, f(b)=b and f(c)=a. $f^{-1}\{a\}=\{b\}$ is ωg-closed in (X, τ). Therefore f is ωg-continuous. {a} is $(sp)^*$-closed sets in Y. $f^{-1}\{a\}=\{b\}$ is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

Theorem 4.44: Every $(sp)^*$-irresolute function is α^*-continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.45: Let X={a,b,c}=Y, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ $\sigma = \{\phi, \{a\}, Y\}$.

Define f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a)=b, f(b)=a and f(c)=c. f^{-1}\{b,c\}=\{a,c\} is g-closed in (X, τ). Therefore f is α^*-continuous. \{b,c\} is $(sp)^*$-closed sets in Y. f^{-1}\{b,c\}=\{a,c\} is not $(sp)^*$-closed in (X, τ). Hence f is not $(sp)^*$-irresolute.

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