Pathos Vertex Semientire Block Graph

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ABSTRACT

The pathos vertex semientire block graph denoted by $P_{vb}(G)$ is the graph whose vertex set is $V(T) \bigcup b_i \bigcup r$ and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the blocks, vertices lie on the regions and the adjacent blocks. We study the characterization of graphs whose pathos vertex semientire block graph is planar, outer planar, Eulerian and Hamiltonian
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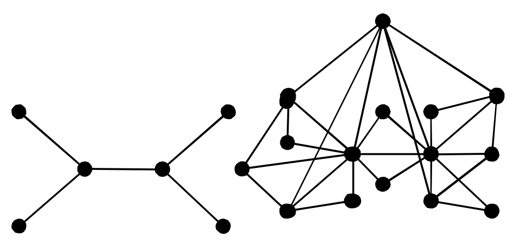
1. INTRODUCTION

Let G(p, q) be a connected planar graph. We refer the terminology of [5]. The concept of pathos of a graph G was introduced by Harary [2], as a collection of minimum number of line disjoint open paths whose union is G. The path number of a graph G is the number of paths in a pathos. A new concept of a graph valued functions called the pathos vertex semientire graph $Pe_v(G)$ of a plane graph G was introduced [5]. For a graph G(p, q) if $B = u_1, u_2, u_3, \cdot\cdot\cdot, u_r; r \geq 2$ is a block of G. Then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut vertex then they are called adjacent blocks. All undefined terminology will conform with that in Harary [2]. All graphs considered here are finite, undirected and without loops or multiple lines.

The edgedegree of an edge e = {a, b} is the sum of degrees of the end vertices a and b. Blockdegree is the number of vertices lies on a block. Blockpath is a path in which each edge in a path becomes a block. Degree of a region is the number of vertices lies on a region. A pendant pathos is a path Pi of pathos having unit length. The inner vertex number i(G) of a planar graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane. A graph G is said to be minimally non-outerplanar if i(G) = 1.

A new concept of a graph valued functions called the pathos vertex semientire graph $Pe_v(G)$ of a plane graph G was introduced [5] and is defined as the graph whose vertex set is $V(T) \bigcup b_i \bigcup r$ and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the path of pathos and vertices lie on the regions. Since the system of pathos for a tree is not unique, the corresponding vertex semientire block graph is also not unique. The vertex semientire block graph is introduced in [6]. The vertex semientire block graph denoted by $e_{vb}(G)$ is the graph whose vertex set is $V(T) \bigcup b_i \bigcup r$ and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the blocks and vertices lie on the regions.

We now define the pathos vertex semientire block graph of a tree T. The pathos vertex semientire block graph of a tree T denoted by $P_{vb}(T)$ is the graph whose vertex set is the union of the vertices , regions and path of pathos of T and the two vertices are adjacent if and only if they are adjacent vertices of T, vertices lie on the blocks of T, vertices lie on the regions of T and the adjacent blocks of T. Clearly the number of regions in a tree is one. The tree T and its pathos vertex semientire block graph $P_{vb}(T)$ is depicted in the figure1.



T Figure 1. $P_{vb}(T)$

2. PRELIMINARY RESULTS

We need the following results to prove further results.

Theorem 1[4]. If G be a connected plane graph then the vertex semientire graph $e_v(G)$ is planar if and only if G is a tree.

Theorem 2[3]. Every maximal outerplanar graph G with p vertices has 2p – 3 edges.

Theorem 3 [4]. For any (p, q) graph G with b blocks and r regions vertex semientire block graph $e_{vb}(G)$ has (p + b +r) vertices and $q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)$ edges, where $d(b_i)$ is the block degree of a block b_i and $d(r_j)$ is the degree of a region r_j.

3. MAIN RESULTS

We start with a preliminary result.

Remark 1. For any graph G, $G \subseteq e_{vb}(G) \subseteq P_{vb}(T)$.

In the following theorem we obtain the number of vertices and edges in pathos vertex semientire block graph.

Theorem 4. For any (p, q) graph T with b blocks and r regions pathos vertex semientire block graph $P_{vb}(T)$ has (2p +k) vertices and $4p-3+\sum_{i=1}^{k} v(p_i)$ edges, where $v(p_i)$ be the number of vertices lies on the path p_i.

Proof. By the Theorem 3, the number of vertices in $e_{vb}(T)$ is (p + b +r). By the definition of pathos vertex semientire block graph $P_{ev}(T)$ it follows that the number of vertices is the union of the vertices, blocks, the regions and the path of pathos of T. Since in a tree T, each edge is a block and it contains only one region. Hence
p+b+r+k=p+q+1+k
=p+p-1+1+k
=2p+k vertices. Hence the number of vertices in pathos vertex semientire block graph $P_{vb}(T)$ is (2p + k).

Further, by Theorem 3, the number of edges vertex semientire block graph $e_{vb}(G)$ is $q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)$. By the Remark 1 it follows that $e_{vb}(G)$ is subgraph of $P_{vb}(T)$. Also the number of edges in $P_{vb}(T)$ is the sum of the edges in $e_{vb}(G)$ and the edges formed by the pathos vertices , which is $\sum v(p_i)$. Hence the number of edge in

$$P_{vb}(G) = q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)+\sum_{i=1}^{k} v(p_i)$$

$$= q+2q+p+\sum_{i=1}^{k} v(p_i) = 3p-3+p+\sum_{i=1}^{k} v(p_i)$$

$$= 4p-3+\sum_{i=1}^{k} v(p_i).$$

Theorem 5. For any tree T, pathos vertex semientire block graph $P_{vb}(T)$ is always nonseparable.

Proof. We have the following cases.

Case 1. Suppose T be a path. All internal vertices of T are the cut vertices C_i. These cut vertices lies on the region as well as on two blocks. Clearly C_i is not a cut vertex in $P_{vb}(T)$. Hence $P_{vb}(T)$is nonseparable.

Case 2. Suppose T be any tree. Since cut-vertex C_i lies on at least two blocks and one region. Hence in $P_{vb}(T)$, C_i becomes non-cut-vertex. Also the pathos vertex is adjacent to all vertices v_i of T. Hence $P_{vb}(T)$ is always nonseparable.

Theorem 6. For any tree T, pathos vertex semientire block graph $P_{vb}(T)$ is planar.

Proof. Suppose a graph T be a tree. By definition of vertex semientire block graph, for each edge of a tree G, there is a $K_4 - e$ in $e_{vb}(G)$. Hence in $P_{vb}(T)$, the pathos vertices are adjacent to the vertices those are lies on the path. Clearly $P_{vb}(T)$ is a graph which is homeomorphic to K_4. Hence $P_{vb}(T)$ is planar.

Theorem 7. For any tree T the pathos vertex semientire block graph $P_{vb}(G)$ always non-outerplanar.

Proof. Consider a tree T be a path P_n. Suppose n=2. Since each edge is a block and both end vertices lies on a block. These end vertices and a blockvertex form a graph K_3 in $e_{vb}(G)$. Further the regionvertex vertex is adjacent to all vertices of G to form K_4-x. Also the pathos vertex is adjacent to all vertices of $e_{vb}(T)$ to form a graph with one inner vertex, which is non- outerplanar. Hence $P_{vb}(T)$ is always non-outerplanar.

Theorem 8. For any tree T, pathos vertex semientire block graph $P_{vb}(T)$ is minimally non- outerplanar if and only if T is a path P_2.

Proof. Proof follows from the Theorem 7.

Theorem 9. For any tree T, pathos vertex semientire block graph $P_{vb}(G)$ is Eulerian if and only if T is a path P_n for n is even.

Proof. Suppose $P_{vb}(G)$ is Eulerian. Assume that a tree T be a path P_n for n is odd. By the definition of $P_{vb}(T)$, the regionvertex is adjacent to all vertices of T, the pathos vertex is adjacent to all vertices of T. Also each edge is a block and the block vertex is adjacent of exactly two vertices . Lastly the pathos vertex is adjacent to all vertices of T. Clearly each vertex v_i is adjacent to the corresponding block vertex b_i, regionvertex r_1 and the pathos vertex p_1. It follows that degree of each v_k is even. Since the regionvertex is adjacent to all vertices v_i for i= 1,2,…n which is odd. Hence degree of regionvertex becomes odd. Similarly the degree of pathos vertex becomes odd. Hence $P_{vb}(T)$ is non- Eulerian, a contradiction.

Conversely suppose T be a path P_n for n is even. By the definition of $P_{vb}(T)$. The degree of all v_i in $P_{vb}(T)$ becomes even. Since the regionvertex is adjacent to all even number of v_i such that degree of regionvertex becomes even. Lastly the pathos vertex is adjacent to all even numbers of vertices such that degree of regionvertex becomes even. Clearly all vertices is of even degree. Hence $P_{vb}(G)$ is Eulerian.

Theorem 10. For any tree T, the pathos vertex semientire block graph $P_{vb}(T)$ is non-Hamiltonian.

Proof. Suppose T be a tree. Without loss of generality consider a path P_n for n=2. Let v_1 and v_2 be the vertices of T. By the definition of vertex edge semientire graph $e_{vb}(T)$, the vertices v_1 and v_2 are adjacent to the block vertex b_1 and the regionvertex r_1. Clearly $e_{vb}(P_2) = K_4\_x$, for any edge x. In pathos vertex edge semientire graph $P_{vb}(T)$, the pathos vertex P_1 is adjacent to v_1 and v_2 . Clearly v_1 , b_1, v_2, r_1, v_1 form a Hamiltonian cycle in $e_{vb}(T)$. But in $P_{vb}(T)$, the vertex P_1 is not lies on the Hamiltonian cycle. Hence very edge forms this type of graph and it is non- Hamiltonian.

4. CONCLUSION

In this paper the relation between the line graph and the pathos vertex semientire block graph is introduced. Further the conditions of planarity, Hamiltonian and Eulerian are established.

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