Some Results on E - Cordial Graphs

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Abstract

Let $G(V, E)$ be a finite simple graph. Let φ be a function from the edge set E to $\{0, 1\}$. For each vertex $v \in V$, define $\varphi(v) = \sum\{\varphi(uv): uv \in E\}(\text{mod } 2)$. The function φ is called an $E-$ cordial labeling of G, if the number of edges labeled 0 and the number of edges labeled 1 differs by at most one and the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most one. A graph that admits $E-$ cordial labeling is said to E - Cordial. In this paper, we prove that Odd Snakes and $C_m^{(t)}$, that is one vertex union of t copies of C_m, for t –even and m – odd are E - Cordial.

Keywords— **Snakes, One vertex union of cycles, E - Cordial labeling, E - Cordial graphs.**

1. Introduction

Let $G(V, E)$ be any finite simple graph. For any graph-theoretic notation we follow West [1].A vertex labeling or valuation of a graph G is an assignment f of labels to the vertices of G, that induces for each edge $e = xy$, a label depending on the vertex labels $f(x)$ and $f(y)$. Yilmaz and Cahit [4] introduced edge-cordial labeling as a weaker version of edge graceful labeling. Let φ be a function from the edge set $E$to $\{0,1\}$. For each vertex $v \in V$, define $\varphi(v) = \sum\{\varphi(uv) : uv \in E\}(mod\ 2)$. The function φ is called an E–cordial labeling of G, if the number of edges labeled 0 and number of edges labeled 1 differ by at most one and the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one. A graph that admits an E –cordial labeling is said to be E – Cordial graph. It is observed that graphs with $n \equiv 2\ (mod\ 4)$ vertices cannot be E - Cordial. For more related results on E - Cordial graphs one can refer Gallian [2].

2. Odd Snakes are E - Cordial.

Recall that snakes are the graphs whose block cut point graph is a path and further odd snake is a snake whose blocks are isomorphic to a cycle C_m of length $m - odd(= 2n + 1)$. Refer Figure. 1. In this section we prove that odd snake is E - Cordial.

2.1. Theorem. $Odd\ Snake\ is\ edge\ Cordial.$

Proof:

Let$G$ be an odd snakeas given in figure 1. Then by definition, G contains r blocks $B_i, 1 \leq i \leq r$ and each block B_i of G is isomorphic to a cycle of length $m(= 2n + 1)$, that is, $C_{2n+1} :\ v_{i,1}e_{i,2}v_{i,2}e_{i,3} \dots\ v_{i,2n}e_{i,2n+1}v_{i,2n+1}$.

For convenience of the labeling, for $1 \leq i \leq r - 1$, let $v_{i,2n+1} = v_{i+1,1}$.Thus from the above, we observe that the blocks B_i and B_{i+1} have a common cut vertex $v_{i,1}$for $2 \leq i \leq r$.

Arrange the edges of all the blocks B_i of G in the following order $\{e_{i,1}, e_{i,2}, e_{i,3}, \dots \dots\ e_{i,2n+1}\}$ as shown in figure. 1.

Let $E_0$and E_1denote the set of all edges assigned the label 0 and 1 respectively. In the same way, let V_0 and V_1denote the set of all vertices assigned the label 0 and 1 respectively.

Define the labeling $\varphi : E(G) \rightarrow \{0,1\}$corresponding to the edges of G in following cases,

Case : 1. If $m = 3$, then for $1 \le i \le r$ and $1 \le j \le 3$, define $\varphi(e_{i,j}) = \begin{cases} 110, & if\ r\ is\ odd \\ 100, & if\ r\ is\ even \end{cases}$

The induced vertex labeling is as follows,

$$v_{1,1} = 0, v_{1,2} = 1, v_{r,3} = 1.$$

For $2 \le i \le r, i -$ even, $v_{i,1} = 0, v_{i,2} = 0.$

For $3 \le i \le r, i -$ odd, $v_{i,1} = 1, v_{i,2} = 1.$

Hence, if $m = 3$, it is observed that, if the number of blocks r is odd, then $|E_1| = |E_0| + 1$ and $|V_1| = |V_0| + 1$ and if r is even, then $|E_1| = |E_0|$ and $|V_0| = |V_1| + 1$

Case : 2. If $m = 4k + 1$, then for $1 \le i \le r$ and $1 \le j \le 2n + 1$,

$$\text{Define } \varphi(e_{i,j}) = \begin{cases} (1100)^k 1, & if\ r\ is\ odd \\ (0011)^k 0, & if\ r\ is\ even \end{cases}$$

The induced vertex labeling is as follows,

$$v_{r,2n+1} = 0.$$

$$\text{For } 1 \le i \le r, v_{i,j} = \begin{cases} 0, if\ 1 \le j \le 2n, j - odd \\ 1, if\ 2 \le j \le 2n, j - even \end{cases}.$$

Hence if $m = 4k + 1$, it is observed that, if the number of blocks r is odd then, $|E_1| = |E_0| + 1$ and $|V_0| = |V_1| + 1$ and if r is even, then $|E_1| = |E_0|$and $|V_0| = |V_1| + 1$.

Case : 3. If $m = 4k + 3$, then for $1 \le i \le r$ and $1 \le j \le 2n + 1$,

$$\text{Define } \varphi(e_{i,j}) = \begin{cases} (1100)^k 110, & if\ r\ is\ odd \\ 100(0011)^k, & if\ r\ is\ even \end{cases}$$

The induced vertex labeling is as follows,

$$v_{1,1} = 0, \ v_{r,2n+1} = 0$$

$$\text{and } v_{i,1} = \begin{cases} 1, if\ 3 \le i \le r - 1, i - odd \\ 0, if\ 2 \le i \le r - 1, i - even \end{cases}$$

$$\text{For } 1 \le i \le r, \text{ and } i \text{ is odd}, v_{i,j} = \begin{cases} 1, if\ 2 \le j \le 2n, j - even \\ 0, if\ 3 \le j \le 2n, j - odd \end{cases}$$

$$\text{For } 2 \le i \le r, \text{ and } i \text{ is even}, v_{i,j} = \begin{cases} 0, if\ 2 \le j \le 2n, j - even \\ 1, if\ 3 \le j \le 2n, j - odd \end{cases}$$

If $m = 4k + 3$, it is observed that, if the number of blocks r is odd, then$|E_1| = |E_0| + 1$ and $|V_1| = |V_0| + 1$ and if r is even, then $|E_1| = |E_0|$ and $|V_0| = |V_1| + 1$.

Hence it is clear that G is E - Cordial.■

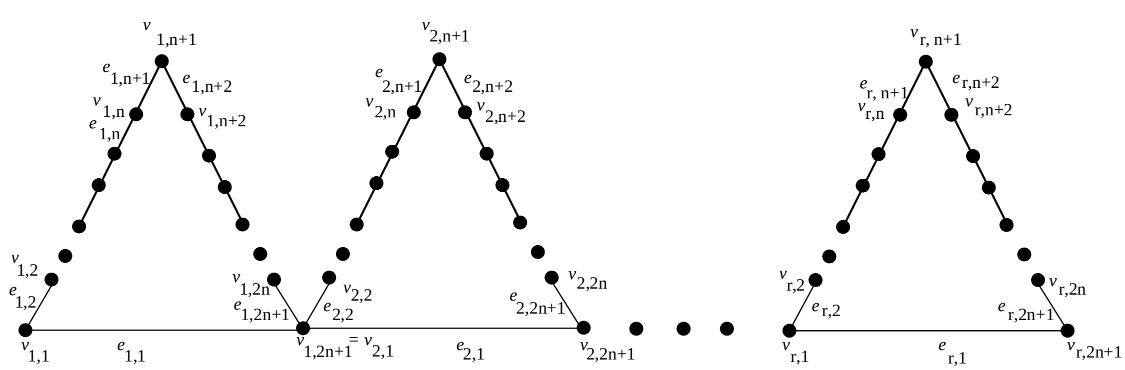

Figure. 1. Odd Snake

2.2. Illustrations to Theorem. 2.1.

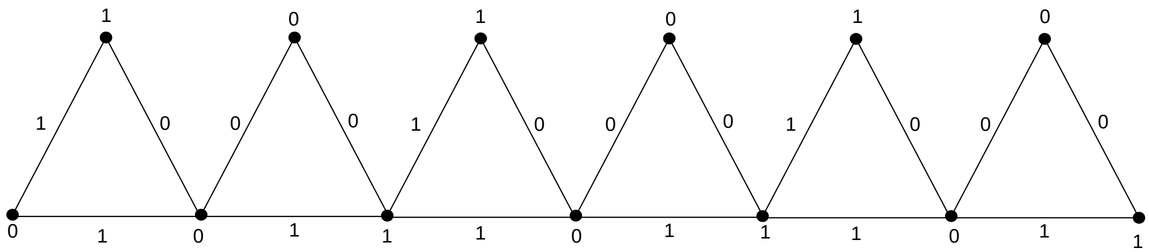

Fig.2. E – Cordial labeling of $C_3 - snake$

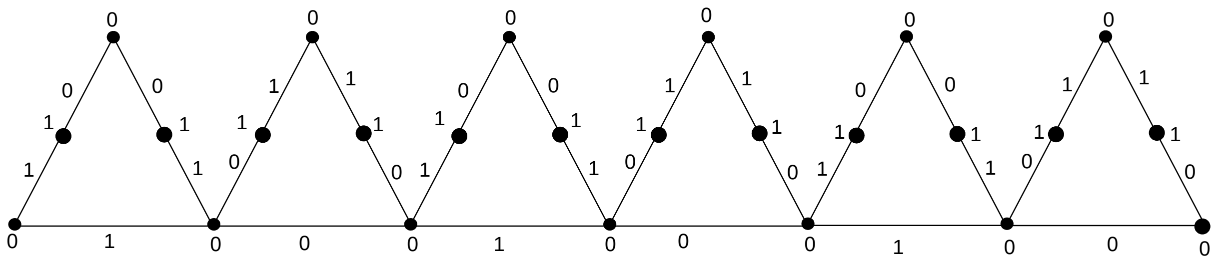

Fig.3. E – Cordial labeling of $C_5 - snake$

3. $C_m^{(t)}$, ONE VERTEX UNION OF EVEN NUMBER OF ODD CYCLES ARE E - CORDIAL.

Recall that a graph G is said to be one vertex union of graphs $G_1, G_2, G_3, \dots, G_t$ if G is obtained by adjoining $G_1, G_2, G_3, \dots, G_t$ at a common vertex v, that is, $G = \cup G_i$, for $i = 1,2,3,\dots,t$. In this section it is proved that $C_m^{(t)}$, the one vertex union of t isomorphiccopies of cycles C_m , for t –even and m – odd are E – Cordial.

3.1. Theorem. $C_m^{(t)}$, one vertex union of t isomorphic copies of cycles C_m , for t –even and m – odd is E - Cordial.

Proof:

Let G be the one vertex union of $G_i, 1 \le i \le t, t - even$, where $G_i \cong C_{m(=2n+1)}$. For $1 \le i \le t$, order the copies G_i as $G_1, G_2, \dots, G_t$.

For $1 \le i \le t$, name the vertices of G_i as follows, $\{ v_{i,1}, v_{i,2}, \dots, v_{i,2n}, v_{i,2n+1} (= v)\}$, where v is a common vertex.

For $1 \le i \le t$, and name the edges of G_i as follows,
$e_{i,1} = v\, v_{i,2}$; $e_{i,2n+1} = v_{i,2n} v$ and $e_{i,j} = v_{i,j}\, v_{i,j+1}$, for $2 \le j \le 2n$

From the above it is observed that for $1 \le i \le t$, the common vertex v is incident with the edges $e_{i,1}$, $e_{i,2n+1}$ and $\langle e_{i,1}, e_{i,2}, e_{i,3}, \dots, e_{i,2n}, e_{i,2n+1} \rangle$ induces the cycle C_{2n+1} : $v\, e_{i,1}\, v_{i,2}\, e_{i,2}\, v_{i,3}\, e_{i,3} \dots\dots v_{i,2n}\, e_{i,2n+1} v$. Refer Figure. 4.

For the convenience of the labeling arrange the edges $e_{i,j}$, for $1 \le i \le t, 1 \le j \le 2n + 1$ of G_i as a sequence in the form $e_{i,1}, e_{i,2}, e_{i,3}, \dots, e_{i,2n+1}$.

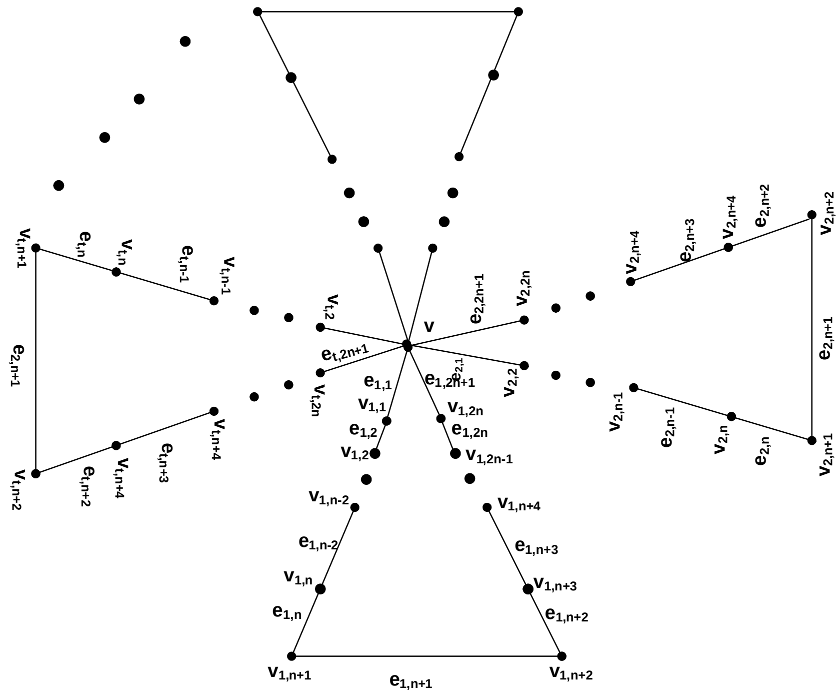

Fig.4. $G = \cup G_i$, for $i = 1,2,3,\ldots,t$

Let E_0 and E_1 denote the set of all edges assigned the label 0 and 1 respectively. In the same way, let V_0 and V_1 denote the set of all vertices assigned the label 0 and 1 respectively.

Define the labeling $\varphi : E(G) \rightarrow \{0,1\}$ corresponding to the edges of G in following cases,

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The induced vertex labeling is as follows,

For $1 \le i \le t, i -$ odd, $v_{i,1} = 0, v_{i,2} = 1$.

For $3 \le i \le t, i -$ even, $v_{i,1} = 1, v_{i,2} = 0$.

Hence, if $m = 3$, it is observed that, $|E_1| = |E_0|$ and $|V_0| = |V_1| + 1$.

Case : 2. If $m = 4k + 1$, then for $1 \le i \le t$ and $1 \le j \le 2n + 1$,

Define $\varphi(e_{i,j}) = \begin{cases} (1100)^k 1, & if\ t\ is\ odd \\ (0011)^k 0, & if\ t\ is\ even \end{cases}$

The induced vertex labeling is as follows,

For $1 \le i \le t, v_{i,j} = \begin{cases} 0, if\ 1 \le j \le 2n\ , j - odd \\ 1, if\ 2 \le j \le 2n\ , j - even \end{cases}$.

Hence if $m = 4k + 1$, it is observed that $|E_1| = |E_0|$ and $|V_0| = |V_1| + 1$.

Case : 3. If $m = 4k + 3$, then for $1 \le i \le t$ and $1 \le j \le 2n + 1$,

Define $\varphi(e_{i,j}) = \begin{cases} (1100)^k 110, & if\ t\ is\ odd \\ 100(0011)^k, & if\ t\ is\ even \end{cases}$

The induced vertex labeling is as follows, For $1 \leq i \leq t$, and i is odd, $v_{i,j} = \begin{cases} 1, if \ \ j - even \\ 0, if \ \ j - odd \end{cases}$

For $2 \leq i \leq t$, and i is even, $v_{i,j} = \begin{cases} 0, if \ \ j - even \\ 1, if \ \ j - odd \end{cases}$

If $m = 4k + 3$, it is observed that, $|E_1| = |E_0|$ and $|V_0| = |V_1| + 1$ and hence G is E - Cordial.■

4. Conclusions

Yilmaz and Cahit have discussed E – Cordial labeling of $C_m^{(3)}$ for all values n, while in this paper E – Cordial labeling of $C_m^{(t)}$, for t –even and m – odd have been investigated.

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