Some New Families of Divisor Cordial Graphs

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Abstract - **In this paper, the duplication of arbitrary vertex v_k of cycle C_n ($n \geq 3$), $< S_n^{(1)} : S_n^{(2)} >$, $< W_n^{(1)} : W_n^{(2)} >$, the graph obtained by joining two copies of cycle W_n by a path P_k ($n \geq 3$), $G_v \odot K_1$, where G_v denotes graph obtained by switching of any vertex v of C_n ($n \geq 4$) and Pl_n ($n \geq 5$) are shown to be divisor cordial.**

AMS subject classifications : 05C78

Keywords **- Cordial labeling, divisor cordial labeling, divisor cordial graph.**

I. INTRODUCTION

We begin with simple, finite, connected and undirected graph G = (V(G),E(G)) with p vertices and q edges. For standard terminology and notations related to graph theory we refer to Harary [4] while for number theory we refer to Burton [2]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

1) Definition : If the vertices are assigned values subject to certain condition(s) then it is known as graph labeling.

Graph labelings have enormous applications within mathematics as well as to several areas of computer science and communication networks. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian [5].

2) Definition : A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f (v) is called the label of the vertex v of G under f .

3) Notation : If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

4) Definition : A binary vertex labeling f of a graph G is called a cordial labeling if $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is cordial if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit [2]. After this many labeling schemes are also introduced with minor variations in cordial theme. The product cordial labeling, total product cordial labeling and prime cordial labeling are among mention a few. The present work is focused on some new families of divisor cordial graphs.

5) Definition : Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a | b. If a does not divide b, then we denote $a \nmid b$.

6) Definition : A prime cordial labeling of a graph G with vertex set V(G) is a bijection $f : V(G) \rightarrow \{1, 2, 3, \ldots, |V(G)|\}$ and the induced function $f^* : E(G) \rightarrow \{0,1\}$ is defined by $f^*(e = uv) = \begin{cases} 1, & \text{if } \gcd(f(u), f(v)) = 1; \\ 0, & \text{otherwise} \end{cases}$

which satisfies the condition $| e_f(0) - e_f(1) | \leq 1$. A graph which admits prime cordial labeling is called a prime cordial graph.

The concept of prime cordial labeling was introduced by Sundaram et al. [6] and in the same paper they have investigated several results on prime cordial labeling. Motivated through the concept of prime cordial labeling Varatharajan et al.[9] introduced a new concept called divisor cordial labeling which is a combination of divisibility of numbers and cordial labelings of graphs.

7) Definition : Let G = (V(G), E(G)) be a simple graph and $f : \rightarrow \{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if f (u) | f(v) or f(v) | f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Varatharajan et al. [9] have proved that path, cycle, wheel, star, $K_{2,n}$ and $K_{3,n}$ are divisor cordial graphs while K_n is not divisor cordial for $n \geq 7$. The divisor cordial labeling of full binary tree as well as some star related graphs are reported by Varatharajan et al. [10] while some star and bistar related graphs are proved to be divisor cordial graphs by Vaidya and Shah [7]. Vaidya and Shah [8] have proved that helm H_n, flower graph Fl_n, Gear graph G_n, switching of a vertex in cycle C_n, switching of a rim vertex in wheel W_n and switching of the apex vertex in helm H_n are divisor cordial graphs.

8) Definition : The shell S_n is the graph obtained by taking $n - 3$ concurrent chords in cycle C_n. The vertex at which all the chords are concurrent is called the apex vertex. The shell S_n is also called the fan f_{n-1}.

9) Definition : A wheel graph W_n is a graph with n+1 vertices, formed by connecting a single vertex to all the vertices of n cycle. It is denoted by $W_n = C_n + K_1$.

10) Definition : Let G and H be two graphs. The corona $G \odot H$ is a graph formed from a copy of G and $|V(G)|$ copies of H by joining the i^{th} vertex of G to every vertex in the i^{th} copy of H.

11) Definition : Consider two copies of graph G namely G_1 and G_2. Then the graph $G' = \langle G_1 : G_2 \rangle$ is a graph obtained by joining the apex vertices of G_1 and G_2 by a new vertex x.

12) Definition : Duplication of arbitrary vertex v of graph G produces a new graph G_1 by adding a new vertex v' with all the vertices which are adjacent to v in G.

13) Definition : A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

In [1], Babujee defines a class of planar graph as graphs obtained by removing certain edges from the corresponding complete graphs. This is denoted by Pl_n.

14) Definition : Let K_n be the complete graph on n vertices $V_n = \{1,2,...,n\}$. The class of graphs Pl_n has the vertex set V_n and the edge set $E_n = E(K_n) \setminus \{(k,l) : 3 \le k \le n-2,\ k+2 \le l \le n\}$.

The embedding we use for Pl_n is described as follows. Place the vertices $v_1, v_2, ..., v_{n-2}$ along a vertical line in that order with v_1 at the bottom and v_{n-2} at the top as shown in Figure 2.1. Now place the vertices v_{n-1} and v_n as the end points of a horizontal line segment (perpendicular to the line segment used for placing the other n–2 points) with v_{n-1} to the left of v_n so that the vertices v_n, v_{n-1}, and v_{n-2} form a triangular face.

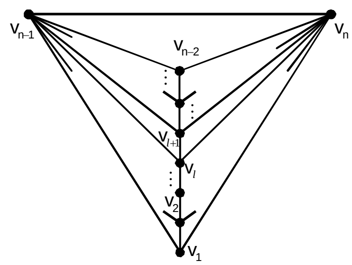

Fig.1: The graph Pl_n

II. Main Theorems

15) Theorem : Duplication of arbitrary vertex v_k of cycle C_n with $n \ge 3$ is divisor cordial graph.

Proof. Let C_n be cycle with n vertices $v_1, v_2, ..., v_n$, where $n \ge 3$. Let v_k be arbitrary vertex of C_n. Let G be the graph obtained by duplicating vertex v_k of cycle C_n. Let v'_k be duplicated vertex of v_k. Then $|V(G)| = n+1$ and $|E(G)| = n+2$.

Define vertex labeling $f : V(G) \to \{1, 2, ..., n+1\}$ as follows

$f(v'_k) = 2$.

Label the vertices $v_k, v_{k+1}, ..., v_{n-1}, v_n, v_1, v_2, ..., v_{k-1}$ in the following order.

$$
\begin{array}{lllll}
1, & 2^2, & 2^3, & ..., & 2^{k_1}, \\
3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\
5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\
... & ... & ... & ... & ..., \\
... & ... & ... & ... & ...
\end{array}
$$

where $(2m-1)2^{k_m} \le n+1$ and $m \ge 1$, $k_m \ge 0$.

Also $(2m-1)2^a$ divides $(2m-1)2^b$ $(a < b)$ and $(2m-1)2^{k_i}$ does not divide $2m+1$.

In the above labeling, the consecutive adjacent vertices of $v_k, v_{k+1}, ..., v_{n-1}, v_n, v_1, v_2, ..., v_{k-1}$ having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge. Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge. $f(v_1) | f(v_k)$, $f(v_1) | f(v'_k)$ and $f(v_{k-1}) \nmid f(v'_k)$ $(f(v'_k) \nmid f(v_{k-1}))$.

Thus, $e_f(0) = e_f(1) = \dfrac{n+2}{2}$, if n is even

$e_f(0) = \dfrac{n+1}{2}$ and $e_f(1) = \dfrac{n+3}{2}$, if n is odd.

Hence $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph.

16) Example : Consider a graph obtaned by duplicating vertex v_8 of cycle C_8 and its divisor cordial labeling are given in Fig. 2

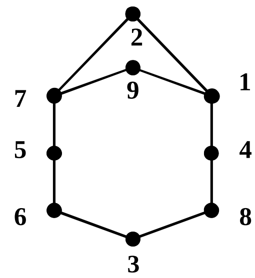

Fig. 2

17) Theorem : Graph $< S_n^{(1)} : S_n^{(2)} >$ is divisor cordial.

Proof : Let $u_1, u_2, ..., u_n$ be the vertices $S_n^{(1)}$ and $v_1, v_2, ..., v_n$ be the vertices of $S_n^{(2)}$. Let u_1 and v_1 be the apex vertices of $S_n^{(1)}$ and $S_n^{(2)}$ respectively which are joined to a vertex x. Let G = $< S_n^{(1)} : S_n^{(2)} >$. Then $|V(G)| = 2n+1$ and $| E(G) | = 4n–4$.

Define vertex labeling $f : V(G) \rightarrow \{1, 2, ..., 2n+1\}$ as follows

$f(x) = 2n+1$,
$f(u_i) = 2i – 1$, $\quad 1 \leq i \leq n – 1$
$f(u_n) = 2n$,
$f(v_i) = 2i$, $\quad 1 \leq i \leq n – 1$
$f(v_n) = 2n – 1$, $\quad 1 \leq i \leq n$.
Thus, $e_f(0) = e_f(1) = 2n – 2$.

Hence $|e_f(0) - e_f(1)| \leq 1$.
Hence G is divisor cordial graph

18) Example : Consider a graph G = $< S_8^{(1)} : S_8^{(2)} >$ and its divisor cordial labeling are given in Fig .3

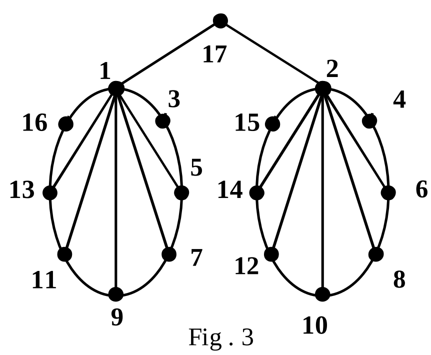

Fig . 3

19) Theorem : Graph $< W_n^{(1)} : W_n^{(2)} >$ is divisor cordial.

Proof: Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the rim vertices of $W_n^{(1)}$ and $W_n^{(2)}$. Let u and v be the apex vertices of $W_n^{(1)}$ and $W_n^{(2)}$ respectively which are adjacent to a common vertex x. Let G = $< W_n^{(1)} : W_n^{(2)} >$. Then $|V(G)| = 2n+3$ and $| E(G) | = 4n+2$.

Define vertex labeling $f : V(G) \rightarrow \{1, 2, ..., 2n+1\}$ as follows

Case 1 : n is even

$f(u) = 1$,
$f(u_n) = p$, where p is the maximum prime number and $p \leq 2n+3$

Label the remaining vertices $u_1, u_2, ..., u_{n–1}$ with odd numbers from 1 to 2n+3 other than 1 and p.

$f(v) = 2$,
$f(x) = 2n+3$,
$f(u_i) = 2i+1$, $\quad 1 \leq i \leq n$
Thus, $e_f(0) = e_f(1) = 2n+1$.

Here $|e_f(0) - e_f(1)| \leq 1$.
Hence G is divisor cordial graph, for n is even.

Case 2 : n is odd

For n = 3

$f(u) = 3$, $f(u_1) = 5$, $f(u_2) = 7$, $f(u_3) = 9$,.

$f(v) = 6$, $f(v_1) = 2$, $f(v_2) = 4$, $f(v_1) = 8$,

$f(x) = 1$.

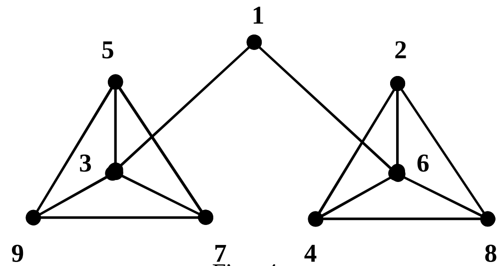

Fig . 4

Thus, $e_f(0) = e_f(1) = 0$.

Here $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph, for n = 3.

For $n \ge 5$

$f(u) = 1$,

$f(v) = 2$,

$f(x) = 2n+3$,

$f(u_n) = p$, where p is the maximum prime number and $p \le 2n+3$

Label the remaining vertices $u_1, u_2, ..., u_{n-1}$ with odd numbers from 1 to 2n+3 other than 1 and p.

$f(v_i) = 2i+2$, $\quad 1 \le i \le n-2$

$f(v_{n-1}) = 2n+2$,

$f(v_n) = 2n$.

Thus, $e_f(0) = e_f(1) = 2n+1$.

Here $|e_f(0) - e_f(1)| \le 1$.

Hence G is divisor cordial graph, for n is odd

Therefore G is divisor cordial graph.

20) Example : Consider a graph $G = \langle W_7^{(1)} : W_7^{(2)} \rangle$ and its divisor cordial labeling are given in Fig .5

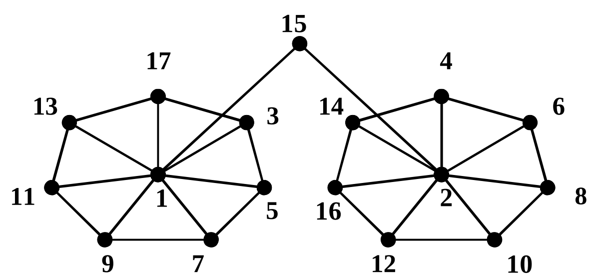

Fig .5

21) Theorem : The graph obtained by joining two copies of cycle W_n by path P_k admits divisor cordial labeling where $n \ge 3$.

Proof. Let G be the graph obtained by joining two copies of cycle W_n by path P_k. Let $u, u_1, u_2, ..., u_n$ be the vertices of first copy of cycle W_n and $v, v_1, v_2, ..., v_n$ be the vertices of second copy of cycle W_n. Let $w_1, w_2, ..., w_k$ be the vertices of path P_k with $u_1 = w_1$ and $v_1 = w_k$. Then $|V(G)| = 2n + k$ and $|E(G)| = 4n+k-1$.

Define vertex labeling $f : V(G) \to \{1, 2, ..., 2n+k\}$ as follows

Case 1: n is odd

$f(u) = 2$,

$f(v) = 1$,

$f(u_i) = k + 2 + 2i$, $\quad 2 \le i \le n \quad$ if k is even

$f(u_i) = k + 1 + 2i$, $\quad 2 \le i \le n \quad$ if k is odd

For k + 1 + 2n is prime

$f(v_i) = k + 1 + 2i, \quad 2 \le i \le n, \quad$ if k is even
For $k + 2 + 2n$ is prime
$f(v_i) = k + 2 + 2i, \quad 2 \le i \le n, \quad$ if k is odd
For $k + 1 + 2n$ is non-prime
$f(v_i) = k + 1 + 2i, \quad 2 \le i \le n-3, \quad$ if k is even
$f(v_{n-1}) = k + 1 + 2n$ and $f(v_n) = k - 1 + 2n, \quad$ if k is odd
For $k + 2 + 2n$ is non-prime
$f(v_i) = k + 2 + 2i, \quad 2 \le i \le n-3, \quad$ if k is odd
$f(v_{n-1}) = k + 2 + 2n$ and $f(v_n) = k + 2n, \quad$ if k is odd
Label the vertices $w_1, w_2, ..., w_k$ in the following order.

$$\begin{array}{lllll} 2^2, & 2^3, & 2^4, & ..., & 2^{k_1}, \\ 3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\ ... & ... & ... & ... & ..., \\ ... & ... & ... & ... & ... \end{array}$$

where $(2m-1)2^{k_m} \le k+2$ and $m \ge 1$, $k_m \ge 0$.

Also $(2m - 1)2^a$ divides $(2m - 1)2^b$ $(a < b)$ and $(2m-1)2^{k_i}$ does not divide $2m+ 1$.

In the above labeling, the consecutive adjacent vertices of $w_1, w_2, ..., w_k$ having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge. Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge. $f(u)|\ f(u_i)$ and $f(v)|f(v_i)$ for $1 \le i \le n$. $f(v_i)\nmid f(v_{i+1})$ for $1 \le i \le n-1$ and $f(v_n) \nmid f(v_1)$ $(f(v_1) \nmid f(v_n))$. But $f(u_i) \nmid f(u_{i+1})$ for $1 \le i \le n-2$ and either $f(v_n) \mid f(v_1)$ or $f(v_n) \mid f(v_{n-1})$.

Thus, $e_f(0) = e_f(1) = \dfrac{4n+k-1}{2}$, if k is odd

$e_f(0) = \dfrac{4n+k-2}{2}$ and $e_f(1) = \dfrac{4n+k}{2}$, if k is even.

Hence $|e_f(0) - e_f(1)| \le 1$.
Hence G is divisor cordial graph

Case 2 : n is even
For n = 4 and k = 4,5

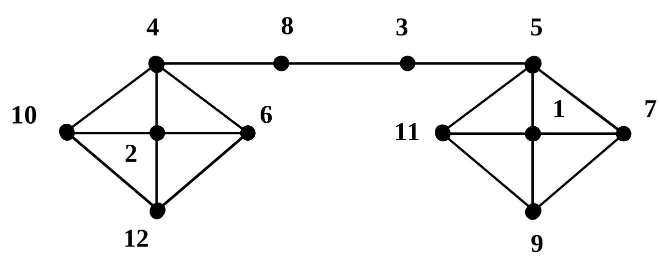

Fig .6

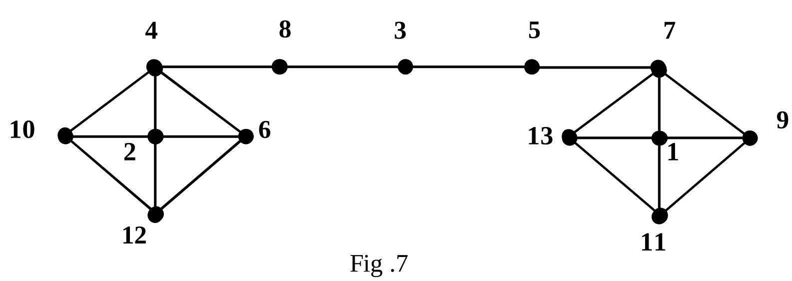

Fig .7

For n is even and $k \ne 4,5$
$f(u) = 2,$

$f(v) = 1,$
$f(u_i) = k + 2 + 2i,$ $\quad 2 \leq i \leq n - 2$ $\quad$ if k is even
$f(u_{n-1}) = k + 2 + 2n$ and $f(u_n) = k + 2n,$ $\quad$ if k is even
$f(u_i) = k + 1 + 2i,$ $\quad 2 \leq i \leq n - 2$ $\quad$ if k is odd
$f(u_{n-1}) = k + 1 + 2n$ and $f(u_n) = k - 1 + 2n,$ $\quad$ if k is odd
For $k + 1 + 2n$ is prime
$f(v_i) = k + 1 + 2i,$ $\quad 2 \leq i \leq n,$ $\quad$ if k is even
For $k + 2 + 2n$ is prime
$f(v_i) = k + 2 + 2i,$ $\quad 2 \leq i \leq n,$ $\quad$ if k is odd
For $k + 1 + 2n$ is non-prime
$f(v_i) = k + 1 + 2i,$ $\quad 2 \leq i \leq n-3,$ $\quad$ if k is even
$f(v_{n-1}) = k + 1 + 2n$ and $f(v_n) = k - 1 + 2n,$ $\quad$ if k is odd
For $k + 2 + 2n$ is non-prime
$f(v_i) = k + 2 + 2i,$ $\quad 2 \leq i \leq n-3,$ $\quad$ if k is odd
$f(v_{n-1}) = k + 2 + 2n$ and $f(v_n) = k + 2n,$ $\quad$ if k is odd
Label the vertices $w_1, w_2, ..., w_k$ in the following order.

$$\begin{array}{ccccc} 2^2, & 2^3, & 2^4, & ..., & 2^{k_1}, \\ 3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\ 5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\ ... & ... & ... & ... & ..., \\ ... & ... & ... & ... & ... \end{array}$$

where $(2m-1)2^{k_m} \leq k+2$ and $m \geq 1$, $k_m \geq 0$.

Also $(2m - 1)2^a$ divides $(2m - 1)2^b$ $(a < b)$ and $(2m-1)2^{k_i}$ does not divide $2m+ 1$.

In the above labeling, the consecutive adjacent vertices of $w_1, w_2, ..., w_k$ having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge. Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge. $f(u) | f(u_i)$ and $f(v)|f(v_i)$ for $1 \leq i \leq n$. $f(v_i) \nmid f(v_{i+1})$ for $1 \leq i \leq n-1$ and $f(v_n) \nmid f(v_1)$ $(f(v_1) \nmid f(v_n))$. But $f(u_i) \nmid f(u_{i+1})$ for $1 \leq i \leq n-2$ and either $f(v_n) | f(v_1)$ or $f(v_n) | f(v_{n-1})$.

Thus, $e_f(0) = e_f(1) = \dfrac{4n+k-1}{2}$, if k is odd

$e_f(0) = \dfrac{4n+k-2}{2}$ and $e_f(1) = \dfrac{4n+k}{2}$, if k is even.

Hence $|e_f(0) - e_f(1)| \leq 1$.

Hence G is divisor cordial graph

22) Example : Consider a graph G obtained by joining two copies of cycle W_5 by path P_5 and its divisor cordial labeling are given in Fig .8

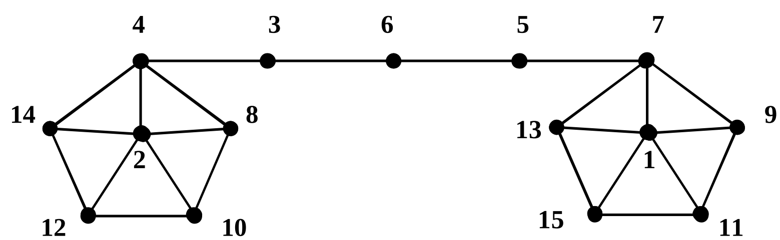

Fig .8

23) Theorem : The $G_v \odot K_1$ is divisor cordial graph, where G_v denotes graph obtained by switching of vertex v of C_n and $n \geq 4$.

Proof. Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n $(n \geq 4)$. G_{v_1} is a graph obtained by switching of vertex v_1 of C_n. Then, $G_v \odot K_1$ be graph with 2n vertices and 3n–5 edges. Let $v_1, v_2, ..., v_n, v'_1, v'_2, ..., v'_n$ be the vertices of $G_v \odot K_1$. Then $|V(G_v \odot K_1)| = 2n$ and $|E(G_v \odot K_1)| = 3n - 5$.

Define vertex labeling $f : V(G_v \odot K_1) \rightarrow \{1, 2, ..., 2n\}$ as follows

Case (i) n is even

$f(v_1) = 1,$

$f(v'_1) = 2,$

$f(v_{1+i}) = 2i + 1, \qquad 1 \le i \le n-1$

$f(v'_{1+i}) = 4i + 2, \qquad 1 \le i \le \frac{n-2}{2}$

$f(v'_{\frac{n-2}{2}+i}) = 4(i-1), \qquad 2 \le i \le \frac{n+1}{2}.$

Thus, $e_f(0) = \frac{3n-4}{2}$ and $e_f(1) = \frac{3n-6}{2}$.

Hence $|e_f(0) - e_f(1)| \le 1$.

Hence $G_v \odot K_1$ is divisor cordial graph

Case (ii) n is odd

$f(v_1) = 1,$

$f(v'_1) = 2,$

$f(v_{1+i}) = 2i + 1, \qquad 1 \le i \le n-1$

$f(v'_{1+i}) = 4i + 2, \qquad 1 \le i \le \frac{n-1}{2}$

$f(v'_{\frac{n-1}{2}+i}) = 4(i-1), \qquad 2 \le i \le \frac{n+1}{2}.$

Thus, $e_f(0) = e_f(1) = \frac{3n-5}{2}$.

Hence $|e_f(0) - e_f(1)| \le 1$.

Hence $G_v \odot K_1$ is divisor cordial graph

24) Example : Consider a graph $G_v \odot K_1$, G_v denotes graph obtained by switching of vertex v_1 of C_7 and its divisor cordial labeling are given in Fig. 9.

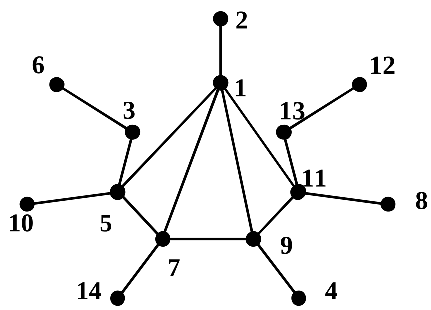

Fig. 9

25) Theorem : The Pl_n is divisor cordial graph, where $n \ge 5$.

Proof. Let Pl_n be a planar graph with n vertices, where $n \ge 5$. Let $v_1, v_2, ..., v_n$ be the vertices of Pl_n. Then $|V(Pl_n)| = n$ and $|E(Pl_n)| = 3n - 6$. Define vertex labeling $f : V(Pl_n) \to \{1, 2, ..., n\}$ as follows

Case (i) n is odd and $n \ge 5$.

Define f by

$f(v_n) = 1,$

$f(v_{n-1}) = p$, where p is the maximum prime numbers between 1 and n. The remaining numbers other than 1 and p from 1 to n are labeled to the remaining vertices $v_1, v_2, ..., v_{n-2}$ as follows. Label of these vertices in the following order.

$$\begin{array}{lllll}
2 & 2^2, & 2^3, & ..., & 2^{k_1}, \\
3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\
5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\
... & ... & ... & ... & ..., \\
... & ... & ... & ... & ...
\end{array}$$

where $(2m-1)2^{k_m} \le n$ and $m \ge 1$, $k_m \ge 0$.

Also $(2m-1)2^a$ divides $(2m-1)2^b$ $(a < b)$ and $(2m-1)2^{k_i}$ does not divide 2m+ 1.

In the above labeling, the consecutive adjacent vertices of $v_1, v_2, ..., v_{n-2}$ having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge. Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge. $f(v_n) \mid f(v_i)$ and $f(v_{n-1}) \nmid f(v_i)$ for $1 \le i \le n-2$ and $f(v_n) \mid f(v_{n-1})$.

Thus, $e_f(0) = \frac{3n-7}{2}$ and $e_f(1) = \frac{3n-5}{2}$.

Hence $|e_f(0) - e_f(1)| \le 1$.

Hence Pl_n is divisor cordial graph if n is odd and $n \ge 5$.

Case (ii) n = 6.

Define f by

$f(v_1) = 3$, $f(v_2) = 4$, $f(v_3) = 6$, $f(v_4) = 3$, $f(v_5) = 5$ and $f(v_6) = 1$.

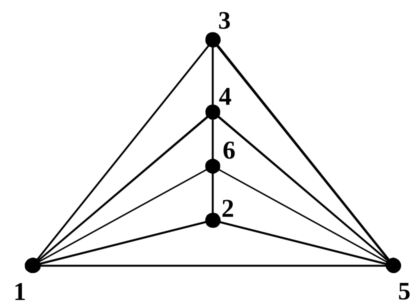

Fig. 10

Thus, $e_f(0) = e_f(1) = 6$.

Hence $|e_f(0) - e_f(1)| \le 1$.

Hence Pl_n is divisor cordial graph if n = 6.

Case (iii) n is even and $n \ge 8$.

Define f by

$f(v_n) = 1$,

$f(v_{n-1}) = p$, where p is the maximum prime number and $p \le n$.

$f(v_{n-2}) = 2$,

The remaining numbers other than 1, 2 and p from 1 to n are labeled to the remaining vertices $v_1, v_2, ..., v_{n-3}$ as follows. Label of these vertices in the following order.

$$
\begin{array}{lllll}
2^2, & 2^3, & 2^4, & ..., & 2^{k_1}, \\
3, & 3\times 2 & 3\times 2^2 & ..., & 3\times 2^{k_2}, \\
5, & 5\times 2 & 5\times 2^2 & ..., & 5\times 2^{k_3}, \\
... & ... & ... & ... & ..., \\
... & ... & ... & ... & ...
\end{array}
$$

where $(2m-1)2^{k_m} \le n$ and $m \ge 1$, $k_m \ge 0$.

Also $(2m-1)2^a$ divides $(2m-1)2^b$ $(a < b)$ and $(2m-1)2^{k_i}$ does not divide 2m+ 1.

In the above labeling, the consecutive adjacent vertices of $v_1, v_2, ..., v_{n-3}$ having the labels even numbers and consecutive adjacent vertices having labels odd and even numbers contribute 1 to each edge. Similarly, the consecutive adjacent vertices having the labels odd numbers and consecutive adjacent vertices having labels even and odd numbers contribute 0 to each edge. $f(v_n) \mid f(v_i)$ and $f(v_{n-1}) \nmid f(v_i)$ $(f(v_i) \nmid f(v_{n-1}))$ for $1 \le i \le n-2$, $f(v_{n-2}) \nmid f(v_{n-1})$ $(f(v_{n-1}) \nmid f(v_{n-2}))$ and $f(v_n) \mid f(v_{n-1})$.

Thus, $e_f(0) = e_f(1) = \frac{3n-6}{2}$.

Hence $|e_f(0) - e_f(1)| \le 1$.

Hence Pl_n is divisor cordial graph if n is even and $n \ge 8$.

Hence Pl_n is divisor cordial graph if $n \ge 5$.

26) Example : Consider a graph Pl_7 and its divisor cordial labeling are given in Fig 2.11.

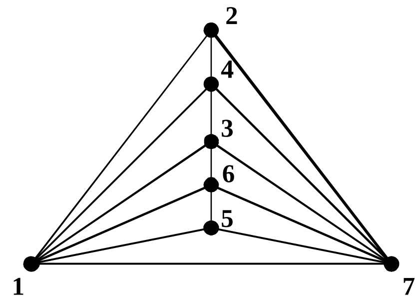

Fig. 11

III. CONCLUSIONS

In this paper, we prove that the duplication of arbitrary vertex v_k of cycle C_n ($n \geq 3$), $< S_n^{(1)} : S_n^{(2)} >$, $< W_n^{(1)} : W_n^{(2)} >$, the graph obtained by joining two copies of cycle W_n by a path P_k ($n \geq 3$), $G_v \odot K_1$, where G_v denotes graph obtained by switching of any vertex v of C_n ($n \geq 4$) and Pl_n ($n \geq 5$) are divisor cordial graph.

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