On Edge Pair Sum Labeling of Graphs

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Abstract - An injective map $f : E(G) \rightarrow \{\pm 1, \pm 2, \dots, \pm q\}$ is said to be an edge pair sum labeling of a graph G(p, q) if the induced vertex function $f^*: V(G) \rightarrow Z - \{0\}$ defined by $f^*(v) = \sum_{e \in E_v} f(e)$ is one – one, where E_v denotes the set of edges in G that are incident with a vertex v and $f^*(V(G))$ is either of the form $\{\pm k_1, \pm k_2, \dots, \pm k_{\frac{p}{2}}\}$ or $\{\pm k_1, \pm k_2, \dots, \pm k_{\frac{p-1}{2}}\} \cup \{k_{\frac{p}{2}}\}$ according as p is even or odd. A graph which admits edge pair sum labeling is called an edge pair sum graph. In this paper we prove that some cycle related graphs are edge pair sum graphs.

Keywords: Pair sum labeling, pair sum graph, edge pair sum labeling, edge pair sum graph, ladder graph.

AMS Subject Classification (2010): 05C78

I. Introduction

All graphs in this paper are finite, simple and undirected. For standard terminology and notations we follow Gross and Yellen [1]. The symbols V(G) and E(G) denote the vertex set and the edge set of a graph. R.Ponraj et.al introduced the concept of pair sum labeling in [7]. An injective map $f: V(G) \rightarrow \{\pm 1, \pm 2, \dots, \pm p\}$ is said to be a pair sum labeling of a graph $G(p,q)$ if the induced edge function $f_e: E(G) \rightarrow Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \dots, \pm k_{\frac{q}{2}}\}$ or $\{\pm k_1, \pm k_2, \dots, \pm k_{\frac{q-1}{2}}\} \cup \{\pm k_{\frac{q+1}{2}}\}$ according as q is even or odd. A graph with a pair sum labeling is called pair sum graph. Analogous to pair sum labeling we define a new labeling called edge pair sum labeling [3]. Let $G(p, q)$ be a graph. An injective map $f : E(G) \rightarrow \{\pm 1, \pm 2, \dots, \pm q\}$ is said to be an edge pair sum labeling if the induced vertex function $f^*: V(G) \rightarrow Z - \{0\}$ *is* defined by $f^*(v) = \sum_{e \in E_v} f(e)$ is one – one where E_v denotes the set of edges in G that are incident with a vertex v and $f^*(V(G))$ is either of the form $\{\pm k_1, \pm k_2, \dots, \pm k_{\frac{p}{2}}\}$ or $\{\pm k_1, \pm k_2, \dots, \pm k_{\frac{p-1}{2}}\} \cup \{k_{\frac{p}{2}}\}$ according as p is even or odd. A graph with an edge pair sum labeling is called an edge pair sum graph. We established that the path, cycle, star graph, $P_m \cup K_{1,n}$, $C_n \odot K_m^c$ if n is even, triangular snake, bistar, $K_{1,n} \cup K_{1,m}$, $c_n \cup c_n$ and complete bipartite graphs $K_{1,n}$ are edge pair sum graph [3-6]. In this paper we prove that some cycle related graphs are edge pair sum graphs.

II EDGE PAIR SUM GRAPH WITH MANY ODD AND EVEN CYCLES:

In [3] it was shown that the cycle C_n is an edge pair sum graph. The star graph $K_{1,n}$ is an edge pair sum graph if and only if n is even. We would like to consider graphs with many odd and even cycles. Let $P_n(+)N_m$ be a graph with $V(P_n(+)\ N_m) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m\}$ $E(P_n(+)\ N_m) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_1u_1, v_1u_2, \ldots, v_1u_m, v_nu_1, v_nu_2, \ldots, v_nu_m\}$. Here p = n + m and q = n+2m-1.

Theorem 2.1: The graph $P_n(+)N_m$ is an edge pair sum graph if m is odd.

Proof: Let $V(P_n(+)N_m) = \{v_i, u_j : 1 \le i \le n, 1 \le j \le m\}$ and $E(P_n(+)N_m) = \{e_i = v_iv_{i+1} : 1 \le i \le (n-1),\ e_j' = v_1u_j \text{ and } e_j'' = v_nu_j : 1 \le j \le m\}$ be the vertices and edges of the graph $P_n(+)N_m$. Define the edge labeling $f : E(P_n(+)N_m) = \{\pm 1, \pm 2, \ldots, \pm(n + 2m - 1)\}$ by considering the following three cases.

Case (i): n = 2

Define $f(e_1) = 2$, $f(e_1') = -1$, $f(e_1'') = -3$,

for $1 \le i \le \frac{m-1}{2}$ $f(e_{1+i}') = (2i + 3)$, $f\left(e'_{\frac{m+1}{2}+i}\right) = -(2i + 3)$, $f(e_{1+i}'') = (m + 2 + 2i)$ and $f\left(e''_{\frac{m+1}{2}+i}\right) = -(m + 2 + 2i)$.

The induced vertex labeling are as follows $f^*(v_1) = f(e_1) + f(e_1') + f(e_{1+i}') + f\left(e'_{\frac{m+1}{2}+i}\right) = 1$, $f^*(v_2) = f(e_1) + f(e_1'') + f(e_{1+i}'') + f\left(e''_{\frac{m+1}{2}+i}\right) = -1$, $f^*(u_1) = f(e_1') + f(e_1'') = -4$, for $1 \le i \le \frac{m-1}{2}$ $f^*(u_{1+i}) = f(e_{1+i}') + f(e_{1+i}'') = (m + 5 + 4i)$ and $f^*\left(u_{\frac{m+1}{2}+i}\right) = f\left(e'_{\frac{m+1}{2}+i}\right) + f\left(e''_{\frac{m+1}{2}+i}\right) = -(m + 5 + 4i)$. From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm 1, \pm(m + 9), \pm(m + 13), \ldots, \pm(3m + 3)\} \cup \{-4\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$. The example for the edge pair sum graph labeling $P_n(+)N_m$ for n = 2 and m = 3 is shown in Figure 1.

Figure 1

Case (ii): n is odd and take n = 2k+1.

Sub case (i): k =1 and k is even

for $1 \le i \le k$ $f(e_i) = (1 + i)$ and $f(e_{k+i}) = -i$, $f(e_1') = 1$, $f(e_1'') = -(k + 1)$, for $1 \le i \le \frac{m-1}{2}$ $f(e_{1+i}') = (k + 1 + 2i)$, $f\left(e'_{\frac{m+1}{2}+i}\right) = -(k + 1 + 2i)$, $f(e_{1+i}'') = (k + m + 2i)$ and $f\left(e''_{\frac{m+1}{2}+i}\right) = -(k + m + 2i)$.

The induced vertex labeling are

$f^*(v_1) = f(e_1) + f(e_1^{'}) + f(e_{1+i}^{'}) + f\left(e_{\frac{m+1}{2}+i}^{'}\right) = 3$, for $1 \le i \le k-1$ $f^*(v_{1+i}) = f(e_i) + f(e_{1+i}) = (2i+3)$, $f^*(v_{k+1}) = f(e_k) + f(e_{k+1}) = k$, for $1 \le i \le k-1$ $f^*(v_{k+1+i}) = f(e_{k+i}) + f(e_{k+1+i}) = -(2i+1)$, $f^*(v_{2k+1}) = f(e_{2k}) + f(e_1^{''}) = -(2k+1)$, $f^*(u_1) = f(e_1^{'}) + f(e_1^{''}) = -k$, for $1 \le i \le \frac{m-1}{2}$ $f^*(u_{1+i}) = f(e_{1+i}^{'}) + f(e_{1+i}^{''}) = (2k+m+1+4i)$ and $f^*\left(u_{\frac{m+1}{2}+i}\right) = f\left(e_{\frac{m+1}{2}+i}^{'}\right) + f\left(e_{\frac{m+1}{2}+i}^{''}\right) = -(2k+m+1+4i)$. From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm k, \pm 3, \pm 5, \pm 7, \dots, \pm(2k+1), \pm(2k+m+5), \pm(2k+m+9), \dots, \pm(2k+3m-1)\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$. Figure 2 illustrates the edge sum graph labeling $P_n(+)N_m$ where m = n = 3.

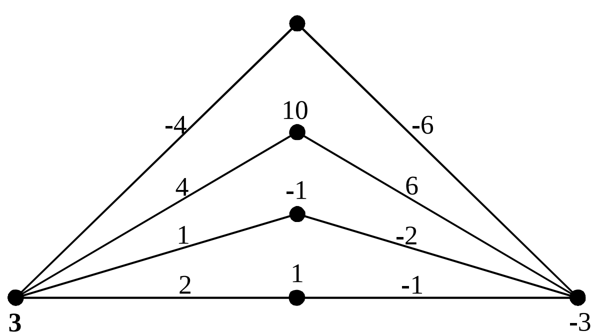

Figure 2

Sub case (ii): k is odd and $k \ge 3$

for $1 \le i \le k$ $f(e_i) = (1+i)$, $f(e_{k+1}) = -2$, $f(e_{k+2}) = -1$, for $1 \le i \le k-2$ $f(e_{k+2+i}) = f(e_{k+i}) - 2$, $f(e_1^{'}) = 1$, $f(e_1^{''}) = -k$, for $1 \le i \le \frac{m-1}{2}$ $f(e_{1+i}^{'}) = (k+2i)$, $f\left(e_{\frac{m+1}{2}+i}^{'}\right) = -(k+2i)$, $f(e_{1+i}^{''}) = (k+m-1+2i)$ and $f\left(e_{\frac{m+1}{2}+i}^{''}\right) = -(k+m-1+2i)$.

The induced vertex labeling are $f^*(v_1) = 3$, for $1 \le i \le k-1$ $f^*(v_{1+i}) = f(e_i) + f(e_{1+i}) = (2i+3)$, $f^*(v_{k+1}) = f(e_k) + f(e_{k+1}) = (k-1)$, for $1 \le i \le k$ $f^*(v_{k+1+i}) = f(e_{k+i}) + f(e_{k+1+i}) = -(2i+1)$, $f^*(u_1) = f(e_1^{'}) + f(e_1^{''}) = -(k-1)$, for $1 \le i \le \frac{m-1}{2}$ $f^*(u_{1+i}) = f(e_{1+i}^{'}) + f(e_{1+i}^{''}) = (2k+m-1+4i)$ and $f^*\left(u_{\frac{m+1}{2}+i}\right) = f\left(e_{\frac{m+1}{2}+i}^{'}\right) + f\left(e_{\frac{m+1}{2}+i}^{''}\right) = -(2k+m-1+4i)$.

From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm 3, \pm 5, \pm 7, \dots, \pm(2k+1), \pm(k-1), \pm(2k+m+3), \pm(2k+m+7), \dots, \pm(2k+3m-3)\}$.

Hence f is an edge pair sum labeling of $P_n(+)N_m$.

Case (iii): n is even and take n = 2k

Sub case (i): k ≡ 0,2 (mod 3)

Define $f(e_1) = -2$, for $1 \le i \le k-1$ $f(e_{1+i}) = (2+i)$, $f(e_{k+1}) = -1$, for $1 \le i \le k-2$ $f(e_{k+1+i}) = -(2+i)$, $f(e_1^{'}) = 1$, $f(e_1^{''}) = -(k+1)$, for $1 \le i \le \frac{m-1}{2}$ $f(e_{1+i}^{'}) = (k+2i)$,

$f\left(e'_{\frac{m+1}{2}+i}\right) = -(k+2i)$, $f(e''_{1+i}) = (k+m-1+2i)$ and $f\left(e''_{\frac{m+1}{2}+i}\right) = -(k+m-1+2i)$. The induced vertex labeling are $f^*(v_1) = -1, f^*(v_2) = 1$, for $3 \le i \le k$ $f^*(v_i) = f(e_i) + f(e_{1+i}) = (2i+1)$, $f^*(v_{k+1}) = k, f^*(v_{k+2}) = -4$, for $3 \le i \le k$ $f^*(v_{k+i}) = -(2i+1)$, $f^*(u_1) = f(e'_1) + f(e''_1) = -k$, for $1 \le i \le \frac{m-1}{2}$ $f^*(u_{1+i}) = f(e'_{1+i}) + f(e''_{1+i}) = (2k+m-1+4i)$ and $f^*\left(u_{\frac{m+1}{2}+i}\right) = f\left(e'_{\frac{m+1}{2}+i}\right) + f\left(e''_{\frac{m+1}{2}+i}\right) = -(2k+m-1+4i)$.

From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm1, \pm k, \pm7, \pm9, \dots, \pm(2k+1), \pm(2k+m+3), \pm(2k+m+7), \dots, \pm(2k+3m-3)\} \cup \{-4\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$.

Sub case (ii): k ≡ 1 (mod 3)

Define $f(e_1) = 2$, $f(e_2) = -3$, for $1 \le i \le k-2$ $f(e_{2+i}) = (3+i)$, $f(e_{k+1}) = -1$, $f(e_{k+2}) = -2$, for $1 \le i \le k-3$ $f(e_{k+2+i}) = -(3+i)$, $f(e'_1) = 1$, $f(e''_1) = -(k+1)$, for $1 \le i \le \frac{m-1}{2}$ $f(e'_{1+i}) = (k+1+2i)$, $f\left(e'_{\frac{m+1}{2}+i}\right) = -(k+1+2i)$, $f(e''_{1+i}) = (k+m+2i)$ and $f\left(e''_{\frac{m+1}{2}+i}\right) = -(k+m+2i)$. The induced vertex labeling are $f^*(v_1) = 3$, $f^*(v_2) = -1$, $f^*(v_3) = 1$, for $4 \le i \le k$ $f^*(v_i) = (2i+1)$, $f^*(v_{k+1}) = k, f^*(v_{k+2}) = -3$, $f^*(v_{k+3}) = -6$, for $4 \le i \le k$ $f^*(v_{k+i}) = -(2i+1)$, $f^*(u_1) = f(e'_1) + f(e''_1) = -k$, for $1 \le i \le \frac{m-1}{2}$ $f^*(u_{1+i}) = f(e'_{1+i}) + f(e''_{1+i}) = (2k+m+1+4i)$ and $f^*\left(u_{\frac{m+1}{2}+i}\right) = f\left(e'_{\frac{m+1}{2}+i}\right) + f\left(e''_{\frac{m+1}{2}+i}\right) = -(2k+m+1+4i)$. From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm1, \pm k, \pm3, \pm9, \pm11, \dots, \pm(2k+1), \pm(2k+m+5), \pm(2k+m+9), \dots, \pm(2k+3m-1) \cup \{-6\}\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$ if m is odd.

Theorem 2.2: The graph $P_n(+)N_m$ is an edge pair sum graph if m is even and $n \ge 3$.

Proof: Let $V(P_n(+)N_m) = \{v_i, u_j : 1 \le i \le n, 1 \le j \le m\}$ and $E(P_n(+)N_m) = \{e_i = v_iv_{i+1} : 1 \le i \le n, e'_j = v_1u_j \text{ and } e''_j = v_nu_j : 1 \le j \le m\}$ be the vertices and edges of the graph $P_n(+)N_m$. Define the edge labeling $f: E(P_n(+)N_m) = \{\pm1, \pm2, \dots, \pm(n+2m-1)\}$ by considering the following four cases.

Case (i): n = 3

Define $f(e_1) = -2$, $f(e_2) = 1$, for $1 \le i \le \frac{m}{2}$ $f(e'_i) = (1+2i)$, $f\left(e'_{\frac{m}{2}+i}\right) = -(1+2i)$, $f(e''_i) = (m+1+2i)$ and $f\left(e''_{\frac{m}{2}+i}\right) = -(m+1+2i)$. The induced vertex labeling are $f^*(v_1) = -2$, $f^*(v_2) = -1$, $f^*(v_3) = 1$, for $1 \le i \le \frac{m}{2}$ $f^*(u_i) = f(e_i^{'}) + f(e_i^{''}) = (m + 2 + 4i)$ and $f^*\left(u_{\frac{m}{2}+i}\right) = f\left(e_{\frac{m}{2}+i}^{'}\right) + f\left(e_{\frac{m}{2}+i}^{''}\right) = -(m + 2 + 4i)$. From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm 1, \pm(m + 6), \pm(m + 10), \dots, \pm(3m + 2) \cup \{-2\}\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$. Figure 3 shows that $P_n(+)N_m$ is an edge pair sum graph labeling for m = 2 and n = 3.

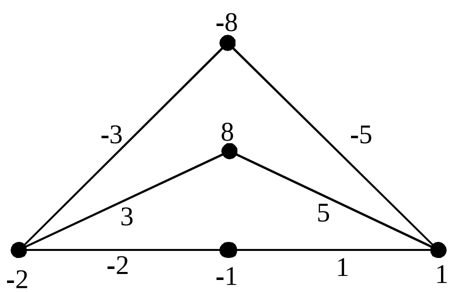

Figure 3

Case (ii): n = 4

$f(e_1) = -2$, $f(e_2) = -1$, $f(e_3) = 3$, for $1 \le i \le \frac{m}{2}$ $f(e_i^{'}) = (3 + 2i)$, $f\left(e_{\frac{m}{2}+i}^{'}\right) = -(3 + 2i)$, $f(e_i^{''}) = (m + 3 + 2i)$ and $f\left(e_{\frac{m}{2}+i}^{''}\right) = -(m + 3 + 2i)$.

The induced vertex labeling are $f^*(v_1) = -2$, $f^*(v_2) = -3$, $f^*(v_3) = 2$, $f^*(v_4) = 3$, for $1 \le i \le \frac{m}{2}$ $f^*(u_i) = f(e_i^{'}) + f(e_i^{''}) = (m + 6 + 4i)$ and $f^*\left(u_{\frac{m}{2}+i}\right) = f\left(e_{\frac{m}{2}+i}^{'}\right) + f\left(e_{\frac{m}{2}+i}^{''}\right) = -(m + 6 + 4i)$. From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm 2, \pm 3, \pm(m + 10), \pm(m + 14), \dots, \pm(3m + 6)\}$.

Hence f is an edge pair sum labeling of $P_n(+)N_m$. Figure 4 shows that $P_n(+)N_m$ is an edge pair sum graph labeling for m = n = 4.

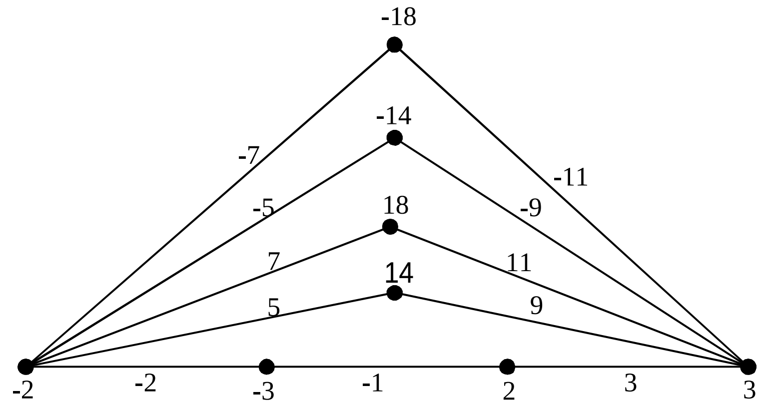

Figure 4

Case (iii): n is even and take n = 2k, $k \ge 3$

Define $f(e_i) = \begin{cases} -2 & if \ i = k - 1 \\ -1 & if \ i = k \\ 3 & if \ i = k + 1 \end{cases}$

$$f(e_i) = \begin{cases} 2k + 1 - 2i & if \ 1 \le i \le k - 2 \\ 2k - 1 - 2i & if \ k + 2 \le i \le 2k - 1 \end{cases}$$

for $1 \le i \le \frac{m}{2}$ $f(e_i^{'}) = (2k + 2i - 1)$, $f\left(e_{\frac{m}{2}+i}^{'}\right) = -(2k + 2i - 1)$, $f(e_i^{''}) = (2k + m - 1 + 2i)$ and $f\left(e_{\frac{m}{2}+i}^{''}\right) = -(2k + m - 1 + 2i)$. The induced vertex labeling are as follows $f^*(v_1) = f(e_1) +$

$f(e_i') + f\left(e'_{\frac{m}{2}+i}\right) = (2k-1)$, $f^*(v_n) = f(e_{2k-1}) + f(e_i'') + f\left(e''_{\frac{m}{2}+i}\right) = -(2k-1)$, for $2 \le i \le k-2$ $f^*(v_i) = f(e_{i-1}) + f(e_i) = 4(k+1-i)$, $f^*(v_{k-1}) = 3, f^*(v_k) = -3,$ $f^*(v_{k+1}) = 2, f^*(v_{k+2}) = -2,$ for $k+3 \le i \le 2k-1$ $f^*(v_i) = f(e_{i-1}) + f(e_i) = 4(k-i)$, for $1 \le i \le \frac{m}{2}$ $f^*(u_i) = f(e_i') + f(e_i'') = (4k+m-2+4i)$ and $f^*\left(u_{\frac{m}{2}+i}\right) = f\left(e'_{\frac{m}{2}+i}\right) + f\left(e''_{\frac{m}{2}+i}\right) = -(4k+m-2+4i)$. From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm 2, \pm 3, \pm 12, \pm 16, \dots, \pm 4(k-1), \pm(2k-1), \pm(4k+m+2), \pm(4k+m+6), \dots, \pm(4k+3m-2)\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$.

Case (iv): n is odd and take n = 2k+1, $k \ge 2$

Define $f(e_i) = \begin{cases} 1 & if\ i = k+1 \\ 2 & if\ i = k \\ -5 & if\ i = k-1 \end{cases}$

$$f(e_i) = \begin{cases} 5 & if\ i = k+2 \\ -(2k+3-2i) & if\ 1 \le i \le k-2 \\ -2k+1+2i & if\ k+3 \le i \le 2k \end{cases}$$

$f(e_1') = 4, f(e_2') = -4, f(e_1'') = 6, f(e_2'') = -6$, for $1 \le i \le \frac{m-2}{2}$ $f(e_{2+i}') = (2k+2i+1), f\left(e'_{\frac{m+2}{2}+i}\right) = -(2k+2i+1)$, $f(e_{2+i}'') = (2k+m-1+2i)$ and $f\left(e''_{\frac{m+2}{2}+i}\right) = -(2k+m-1+2i)$. The induced vertex labeling are as follows $f^*(v_1) = f(e_1) + f(e_i') + f\left(e'_{\frac{m}{2}+i}\right) = -(2k+1)$, $f^*(v_n) = f(e_{2k}) + f(e_i'') + f\left(e''_{\frac{m}{2}+i}\right) = (2k+1)$, for $2 \le i \le k-1$ $f^*(v_i) = f(e_{i-1}) + f(e_i) = 4(-k-2+i)$, $f^*(v_k) = -3, f^*(v_{k+1}) = 3,$ $f^*(v_{k+2}) = 6$, for $k+3 \le i \le 2k$ $f^*(v_i) = f(e_{i-1}) + f(e_i) = -4(k-i)$, $f^*(u_1) = f(e_1') + f(e_1'') = 10$, $f^*(u_2) = f(e_2') + f(e_2'') = -10$, for $1 \le i \le \frac{m-2}{2}$ $f^*(u_{2+i}) = f(e_i') + f(e_i'') = (4k+m+4i)$ and $f^*\left(u_{\frac{m+2}{2}+i}\right) = f\left(e'_{\frac{m}{2}+i}\right) + f\left(e''_{\frac{m}{2}+i}\right) = -(4k+m+4i)$.

From the above arguments we get $f^*(V(P_n(+)N_m)) = \{\pm 3, \pm 12, \pm 16, \dots, \pm 4k, \pm(2k+1), \pm(4k+m+4), \pm(4k+m+8), \dots, \pm(4k+3m-4) \cup \{6\}\}$. Hence f is an edge pair sum labeling of $P_n(+)N_m$. Figure 5 shows that $P_n(+)N_m$ is an edge pair sum graph for m = 2 and n= 6.

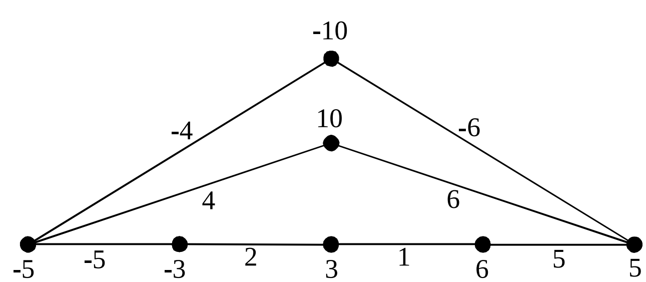

Figure 5

III. LADDER GRAPH:

Let $L_n = P_n \times P_2$ be a ladder with $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le (n-1)\} \cup \{u_i v_i : 1 \le i \le n\}$.

Theorem 3.1: The graph L_n is an edge pair sum graph if n is even.

Proof: let $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and $E(L_n) = \{e_i = u_i u_{i+1} : 1 \le i \le (n-1), e_i' = u_i v_i : 1 \le i \le n \text{ and } e_i'' = v_i v_{i+1} : 1 \le i \le (n-1)\}$ are the vertices and edges of the graph L_n. Define the edge labeling $f: E(L_n) = \{\pm 1, \pm 2, \ldots, \pm(3n-2)\}$ by considering the following two cases.

Case (i): n = 2

Define $f(e_1) = -2$, $f(e_1') = 1, f(e_2') = -1$, $f(e_1'') = 2$. The induced vertex labeling $f^*(u_1) = f(e_1') + f(e_1) = -1, f^*(u_2) = f(e_1) + f(e_2') = -3$, $f^*(v_1) = f(e_1') + f(e_1'') = 3$ and $f^*(v_2) = f(e_1'') + f(e_2') = 1$. From the above arguments we get $f^*(V(L_n)) = \{\pm 1, \pm 3\}$. Hence f is an edge pair sum labeling of L_n.

Case (ii) n = 2k, k ≥ 2

for $1 \le i \le \frac{n}{2} - 1$ $f(e_i) = 2\left(\frac{n}{2} - i\right) + 1$, $f\left(e_{\frac{n}{2}}\right) = -2$, for $1 \le i \le \frac{n}{2} - 1$ $f\left(e_{\frac{n}{2}+i}\right) = -6i + 2$, for $1 \le i \le \frac{n}{2} - 1$ $f(e_i') = 6(\frac{n}{2} - i)$, $f\left(e_{\frac{n}{2}}'\right) = 1$, $f\left(e_{\frac{n}{2}+1}'\right) = -1$, for $1 \le i \le \frac{n}{2} - 1$ $f\left(e_{\frac{n}{2}+1+i}'\right) = -6i$ and $f(e_i'') = 3(n - 2i) - 2$, $f\left(e_{\frac{n}{2}}''\right) = 2$, for $1 \le i \le \frac{n}{2} - 1$ $f\left(e_{\frac{n}{2}+i}''\right) = -(2i + 1)$. The induced vertex labeling are as follows

$f^*(u_1) = f(e_1) + f(e_1') = (4n - 7)$, for $1 \le i \le \frac{n}{2} - 2$ $f^*(u_{i+1}) = f(e_i) + f(e_{i+1}) + f(e_{i+1}') = (5n - 10i - 6)$, $f^*\left(u_{\frac{n}{2}}\right) = f\left(e_{\frac{n}{2}-1}\right) + f\left(e_{\frac{n}{2}}'\right) + f\left(e_{\frac{n}{2}}\right) = 2$, $f^*\left(u_{\frac{n}{2}+1}\right) = f\left(e_{\frac{n}{2}+1}\right) + f\left(e_{\frac{n}{2}+1}'\right) + f\left(e_{\frac{n}{2}}\right) = -7$, for $1 \le i \le \frac{n}{2} - 2$ $f^*\left(u_{\frac{n}{2}+1+i}\right) = f\left(e_{\frac{n}{2}+i}\right) + f\left(e_{\frac{n}{2}+1+i}'\right) + f\left(e_{\frac{n}{2}+1+i}\right) = -(18i + 2)$, $f^*(u_n) = f(e_{n-1}) + f(e_n') = -(6n - 14)$, $f^*(v_1) = f(e_1'') + f(e_1') = (6n - 14)$, for $1 \le i \le \frac{n}{2} - 2$ $f^*(v_{1+i}) = f(e_i'') + f(e_{i+1}'') + f(e_{i+1}') = (9n - 18i - 16)$, $f^*\left(v_{\frac{n}{2}}\right) = f\left(e_{\frac{n}{2}}'\right) + f\left(e_{\frac{n}{2}-1}''\right) + f\left(e_{\frac{n}{2}}''\right) = 7$, $f^*\left(v_{\frac{n}{2}+1}\right) = f\left(e_{\frac{n}{2}+1}'\right) + f\left(e_{\frac{n}{2}+1}''\right) + f\left(e_{\frac{n}{2}}''\right) = -2$, for $1 \le i \le \frac{n}{2} - 2$ $f^*\left(v_{\frac{n}{2}+1+i}\right) = f\left(e_{\frac{n}{2}+1+i}'\right) + f\left(e_{\frac{n}{2}+i}''\right) + f\left(e_{\frac{n}{2}+1+i}''\right) = -(10i + 4)$, $f^*(v_n) = f(e_n') + f(e_{n-1}'') = -(4n - 7)$. From the above labeling we get $f^*(V(L_n)) = \{\pm 2, \pm 7 \pm (4n - 7), \pm(6n - 14), \pm 14, \pm 24, \pm 34, \ldots, \pm(5n - 16), \pm 20, \pm 38, \pm 56, \ldots \pm (9n - 34)\}$. Hence f is an edge pair sum labeling for $L_n$if n is even. The example for the edge pair sum graph labeling of L_n for n = 4 is shown in Figure 6.

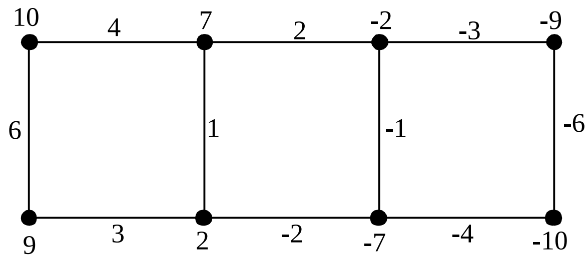

Figure 6

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