$(gs)^*$-Closed Sets in Topological Spaces

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ABSTRACT

In this paper we introduce a new class of sets namely, $(gs)^*$-closed sets, properties of this set are investigated and we introduce $(gs)^*$-continuous maps and $(gs)^*$-irresolute maps.

Keywords:$(gs)^*$-closed sets, $(gs)^*$-continuous and $(gs)^*$-irresolute.

1.INTRODUCTION

Levine.N[9] introduced generalized closed sets in 1970. S.P Arya and T.Nour [3]defined gs-closed sets in 1990. M.K.R.S. Veerakumar [22]introduced the generalized g^*-closed sets in 2000. Pushpalatha.P and Anitha.K [19] introduced the g^*s-closed sets in 2011. M.Pauline Mary Helen,Ponnuthai selvarani,Veronica Vijayan [18] introduced and studied g^{**}-closed sets in 2012. In this paper we introduce and study the concept of $(gs)^*$-closed sets

2.PRELIMINARIES

Throughout this paper (X,τ) and (Y,σ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), Cl(A) and int(A) denote the closure and the interior of A respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called a

1) pre-open set [15] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.

2) semi-open set [10] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.

3)semi-preopen set [1] if $A \subseteq cl(int(cl(A)))$ and a semi preclosed set if $int(cl(int(A))) \subseteq U$.

4) α -open set[16] if $A \subseteq int(cl(int(A)))$ and a α -closed set if $cl(int(cl(A))) \subseteq A$.

Definition 2.2: A subset A of a topological space (X,τ) is said to be a

1) g-closed [9] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in(X,τ).

2) sg-closed [5] set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ).

3) gs-closed set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

4) α g-closed set [11] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

5) g α -closed set [12] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X,τ).

6) gsp-closed set [7] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

7) gp-closed set [13] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

8) g^*-closed set [22] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).

9) g^*s- closed set [19] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in (X, τ).

10) ψ - closed set [23] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is sg-open in (X, τ).

11) ψ^*- closed set [27] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ψ -open in (X, τ).

12) g^{**}-closed set [18] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g^*-open in (X, τ).

13) α^*-closed set [26] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α -open in (X, τ).

14) s α g^*-closed set [14] if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g^*-open in (X, τ).

15) α g^{**}-closed set [20] if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g^{**}-open in (X, τ).

16) sg^{**}-closed set [17] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g^{**}-open in (X, τ).

17) $(g\alpha)^*$-closed set [25] if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g α -open in (X, τ).

18) ω -closed set [21] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

19) $g^\#$ semi closed set [24] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α g-open in (X, τ).

Definition 2.3: A function f:(X, τ) $\rightarrow$ (Y, σ) is called a

1) g-continuos [4] if f^{-1}(V) is a g-closed set of (X, τ) for every closed set V of (Y, σ).

2) sg-continuous [5] if f^{-1}(V) is a sg-closed set of (X, τ) for every closed set V of (Y, σ).

3) gs-continuous [6] if f^{-1}(V) is a gs-closed set of (X, τ) for every closed set V of (Y, σ).

4) α g-continuous [8] if f^{-1}(V) is an α g-closed set of (X, τ) for every closed set V of (Y, σ).

5) g α -continuous [12] if f^{-1}(V) is a g α -closed set of (X, τ) for every closed set V of (Y, σ).

6) gsp-continuous [7] if f^{-1}(V) is a gsp-closed set of (X, τ) for every closed set V of (Y, σ).

7) gp-continuous [2] if f^{-1}(V) is a gp-closed set of (X, τ) for every closed set V of (Y, σ).

8) g^*-continuous if [22] f^{-1}(V) is a g^*-closed set of (X, τ) for every closed set V of (Y, σ).

9) g^*s- continuous [19] if f^{-1}(V) is a g^*s-closed set of (X, τ) for every closed set V of (Y, σ).

10) ψ - continuous[23] if f^{-1}(V) is a ψ -closed set of (X, τ) for every closed set V of (Y, σ).

11) ψ^*- continuous[27] if $f^{-1}(V)$ is a ψ^*-closed set of (X,τ) for every closed set V of (Y,σ).

12) g^{**}-continuous [18] if $f^{-1}(V)$ is a g^{**}-closed set of (X,τ) for every closed set V of (Y,σ).

13) α^*-continuous [26] if $f^{-1}(V)$ is a α^*-closed set of (X,τ) for every closed set V of (Y,σ).

14) $s\alpha g^*$-continuous [14] if $f^{-1}(V)$ is a $s\alpha g^*$-closed set of (X,τ) for every closed set V of (Y,σ).

15) αg^{**}-continuous [20] if $f^{-1}(V)$ is a αg^{**}-closed set of (X,τ) for every closed set V of (Y,σ).

16) sg^{**}-continuous [17] if $f^{-1}(V)$ is a sg^{**}-closed set of (X,τ) for every closed set V of (Y,σ).

17) $(g\alpha)^*$-continuous [25] if $f^{-1}(V)$ is a $(g\alpha)^*$-closed set of (X,τ) for every closed set V of (Y,σ).

18) ω-continuous [21] if $f^{-1}(V)$ is a ω-closed set of (X,τ) for every closed set V of (Y,σ).

19) $g^{\#}$-semi continuous [24] if $f^{-1}(V)$ is a $g^{\#}$-semi-closed set of (X,τ) for every closed set V of (Y,σ).

3. BASIC PROPERTIES OF $(gs)^*$-CLOSED SETS

We now introduce the following definition.

Definition 3.1: A subset A of a topological space(X,τ) is called $(gs)^*$-closed set if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is gs-open in X.

Theorem 3.2: Every closed set is a $(gs)^*$-closed set.

Proof follows from the definition.

The following example supports that a $(gs)^*$-closed set need not be closed in general.

Example 3.3: Let X={a,b,c} , $\tau=\{\phi,X,\{b,c\}\}$. Then A={a,b} is $(gs)^*$-closed but not a closed set of (X,τ).

Theorem 3.4: Every $(gs)^*$-closed set is (i) g-closed,(ii) g^*-closed ,(iii) g^{**}-closed.

Proof follows from the definition.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.5: Let X={a,b,c} and $\tau=\{\phi,X,\{a\},\{a,b\}\}$.Then A={a,c} is $(gs)^*$-closed but not g-closed ,g^*-closed and g^{**}-closed.

Theorem 3.6: Every $(gs)^*$-closed set is α g-closed but not conversely.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $\alpha\, cl(A)\subseteq cl(A)\subseteq U$. Hence A is α g-closed.

Example 3.7: Let X={a,b,c} , $\tau=\{\phi,X,\{b\}\}$. Then A={a} is α g-closed but not $(gs)^*$-closed in (X,τ) .

Theorem 3.8: Every $(gs)^*$-closed set is $g\alpha$-closed but the converse is not true.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be α-open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $\alpha\, cl(A)\subseteq cl(A)\subseteq U$. Hence A is $g\alpha$-closed.

Example 3.9: Let X={a,b,c}, τ ={ϕ,X,{b}}. Then A={c} is $g\alpha$-closed but not $(gs)^*$-closed in (X,τ).

Theorem 3.10: Every $(gs)^*$-closed set is sg-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be semi-open in. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $Scl(A)\subseteq cl(A)\subseteq U$. Hence A is sg-closed.

The following example supports that a sg-closed set need not be $(gs)^*$-closed in general.

Example 3.11: Let X={a,b,c} , τ ={ϕ,X,{c},{a,c}}. Then A= {a} is sg-closed but not a $(gs)^*$-closed set in (X,τ).

Theorem 3.12: Every $(gs)^*$-closed set is gs-closed but not conversely.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $Scl(A)\subseteq cl(A)\subseteq U$. Hence A is gs-closed.

Example 3.13: Let X={a,b,c} , τ ={ϕ,X,{c},{a,c}}. Then A={b,c} is gs-closed but not $(gs)^*$-closed in (X,τ).

Theorem 3.14: Every $(gs)^*$-closed set is gsp-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $Spcl(A)\subseteq cl(A)\subseteq U$. Hence A is gsp-closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.15: Let X={a,b,c}, τ ={ϕ,X,{a,b}}. Then A={a} is gsp-closed but not a $(gs)^*$-closed set in (X,τ).

Theorem 3.16: Every $(gs)^*$-closed set is gp-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $pcl(A)\subseteq cl(A)\subseteq U$. Hence A is gp-closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.17: Let X={a,b,c}, τ ={ϕ,X,{b},{b,c}}. Then A={a,b} gp-closed but not $(gs)^*$-closed in (X,τ).

Theorem 3.18: Every $(gs)^*$-closed set is α^*-closed but not conversely.

Proof follows from the definition.

Example 3.19: Let X={a,b,c} , τ ={ϕ,X,{a},{b,c}}. Then A= {b} is α^*-closed but not $(gs)^*$-closed in (X,τ).

Theorem 3.20: Every $(gs)^*$-closed set is $s\alpha g^*$-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be g^*-open . Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $\alpha cl(A)\subseteq cl(A)\subseteq U$. Hence A is $s\alpha g^*$-closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.21: Let X={a,b,c} , τ ={ ϕ ,X,{a},{a,b}}. Then A={a,c} is $s\alpha g^*$-closed but not a $(gs)^*$-closed set in (X, τ).

Theorem 3.22: Every $(gs)^*$-closed set is αg^{**}-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be g^{**}-open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $\alpha cl(A)\subseteq cl(A)\subseteq U$. Hence A is αg^{**}-closed.

The following example supports that the converse of the above theorem is not true in general.

Example 3.23: Let X={a,b,c}, τ ={ ϕ ,X,{a},{a,b}}. Then A={b} is αg^{**}-closed but not $(gs)^*$-closed in (X, τ).

Theorem 3.24: Every $(gs)^*$-closed set is sg^{**}-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be g^{**}-open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $scl(A)\subseteq cl(A)\subseteq U$. Hence A is sg^{**}-closed.

The following example supports that a sg^{**}-closed set need not be $(gs)^*$-closed in general.

Example 3.25: Let X={a,b,c}, τ ={ ϕ ,X,{a},{a,b}}. Then A={b} is sg^{**}-closed but not $(gs)^*$-closed in (X, τ).

Theorem 3.26: Every $(gs)^*$-closed set is $(g\alpha)^*$-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be $g\alpha$ -open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $\alpha cl(A)\subseteq cl(A)\subseteq U$. Hence A is $(g\alpha)^*$-closed.

The following example supports that a $(g\alpha)^*$-closed set need not be $(gs)^*$-closed in general.

Example 3.27: Let X={a,b,c}, τ ={ ϕ ,X,{b}}. Then A= {a} is a $(g\alpha)^*$-closed set but not $(gs)^*$-closed in (X, τ).

Theorem 3.28: Every $(gs)^*$-closed set is ω -closed.

Proof follows from the definition.

Example 3.29: Let X={a,b,c} , τ ={ ϕ ,X,{a},{a,b}}. Then A={a,b} is ω -closed but not a $(gs)^*$-closed set in (X, τ).

Theorem 3.30: Every $(gs)^*$-closed set is ψ -closed but not conversely.

Proof: Let A be $(gs)^*$-closed set. Let $A\subseteq U$ and U be sg-open. Then $A\subseteq U$ and U is gs-open and $cl(A)\subseteq U$, since A is $(gs)^*$-closed. Then $sc\ l(A)\subseteq cl(A)\subseteq U$. Hence A is ψ -closed.

Example 3.31: Let X={a,b,c}, τ ={ ϕ ,X,{c},{a,c}}.Then A={a} is ψ -closed but not $(gs)^*$-closed in (X, τ).

Theorem 3.32: Every $(gs)^*$-closed set is ψ^*-closed.

Proof: Let A be $(gs)^*$-closed set. Let $A \subseteq U$ and U be ψ -open. Then $A \subseteq U$ and U is gs-open and $cl(A) \subseteq U$, since A is $(gs)^*$-closed. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is ψ^*-closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.33: Let X={a,b,c} , τ ={ ϕ ,X,{c},{a,c}}.Then A={a} is ψ^*-closed but not $(gs)^*$-closed in (X, τ).

Theorem 3.34: Every $(gs)^*$-closed set is g^*s-closed but not conversely.

Proof follows from the definition.

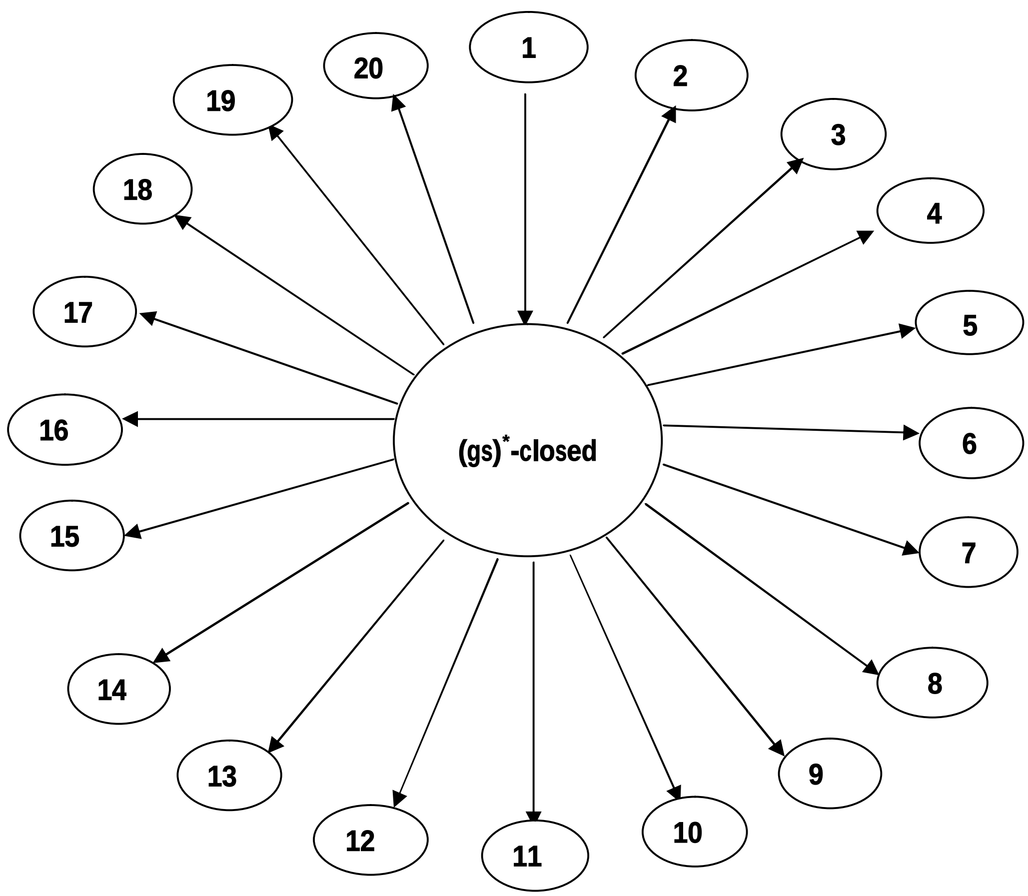
Example 3.35: Let X={a,b,c} and τ ={ ϕ ,X,{b}}.Then A= {a} is g^*s-closed but not $(gs)^*$-closed in (X, τ).

Theorem 3.36: Every $(gs)^*$-closed set is $g^\#$-semi closed but not conversely.

Proof: Let A be $(gs)^*$-closed set. Let $A \subseteq U$ and U be α g-open. Then $A \subseteq U$ and U is gs-open and $cl(A) \subseteq U$, since A is $(gs)^*$-closed. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is $g^\#$-semi closed.

Example 3.37: Let X={a,b,c} , τ ={ ϕ ,X,{b}}. Then A={c} is $g^\#$-semi closed but not a $(gs)^*$-closed set in (X, τ).

The above results can be represented in the following figure.



Where A$\rightarrow$B represents A implies B but B does not implies A.

1. Closed	**6.gα -closed**	**11.sα g*-closed**	**16.g*s-closed**
2.g-closed	**7.sg-closed**	**12.α g**-closed**	**17.ψ -closed**
3.g*-closed	**8.gs-closed**	**13.sg**-closed**	**18.ψ *-closed**
4.g**-closed	**9.gsp-closed**	**14.(gα)*-closed**	**19.ω -closed**
5.α g-closed	**10.gp-closed**	**15. g$^\#$-semi closed**	**20.α *-closed**

4.(gs)*-CONTINUOUS AND (gs)*-IRRESOLUTE MAPS

Definition 4.1: A function f:(X,τ)$\rightarrow$(Y,σ) is called (gs)*-continuous if $f^{-1}(V)$ is a (gs)*-closed set of (X,τ) for every closed set V of (Y,σ).

Definition 4.2: A function f:(X,τ)$\rightarrow$(Y,σ) is called (gs)*-irresolute if $f^{-1}(V)$ is a (gs)*-closed set of (X,τ) for every (gs)*- closed set V of (Y,σ).

Theorem 4.3: Every continuous map is (gs)*-continuous but not conversely.

Example 4.4: Let X=Y={a,b,c},τ ={ϕ,X,{b,c}} and σ ={ϕ,Y,{a}}. Define f:(X,τ)$\rightarrow$(Y,σ) by f(a)=b, f(b)=a , f(c)=c. f is (gs)*-continuous but not continuous since {b,c} is a closed set of (Y,σ) but $f^{-1}\{b,c\}$={a,c} is not closed in (X,τ) .

Theorem 4.5: Every (gs)*-continuous map is (i) g-continuous (ii) g*-continuous (iii)g**-continuous.

Proof: Let f:(X,τ)$\rightarrow$(Y,σ) be (gs)*-continuous. Let V be a closed set in (Y,σ). Then $f^{-1}(V)$ is (gs)*-closed in (X,τ), since f is (gs)*-continuous and hence by theorem 3.4(resp.3.6, 3.8), it is g-closed(resp.g*-closed,g**-closed). Hence f is g-continuous(resp. g*-continuous, g**-continuous) but the converse is not true.

Example 4.6: Let X=Y={a,b,c},τ ={ϕ,X,{a},{a,b}} and σ ={ϕ,Y,{b}}**.** Let f:(X,τ)$\rightarrow$(Y,σ) be the identity mapping. Then $f^{-1}\{a,c\}$={a,c} which is g-closed(resp. g*-closed, g**-closed) but not (gs)*-closed. Hence f is g-continuous (resp. g*-continuous, g**-continuous) but not (gs)*-continuous.

Theorem 4.7: Every (gs)*-continuous map is (i)α g-continuous,(ii) gα -continuous (iii) (gα)*- continuous, (iv) g*s-continuous,(v) g$^\#$-semi-continuous.

Proof: Let f:(X,τ)$\rightarrow$(Y,σ) be (gs)*-continuous. Let V be a closed set in (Y,σ).Then $f^{-1}(V)$ is (gs)*-closed in (X,τ), since f is (gs)*-continuous and hence by theorem 3.6 (resp .3.8, 3.26, 3.34, 3.36), it is α g-closed(resp. gα -closed, (gα)*-closed, g*s- closed, g$^\#$ -semi- closed). Hence f is α g-continuous(resp. gα -continuous, (gα)*-continuous, g*s-continuous, g$^\#$-semi-continuous)

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.8: Let X=Y={a,b,c},τ ={ϕ,X,{b}} and σ ={ϕ,Y,{c},{b,c}}**.** Let f:(X,τ)$\rightarrow$(Y,σ) be the identity mapping. Then $f^{-1}\{a\}$={a} is α g-closed(resp. gα -closed, (gα)*-closed, g*s- closed, g$^\#$ -semi- closed) but not (gs)*-closed.

Hence f is α g-continuous(resp. gα -continuous, (gα)*-continuous, g*s-continuous, g$^\#$-semi-continuous) but not (gs)*-continuous.

Theorem 4.9: Every (gs)*-continuous map is (i) sg-continuous, (ii) gs-continuous, (iii) ψ -continuous ,(iv) ψ^*-continuous.

Proof: Let f:(X, τ)$\rightarrow$(Y, σ) be (gs)*-continuous. Let V be a closed set in (Y, σ).Then $f^{-1}(V)$ is (gs)*-closed in (X, τ), since f is (gs)*-continuous and hence by theorem 3.10(resp .3.12,3.30,3.32),it is sg-closed(resp. gs-closed, ψ -closed, ψ^*-closed). Hence f is sg-continuous(resp. gs-continuous, ψ -continuous, ψ^*-continuous)

The following example supports that the converse of the above theorem is not true in general.

Example 4.10: Let X=Y={a,b,c}, τ ={ ϕ ,X,{c},{a,c}} and σ ={ ϕ ,Y,{b,c}}.Let f:(X, τ)$\rightarrow$(Y, σ) be the identity mapping. Then $f^{-1}\{a\}$={a} which is gs-closed(resp sg-closed, ψ -closed, ψ^*-closed) but not (gs)*-closed .Hence f is gs-continuous (resp.sg-continuous, ψ -continuous , ψ^*-continuous)but not (gs)*-continuous.

Theorem 4.11: Every (gs)*-continuous map is gsp-continuous.

Proof: Let f:(X, τ)$\rightarrow$(Y, σ) be (gs)*-continuous. Let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is (gs)*-closed in (X, τ), since f is (gs)*-continuous and hence by theorem 3.14, it is gs-closed. Hence f is gsp-continuous but the converse need not true.

Example 4.12: Let X=Y={a,b,c}, τ ={ ϕ ,X,{a,b}} and σ ={ ϕ ,Y,{a,c}}**.** Let f:(X, τ)$\rightarrow$(Y, σ) defined by f(a)=b,f(b)=c, f(c)=b. Then $f^{-1}\{b\}$={a} is gsp-closed but not (gs)*-closed. Hence f is gsp-continuous but not (gs)*-continuous.

Theorem 4.13: Every (gs)*-continuous map is gp-continuous.

Proof: Let f:(X, τ)$\rightarrow$(Y, σ) be (gs)*-continuous. Let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is (gs)*-closed in (X, τ), since f is (gs)*-continuous and hence by theorem 3.16, it is gp-closed. Hence f is gp-continuous.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.14: Let X=Y={a,b,c}, τ ={ ϕ ,X,{b},{b,c}} and σ ={ ϕ ,Y,{a,c}}**.** Let f:(X, τ)$\rightarrow$(Y, σ) defined by f(a)=a , f(b)=c ,f(c)=b. Then $f^{-1}\{b\}$={c} is gp-closed but not (gs)*-closed. Hence f is gp-continuous but not (gs)*-continuous.

Theorem 4.15: Every (gs)*-continuous map is (i) α^*-continuous (ii) ω -continuous but not conversely.

Proof: Let f:(X, τ)$\rightarrow$(Y, σ) be (gs)*-continuous. Let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is (gs)*-closed in (X, τ), since f is (gs)*-continuous and hence by theorem 3.18,(resp.3.28), it is α^*-closed(resp. ω -closed). Hence f is α^*-continuous (resp. ω -continuous).

Example 4.16: Let X=Y={a,b,c}, τ ={ ϕ ,X,{a},{b,c}} and σ ={ ϕ ,Y,{a,b}}**.** Let f:(X, τ)$\rightarrow$(Y, σ) defined by f(a)=b , f(b)=c ,f(c)=a. Then $f^{-1}\{c\}$={b} is α^*-closed(resp. ω -closed) but not (gs)*-closed. Hence f is α^*-continuous (resp. ω -continuous) but not (gs)*-continuous.

Theorem 4.17: Every (gs)*-continuous map is (i)sα g*-continuous (ii)α g**-continuous (iii)sg**-continuous**.**

Proof: Let f:(X, τ)$\rightarrow$(Y, σ) be (gs)*-continuous. Let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is (gs)*-closed in (X, τ), since f is (gs)*-continuous and hence by theorem 3.20(resp. (3.22),(3.24)), it is sα g* closed (resp.α g**-closed, sg**-closed). Hence f is sα g*-continuous (resp.α g**-continuous, sg**-continuous).

The following example supports that the converse of the above results are not true in general.

Example 4.18: Let X=Y={a,b,c}, τ ={ ϕ ,X,{a},{a,b}} and σ ={ ϕ ,Y,{a}}. Let f:(X, τ) $\rightarrow$ (Y, σ) defined by f(a)=b , f(b)=a ,f(c)=c. Then $f^{-1}\{b,c\}$ ={a,c} is s α g^*- closed(resp. α g^{**}-closed, sg^{**}-closed) but not $(gs)^*$-closed. Hence f is s α g^*- continuous (resp. α g^{**}-continuous, sg^{**}-continuous) but not $(gs)^*$-continuous.

Theorem 4.19: Every $(gs)^*$-irresolute function is $(gs)^*$-continuous but not conversely.

Example 4.20: Let X=Y={a,b,c}, τ ={ ϕ ,X,{b,c}} and σ ={ ϕ ,Y,{a},{b,c}}. Define g: (X, τ) $\rightarrow$ (Y, σ) by g(a)=b, g(b)=a, g(c)=c. $g^{-1}\{b,c\}$ ={a,c} is $(gs)^*$ closed in (X, τ) . Therefore g is $(gs)^*$ continuous. {a} is $(gs)^*$-closed in (Y, σ) but $g^{-1}\{a\}$ ={b} is not a $(gs)^*$ closed set in (X, τ). Therefore g is not $(gs)^*$ irresolute. Hence g is $(gs)^*$-continuous but not $(gs)^*$-irresolute.

Theorem 4.21: Every $(gs)^*$-irresolute function is (i)g-continuous,(ii) g^*-continuous,(iii)g^{**}-continuous.

The following example supports that the converse of the above results are not true in general.

Example 4.22: Let X=Y={a,b,c}, τ ={ ϕ ,X,{a},{a,b}} and σ ={ ϕ ,Y,{a},{b,c}}. Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a)=b, f(b)=a, f(c)=c. $f^{-1}\{b,c\}$ ={a,c} is g-closed(resp. g^*-closed, g^{**}-closed) in (X, τ) . Therefore f is g-continuous (resp . g^*-continuous, g^{**}-continuous). {b,c} is $(gs)^*$-closed in (Y, σ) but $f^{-1}\{b,c\}$ ={a,c} is not $(gs)^*$-closed set in (X, τ). Therefore f is not $(gs)^*$ irresolute. Hence f is g-continuous(resp. g^*-continuous, g^{**}-continuous) but not $(gs)^*$-irresolute.

Theorem 4.23: Every $(gs)^*$-irresolute function is (i) gs-continuous, (ii) sg-continuous, (iii) ψ -continuous, (iv) ψ^*-continuous .

The following example supports that the converse of the above results are not true in general.

Example 4.24: Let X=Y={a,b,c}, τ ={ ϕ ,X,{c},{a,c}} and σ ={ ϕ ,Y,{a},{b,c}}. Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a)=b, f(b)=a, f(c)=c. $f^{-1}\{a\}$ ={b} is gs-closed(resp.sg-closed, ψ -closed, ψ^*-closed) in (X, τ). Therefore f is gs-continuous (resp. sg-continuous, ψ -continuous, ψ^*-continuous). {b,c} is $(gs)^*$-closed in (Y , σ) but $f^{-1}\{b,c\}$ ={a,c} is not $(gs)^*$ closed set in (X, τ). Therefore f is not $(gs)^*$ irresolute. Hence f is gs-continuous(resp.sg-continuous, ψ -continuous, ψ^*-continuous) but not $(gs)^*$-irresolute.

Theorem 4.25: Every $(gs)^*$-irresolute function is gsp-continuous but not conversely.

Example 4.26: Let X=Y={a,b,c}, τ ={ ϕ ,X,{a,b}} and σ ={ ϕ ,X,{b},{b,c}}.Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a)=c, f(b)=a, f(c)=b. $f^{-1}\{a\}$ ={c} is gsp-closed in (X, τ) . Therefore f is gsp-continuous. {a,c} is $(gs)^*$-closed in (Y, σ) but $f^{-1}\{a,c\}$ ={a,b} is not $(gs)^*$ closed set in(X, τ). Therefore f is not $(gs)^*$ irresolute. Hence f is gsp-continuous but not $(gs)^*$-irresolute.

Theorem 4.27: Every $(gs)^*$-irresolute function is gp-continuous.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.28: Let X=Y={a,b,c}, τ ={ ϕ ,X,{b},{b,c}} and σ ={ ϕ ,X,{b}}..Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a)=a, f(b)=c, f(c)=b. $f^{-1}\{a,c\}$ ={a,b} is gp-closed in (X, τ).Therefore f is gp-continuous. {a,c} is $(gs)^*$-closed in (Y, σ) but

$f^{-1}\{a,c\}$ = {a,b} is not $(gs)^*$ closed set in (X, τ). Therefore f is not $(gs)^*$ irresolute. Hence f is gp-continuous but not $(gs)^*$-irresolute.

Theorem 4.29: Every $(gs)^*$-irresolute function is (i) α^*-continuous, (ii) ω-continuous.

The converse of the above results are not true in general.

Example 4.30: Let X=Y={a,b,c}, τ ={ϕ,X,{a},{b,c}}} and σ ={ϕ,X,{c},{a,c}}.Define f: (X,τ)→(Y,σ) by f(a)=a, f(b)=c, f(c)=b. $f^{-1}\{b\}$ ={c} is α^*-closed(resp. ω-closed) in (X, τ) .Therefore f is α^*-continuous(resp. ω-continuous). {b} is $(gs)^*$-closed in (Y, σ) but $f^{-1}\{b\}$ ={c} is not $(gs)^*$ closed set in (X,τ). Therefore f is not $(gs)^*$ irresolute. Hence f is α^*-continuous(resp. ω-continuous) but not $(gs)^*$-irresolute.

Theorem 4.31: Every $(gs)^*$-irresolute function is (i) gα-continuous, (ii) $(g\alpha)^*$- continuous,(iii) g^*s-continuous, (iv) $g^\#$-semi-continuous (v) αg-continuous.

The following example supports that the converse of the above results are not true in general.

Example 4.32: Let X=Y={a,b,c}, τ ={ϕ,X,{b}} and σ ={ϕ,Y,{c},{a,c}}. Define f: (X,τ)→(Y,σ) by f(a)=c, f(b)=a, f(c)=b. $f^{-1}\{b\}$ ={c} is gα-closed (resp.$(g\alpha)^*$-closed, g^*s- closed, $g^\#$-semi-closed) in (X, τ). Therefore f is gα-continuous (resp.$(g\alpha)^*$-continuous, g^*s-continuous, $g^\#$-semi-continuous). {a,b} is $(gs)^*$-closed in (Y, σ) but $f^{-1}\{a,b\}$ ={b,c} is not $(gs)^*$ closed set in (X, τ). Therefore f is not $(gs)^*$ irresolute. Hence f is gα-continuous (resp. αg-continuous,$(g\alpha)^*$-continuous, g^*s-continuous, $g^\#$-semi-continuous) but not $(gs)^*$-irresolute.

Theorem 4.33 : Every $(gs)^*$-irresolute function is (i) s$\alpha$$g^*$-continuous, (ii) $\alpha$$g^{**}$- continuous,(iii) sg^{**}-continuous.

The following example supports that the converse of the above results are not true in general.

Example 4.34: Let X=Y={a,b,c}, τ ={ϕ,X,{a},{a ,b}} and σ ={ϕ,Y,{a},{b ,c}}. Define f: (X,τ)→(Y,σ) by f(a)=c, f(b)=a, f(c)=b. $f^{-1}\{a\}$ ={b} is s$\alpha$$g^*$-closed(resp.$\alpha$$g^{**}$-closed, sg^{**}- closed) in (X, τ) .Therefore f is s$\alpha$$g^*$-continuous (resp.$\alpha$$g^{**}$-continuous, sg^{**}-continuous). {b,c} is $(gs)^*$-closed in (Y, σ) but $f^{-1}\{b,c\}$ ={a,c} is not $(gs)^*$ closed set in (X, τ). Therefore f is not $(gs)^*$ irresolute. Hence f is s$\alpha$$g^*$-continuous(resp.$\alpha$$g^{**}$-continuous, sg^{**}-continuous) but not $(gs)^*$-irresolute.

REFERENCES

[1] Andrijevic , Semi-preopen sets, Mat. Vesnik,38(1) (1986),24-32.

[2] Arokiarani .I , Balachandran.K and Dontchev. J, Some characterizations of gp-continuous maps between topological spaces, Mem.Fac.Sci.Kochi.Univ.Ser.A. Math., 20(1999),93-104.

[3] Arya S.P and Nour.T, characterizations of s-normal spaces, Indian J.Pure.Appl.Math., 21(8)(1990), 717-719.

[4] Balachandran .K, Sundaram .P and Maki.H, On generalized continuous maps in topological spaces, Mem. Fac. Kochi Univ. Ser. A. Math., 12(1991), 5-13.

[5] Bhattacharya and Lahiri. B.K, Semi-generalized closed sets in topology, Indian J.Math., 29(3) (1987), 375-382.

[6] Devi R , Maki.H and Balachandran.K, Semi-generalized homeomorphisms and generalized semi-homeomorphisms in topological spaces,Indian J.Pure.Appl.Math., 26(3) (1995), 271-284.

[7] Dontchev.J, On generalizing semi-preopen sets,Mem.Fac.Sci.Kochi Ser.A,Math.,16(1995), 35-48.

[8] Gnanambal.Y, On generalized preregular closed sets in topological spaces, Indian J.Pure.Appl.Math.,28(3)(1997), 35-48.

[9] Levine N,Generalized closed sets topology,Rend.Circ.Math.Palermo,19(2)(1970), 89-96.

[10] Levine N,Semi-open sets and semi-continuity in topological spaces,Amer.Math.Monthly,70(1963) 36-41.

[11] Maki.H,Devi.R and Balachandran.K,Associated topologies of generalized α -closed sets and α -generalized closed sets,Mem.Fac.Sci.Kochi Univ.Ser.A,Math.,15(1994),51-63.

[12] Maki.H,Devi.R and Balachandran.K,Generalized α -closed sets in topology,Bull.Fukuoka Univ.Ed.PartIII,42(1993), 13-21.

[13] Maki.H, Umehara.J and Noiri.T,Every topological spaces is pre- $T_{1/2}$,Mem . Fac .Sci .Kochi .Univ .Ser.A ,Math .,17(1996),33-42.

[14] Maragathavalli.S and Sheik John.M, On $s\alpha g^*$-closed sets in topological spaces,ACTA CIENCIA INDICA,Vol XXX1 2005 No.3,(2005),805-815.

[15] Mashhour.A.S,Abd.M.E,E1-Monsef and S.N.E1-Deep, On pre-continuous and weak-pre continuous mapping ,Proc. Math.and Phys.Soc.Egypt,53(1982), 47-53.

[16] Njastad.O, On Some classes of nearly open sets,Pacific J.Math.,15(1965),961-970.

[17] Pauline Mary Helen.M, Monica.P ,sg^{**}-closed sets in topological spaces,IJCA,Issue 3,Volume 2(April 2013),ISSN: 2250-1797,20-30.

[18] Pauline Mary Helen.M, Veronica Vijayan, Ponnuthai Selvarani.S, g^{**}-closed sets in topologicalspaces,IJMA,3(5),2012,1-15.

[19] Pushpalatha .P and Anitha.K, g^*s-closed sets in topological space,Int.J. contemp.Math.Sciences,Vol.6.,March 2011,no 19,917-929.

[20] Dr.Saravanakumar.D,Sathiskumar.K.M, On a class of αg^{**}-closed sets in topological spaces and some mapping,International Journal of Scientific & Research Publications,Volume 2,Issue 6,June 2012 ISSN 2250-3153.

[21] Sundaram.P and Sheik John.M(1995) Weakly closed sets and weak continuous maps in topological spaces Proc.82nd Indian Sci.cong.49.(50-58).

[22] Veerakumar M.K.R.S, Between closed sets and g-closed sets,Mem.Fac.Sci.Kochi .Univ.Ser.A.Math.,17(1996),33-42.

[23] Veerakumar M.K.R.S, Between Semiclosed sets and semi pre-closed sets,Rend.Instint.Univ.Trieste(Italy) XXXII,25-41(2000).

[24] Veerakumar M.K.R.S, $g^{\#}$semi-closed sets in topological spaces. International Journal of Scientific & Research Publications,Vol 2,Issue 6,June 2012 ISSN 2250-3153.

[25] Veronica Vijayan ,Daffiny Swarnakumari.E,$(g\alpha)^*$-closed sets in topological spaces,IJCA,Issue 3,Volume 3(May-June 2013),ISSN:2250-1797.

[26] Veronica Vijayan ,Priya.F, α^*-closed sets in topological spaces,IJCA,Issue 3,Volume 4(July-Aug 2013),ISSN:2250-1797,49-60.

27] Veronica Vijayan ,SelvaPriya.K, A study on ψ^* ,$\overline{\psi}$ and $\overline{\psi}^*$ -closed sets in topological spaces, IJCA, Issue 3,Volume 2(April 2013),ISSN:2250-1797.