Transient Behavior of $M^{[X]}/G/1$ Retrial Queueing Model with Non Persistent Customers, Random Break Down, Bernoulli Vacation and Orbital Search

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***Abstract* - We investigate a single server batch arrival retrial queueing system with Bernoulli vacation and orbital search. Batches are arriving in accordance with Poisson process with arrival rate λ and are served one by one with first come first served basis. At the arrival epoch, if the customer finds the server busy, breakdown or vacation can either join the orbit with probability p or he/she can leave the system with probability 1-p, such customers are called non-persistent. As the server is considered as unreliable one, it may encounter break down at any time. For to resume its service it has to go for a immediate repair process. At the completion epoch of a service, the server may go for a vacation with probability θ or stay back in the system to serve a next customer with probability $1-\theta$, if any. We obtain the transient solution and steady solution of the model by using supplementary variable technique. Also we derive the system performance measures and reliability indices.**

Keywords— **Batch size, Bernoulli, Orbit Search, Reliability Indices, Steady Solution, Transient Solution.**

I. Introduction

Due to wide applications in telephone switching systems, telecommunication and computer networks, the study of retrial queues has become popular in the research of queueing models. Single server retrial queues with batch arrivals was introduced by Falin (1976) by considering a policy that "if the server is busy at the arrival epoch, then the whole batch joins the retrial group, whereas if the server is free, then one of the arriving units starts its service and the rest join the retrial group". Yang and Templeton (1987), Fallin (1990) and Kulkarni (1997). Artalejo (1999) and Gomez (2006) presented excellent surveys on retrial queues. Artalejo and Falin (2002) studied a comparative analysis between standard and retrial queues by considering the similarities and differences between them.

There have been significant contribution in the research study of retrial queues with vacations. Artalejo(1997, 1999) discussed retrial queues with exhaustive vacation. Choi et al.(1990, 1993) studied M/G/1 retrial queue with vacation. Krishna kumar et al.(2002) studied retrial queues with feed back and starting failures. Zhou (2005) discussed the same model with FCFS orbit policy. Retrial queues with unreliable server and repair have also been paid more attention by numerous authors. Aissani(1993,1994) and Kulkarni(1990), Djellab(2002), Peishu Chen(2010) and Zhou(2009) studied retrial queueing system for an unreliable server. Artalejo (1994) found new results for retrial queueing systems with break downs. Djellab (2006) reviewed the stochastic decomposition for the number of customers in M/G/1 retrial queues with reliable server and server subjected to breakdowns. Wang et al.(2001) incorporated reliability analysis on retrial queue with server breakdowns and repairs. The Choudhury (2012) extended his analysis by including delaying time before the repair of the server for batch arrivals. Ke (2009) studied the M/G/1 retrial queue with balking and feedback. Also the same author analyzed $M^{[x]}/(G1,G2)/1$ retrial queue under Bernoulli vacation schedules and starting failures. Jinting Wang et al.(2008) discussed the transient analysis of M/G/1 retrial queue subject to disasters and server failures. The same author (2008) obtained steady state solution of the queue model with two-Phase Service.

Many authors concentrated retrial queue models with all aspects for non- persistent customers. In this context Arumuganathan (2008, 2009), Kasthuri Ramnath(2010) presented papers with different vacation policies. In the context of retrial queueing model, each service is preceded and followed by the server's idle time because of the ignorance of the status of the server and orbital customers by each other. The introduction of search of orbital customers immediately after a service completion helps to reduce the server's idle time. Artalejo et al.(2002) studied orbital search by the server after a service is completed. Dudin et al.(2004) analyzed BMAP/G/1 retrial system with search of customers from the orbit. Chakravarthy et al.(2006) discussed multi server retrial queue with orbital search. Krishnamoorthy et al (2005) studied retrial queue with non-persistent customer and orbital search.

In this paper we consider a single server queueing system in which primary customers arrive according to compound Poisson stream with rate λ. Upon arrival, customer finds the sever busy or down or on vacation the customer may leave the service area as there is no place in front of the server, he/she may join the pool of customers called orbit with probability p or leave the system with probability 1-p. otherwise the server can get service immediately if the server is in idle state Also the server can opt for Bernoulli vacation. The rest of the paper is organized as follows: Section 2, provides a description of the mathematical model. Section 3 deals with transient analysis of the model for which probability generating function of the distribution has been obtained. In section 4 steady state solution has been obtained with somr basic performance measures and reliability indices of this model are derived in Section 5. In section 6, conclusion for the model has been given.

II MATHEMATICAL DESCRIPTION OF THE MODEL

We consider an retrial queue with random break downs and Bernoulli schduled vacation. Customers arrive in batches in batches of variable size in a compound Poisson process. Let $\lambda c_i \Delta t$ (i=1,2,3,....) be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t+\Delta t)$, where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches and the customers are served one-by-one on a "first come-first served" basis. While arriving, if a customer finds the server idle, the customer gets service immediately. Otherwise, the server is found in astate of busy or down or on vacation, the customer is obliged to join a retrial orbit according to an FCFS discipline with probability p or leaves the system with probability 1- p.

The service times of the customers are identically independent random variables with probability distribution function B(x), density function b(x), k^{th} moment $\mu_k (k = 1,2)$. When the server is serving the customers, it may getr break down at any time so that the server will be down for a short duration of time. The server's life times are generated by exogenous Poisson process with rate α. As soon as the server gets break down it is sent for repair so that the server stops providing service to the customers. The repair times $Q_n; n \ge 0$ of the server are identically independent random variables with distribution function Q(y)and k^{th} finite moment $r_k; k \ge 1$. After the repair process is over the server is ready to resume its remaining service to the customers and in this case the service times are cumulative, which we may referred to as generalized service times. After a service completion the server may go for a vacation of random length V with probability θ or with probability $1-\theta$ he may serve the next unit; if any. The vacation completion times $V_n; n \ge 0$ of the server are identically independent random variables with distribution function V(x)and k^{th} finite moment $v_k; k \ge 1$. After completing vacation the server searches for the customers in the orbit (if any) with probability q or remains idle with its complimentary probability 1-q. The search time is assumed to be negligible. All stochastic processes involved in the system are assumed to be independent of each other.

Now we obtain the probability generating function of the joint distribution of the state of the server and the number in the system by treating $I^0(t), B^0(t)$ are the elapsed retial time and service time of the customers at time t respectively also $Q^0(t)$ and $V^0(t)$ are the elapsed elapsed repair time and elapsed vacation time at time t, respectively as supplementary variables. Assuming that the system is empty initially. Let N(t) be the number of customers in the retrial queue at time t, and C(t) be the number of customer in service at time t. In order to have Markov process, we define the state probabilities at time t as follows:

Y(t)=0, if the server is idle at time t,
1, if the server is idle during retrial time at time t,
2, if the server is busy at time t,
3, if the server is on vacation at time t,
4, if the server is under repair at time t.

Introducing the supplementary $I^0(t), B^0(t), Q^0(t)$ and $V^0(t)$ to obtain a bivariate Markov process

$Z(t) = N(t), X(t)$,

where $X(t) = 0$ if Y(t)=0,

$X(t) = I^0(t)$ if Y(t)=1,

$X(t) = B^0(t)$ if Y(t)=2,

$X(t) = V^0(t)$ if Y(t)=3

$X(t) = Q^0(t)$ if Y(t)=4,

Defining the limiting probabilities as follows:

$$I_0(t) = P(N(t) = 0, X(t) = 0);$$

$$I_n(x,t)dx = P(N(t) = n, X(t) = I^0(t); x < I^0(t) \le x + dx); x,t > 0, n \ge 1;$$

$$P_n(x,t)dx = P(N(t) = n, X(t) = P^0(t); x < P_1^0(t) \le x + dx); x,t > 0, n \ge 0,$$

$$V_n(x)dx = P(N(t) = n, X(t) = V^0(t); x < V^0(t) \le x + dx); x,t > 0, n \ge 0,$$

and for fixed values of x and $n \ge 1$

$$Q_n(x,y,t)dy = P(N(t) = n, X(t) = R^0(t); y < R^0(t) \le y + dy); x,y,t > 0.$$

Further it is assumed that I(x),B(x)and V(x) are continuous at x=0 and R(y)are continuous at y=0 respectively, such that

$$\eta(x)dx = \frac{dI(x)}{1 - I(x)}; \mu(x)dx = \frac{dB(x)}{1 - B(x)};$$

$$\nu(x)dx = \frac{dV(x)}{1 - V(x)}; \beta(y)dy = \frac{dQ(y)}{1 - Q(Y)}$$

are the first order differential (hazard rate) functions of I(), B(),V(), and Q()respectively.

III THE TRANSIENT STATE EQUATIONS

we derive the following system of equations that govern the dynamics of the system behavior:

$$\frac{d}{dt} I_0(t) = -\lambda I_0(t) + (1-\theta)\int_0^\infty P_0(x,t)\mu(x)dx + (1-q)\int_0^\infty V_0(x,t)\nu(x)dx \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda + \eta(x)\right) I_n(x,t) = 0; n \ge 0 \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \alpha + \mu(x)\right) P_n(x,t) = \lambda p \Sigma_{i=1}^n c_i P_{n-i}(x,t) + \int_0^\infty Q_n(x,y,t)\beta(y)dy; n \ge 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \beta(y)\right) Q_n(x,y,t) = \lambda p \Sigma_{i=1}^n c_i Q_{n-i}(x,y,t); n \ge 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + p\lambda + \nu(x)\right) V_n(x,t) = \lambda p \Sigma_{i=1}^n c_i V_{n-i}(x,t); n \ge 0 \tag{5}$$

with boundary conditions

$$I_n(0,t) = (1-\theta)\int_0^\infty P_n(x,t)\mu(x)dx + (1-q)\int_0^\infty V_n(x,t)\nu(x)dx \tag{6}$$

$$P_0(0,t) = \int_0^\infty I_1(x,t)\eta(x)dx + q\int_0^\infty V_1(x,t)\nu(x)dx + \lambda c_1 I_0(t) \tag{7}$$

$$P_n(0,t)=\int_0^\infty I_{n+1}(x,t)\eta(x)dx+\lambda\int_0^\infty \Sigma_{i=1}^n c_i I_{n+1-i}(x,t)dx+q\int_0^\infty V_{n+1}(x,t)\nu(x)dx+\lambda c_{n+1}I_n(t);n\geq 1 \tag{8}$$

$$Q_n(x,0,t)=\alpha P_n(x,t);n\geq 0 \tag{9}$$

$$V_n(0,t)=\theta\int_0^\infty P_n(x,t)\mu(x)dx \tag{10}$$

Now we define the probability generating function

$$I_q(x,z,t)=\sum_{n=1}^{\infty}z^n I_n(x,t);I_q(z,t)=\sum_{n=1}^{\infty}z^n I_n(t); \tag{11}$$

$$P_q(x,z,t)=\sum_{n=0}^{\infty}z^n P_n(x,t);P_q(z,t)=\sum_{n=1}^{\infty}z^n P_n(t); \tag{12}$$

$$V_q(x,z,t)=\sum_{n=0}^{\infty}z^n V_n(x,t);V_q(z,t)=\sum_{n=0}^{\infty}z^n V_n(t) \tag{13}$$

$$Q_q(x,y,z,t)=\sum_{n=0}^{\infty}z^n Q_n(x,y,t);Q_q(x,z,t)=\sum_{n=0}^{\infty}z^n Q_n(x,t); \tag{14}$$

$$C(z)=\sum_{n=1}^{\infty}c_n z^n \tag{15}$$

which are convergent inside the circle given by $|z|\leq 1$ and define the Laplace transform of a function f(t) as

$$\bar{f}(s)=\int_0^\infty f(t)e^{-st}dt \tag{16}$$

Taking Laplace transform for equations (18) -(26)

$$(s+\lambda)\bar{I}_0(s)=1+(1-\theta)\int_0^\infty \overline{P_0}(x,s)\mu(x)dx+(1-q)\int_0^\infty \overline{V_0}(x,s)\nu(x)dx \tag{17}$$

$$\left(\frac{d}{dx}+s+\lambda+\eta(x)\right)\bar{I}_n(x,s)=0 \tag{18}$$

$$\left(\frac{d}{dx}+s+\lambda p+\alpha+\mu(x)\right)\overline{P_n}(x,s)=\lambda p\Sigma_{i=1}^n c_i\overline{P}_{n-i}(x,s)+\int_0^\infty \overline{Q_n}(x,y,s)\beta(y)dy;n\geq 0 \tag{19}$$

$$\left(\frac{d}{dx}+s+\lambda p+\beta(y)\right)\overline{Q_n}(x,y,s)=\lambda p\Sigma_{i=1}^n c_i\overline{Q}_{n-i}(x,y,s);n\geq 0 \tag{20}$$

$$\left(\frac{d}{dx}+s+\lambda p+\nu(x)\right)\overline{V_n}(x,s)=\lambda p\Sigma_{i=1}^n c_i\overline{V}_{n-i}(x,s);n\geq 0 \tag{21}$$

$$\bar{I}_n(0,s)=(1-\theta)\int_0^\infty \overline{P_n}(x,s)\mu(x)dx+(1-q)\int_0^\infty \overline{V_n}(x,s)\nu(x)dx \tag{22}$$

$$\overline{P}_0(0,s)=\int_0^\infty \bar{I}_1(x,s)\eta(x)dx+q\int_0^\infty \overline{V}_1(x,s)\nu(x)dx+\lambda c_1\bar{I}_0(s) \tag{23}$$

$$\overline{P}_n(0,s)=\int_0^\infty \bar{I}_{n+1}(x,s)\eta(x)dx+\lambda\int_0^\infty \Sigma_{i=1}^n c_i\bar{I}_{n+1-i}(x,s)dx$$
$$+q\int_0^\infty \overline{V}_{n+1}(x,s)\nu(x)dx+\lambda c_{n+1}\bar{I}_n(s);n\ge 1 \tag{24}$$

$$\overline{Q}_n(x,0,s)=\alpha\overline{P}_n(x,s);n\ge 0 \tag{25}$$

$$\overline{V}_n(0,s)=\theta\int_0^\infty \overline{P}_n(x,s)\mu(x)dx \tag{26}$$

Applying probability generating function for the equations(18)-(26)

$$\left(\frac{d}{dx}+s+\lambda+\eta(x)\right)\bar{I}_q(x,z,s)=0 \tag{27}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\alpha+\mu(x)\right)\overline{P}_q(x,z,s)=\int_0^\infty \overline{Q}_q(x,y,z,s)\beta(y)dy \tag{28}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\beta(y)\right)\overline{Q}_q(x,y,z,s)=0 \tag{29}$$

$$\left(\frac{d}{dx}+s+\lambda p(1-C(z))+\nu(x)\right)\overline{V}_q(x,z,s)=0 \tag{30}$$

$$\bar{I}_q(0,z,s)=(1-\theta)\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx+(1-q)\int_0^\infty \overline{V}_q(x,z,s)\nu(x)dx+1-(s+\lambda)\bar{I}_0(s) \tag{31}$$

$$z\overline{P}_q(0,z,s)=\int_0^\infty \bar{I}_q(x,z,s)\eta(x)dx+\lambda C(z)\int_0^\infty \bar{I}_q(x,z,s)dx+q\int_0^\infty \overline{V}_q(x,z,s)\nu(x)dx+\lambda C(z)\bar{I}_0(s) \tag{32}$$

$$\overline{Q}_q(x,0,z,s)=\alpha\overline{P}_q(x,z,s) \tag{33}$$

$$\overline{V}_q(0,z,s)=\theta\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx \tag{34}$$

solving for equations(27)-(30)

$$\bar{I}_q(x,z,s)=\bar{I}_q(0,z,s)e^{-(s+\lambda)x-\int_0^x \eta(t)dt} \tag{35}$$

$$\overline{P}_q(x,z,s)=\overline{P}_q(0,z,s)e^{-\phi(z,s)x-\int_0^x \mu(t)dt} \tag{36}$$

where

$$\phi(z,s)=s+\lambda p(1-C(z))+\alpha[1-\overline{\phi}(\lambda p(1-C(z)))\overline{Q}(\lambda p(1-C(z)))]$$

$$\overline{Q}_q(x,y,z,s)=\overline{Q}_q(x,0,z,s)e^{-(s+\lambda p(1-C(z))x)-\int_0^y \beta(t)dt} \tag{37}$$

$$\overline{V}_q(x,z,s)=\overline{V}_q(0,z,s)e^{-(s+\lambda p(1-C(z))x)-\int_0^x v(t)dt} \quad (38)$$

Integrate equations (35)-(38)by parts with respect to x

$$\bar{I}_q(z,s)=\bar{I}_q(0,z,s)\left[\frac{1-\bar{I}(s+\lambda)}{(s+\lambda)}\right] \quad (39)$$

$$\overline{P}_q(z,s)=\overline{P}_q(0,z,s)\left[\frac{1-\overline{B}(\phi(z,s))}{(\phi(z,s))}\right] \quad (40)$$

$$\overline{Q}_q(x,z,s)=\overline{Q}_q(x,0,z,s)\left[\frac{1-\overline{Q}(s+\lambda p(1-C(z)))}{(s+\lambda p(1-C(z)))}\right] \quad (41)$$

$$\overline{V}_q(z,s)=\overline{V}_q(0,z,s)\left[\frac{1-\overline{V}(s+\lambda p(1-C(z)))}{(s+\lambda p(1-C(z)))}\right] \quad (42)$$

where $\bar{I}(s+\lambda),\overline{B}(\phi(z,s)),\overline{Q}(s+\lambda p(1-C(z)))$ and $\overline{V}(s+\lambda p(1-C(z)))$ are the Laplace-Stieltjes transforms of the retrial time, service time, repair time and vacation completion time of the server respectively.

Multiply equation (35) by $\eta(x)$ and integrate w.r.t x

$$\int_0^\infty \bar{I}_q(x,z,s)\eta(x)dx=\bar{I}_q(0,z,s)\bar{I}(s+\lambda) \quad (43)$$

Multiply equation (36) by $\mu(x)$ and integrate w.r.t x

$$\int_0^\infty \overline{P}_q(x,z,s)\mu(x)dx=\overline{P}_q(0,z,s)\overline{B}(\phi(z,s)) \quad (44)$$

Multiply equation (37) by $\beta(y)$ and integrate w.r.t x

$$\int_0^\infty \overline{Q}_q(x,y,z,s)\beta(y)dy=\overline{Q}_q(x,0,z,s)\overline{Q}(s+\lambda p(1-C(z))) \quad (45)$$

Multiply equation (39) by $v(x)$ and integrate w.r.t x

$$\int_0^\infty \overline{V}_q(x,z,s)v(x)dx=\overline{V}_q(0,z,s)\overline{V}(s+\lambda p(1-C(z))) \quad (46)$$

Now equation (31) becomes

$$\bar{I}_q(0,z,s)=[1-(s+\lambda)\bar{I}_0(s)]+[(1-\theta)+(1-q)\theta\overline{V}(s+\lambda p(1-C(z)))]\overline{B}(\phi(z,s))\overline{P}_q(0,z,s) \quad (47)$$

$$\bar{I}_q(z,s)=\left[[1-(s+\lambda)\bar{I}_0(s)]+[(1-\theta)+(1-q)\theta\overline{V}(s+\lambda p(1-C(z)))]\overline{B}(\phi(z,s))\overline{P}_q(0,z,s)\right]\left[\frac{1-\bar{I}(s+\lambda)}{(s+\lambda)}\right] \quad (48)$$

Now equation(32) becomes

$$\bar{P}_q(0,z,s) = \frac{\lambda C(z)\bar{I}_0(s) + \left[1-(s+\lambda)\bar{I}_0(s)\right]\left[\lambda C(z)\left(\frac{(1-\bar{I}(s+\lambda))}{(s+\lambda)}\right) + \bar{I}(s+\lambda)\right]}{D(z,s)} \quad (49)$$

where

$$D(z,s) = z - [(1-\theta)+(1-q)\theta\bar{V}(s+\lambda p(1-C(z)))]\bar{B}(\phi(z,s))\left[\lambda C(z)\left(\frac{(1-\bar{I}(s+\lambda))}{(s+\lambda)}\right)+\bar{I}(s+\lambda)\right] - q\theta\bar{V}(s+\lambda p(1-C(z)))\bar{B}(\phi(z,s)) \quad (50)$$

substitute the value for $\bar{P}_q(0,z,s)$ we can obtain the probability generating function of various states of the system like $I_q(z,s), P_q(z,s), Q_q(x,z,s), V_q(z,s)$ in the transient state.

IV STEADY STATE DISTRIBUTION

In this section we shall derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress the argument 't' where ever it appears in the time dependent analysis. By using well known Tauberian property as follows:

$$Lt_{s\to 0} s\bar{f}(s) = Lt_{t\to\infty} f(t) \quad (51)$$

$$I_q(z) = \frac{I_0[1-\bar{I}(\lambda)][(C(z)-1)\bar{I}(\lambda)[(1-\theta)+(1-q)\theta\bar{V}(\lambda p(1-C(z)))]\bar{B}(\phi(z)) - D(z)]}{D(z)} \quad (52)$$

$$P_q(z) = \frac{I_0\bar{I}(\lambda)\lambda(C(z)-1)[1-\bar{B}(\phi(z))]}{D(z)\phi(z)} \quad (53)$$

$$Q_q(x,z) = \frac{\alpha I_0\bar{I}(\lambda)[1-\bar{B}(\phi(z))][\bar{Q}(s+\lambda p(1-C(z)))-1]}{pD(z)\phi(z)} \quad (54)$$

$$V_q(z) = \frac{\theta I_0\bar{I}(\lambda)\bar{B}(\phi(z))[\bar{V}(s+\lambda p(1-C(z)))-1]}{pD(z)} \quad (55)$$

$$D(z) = z - [(1-\theta)+(1-q)\theta\bar{V}(\lambda p(1-C(z)))]\bar{B}(\phi(z))\left[C(z)(1-\bar{I}(\lambda))+\bar{I}(\lambda)\right] - q\theta\bar{V}(\lambda p(1-C(z)))\bar{B}(\phi(z)) \quad (56)$$

using the normalization condition I_0 can be obtained

$$I_0 + Lt_{z\to 1}(I_q(z)+P_q(z)+Q_q(z)+V_q(z)) = 1 \quad (57)$$

$$I_0 = \frac{C_{[1]}[q\theta(1-\bar{I}(\lambda))]+[1-C_{[1]}[1-\bar{I}(\lambda)]] - p\rho}{\bar{I}(\lambda)(1+\rho(1-p))} \quad (58)$$

$$\rho = \lambda C_{[1]}[\mu_1(1+\alpha r_1)+\theta v_1] \quad (59)$$

In addition, various system state probabilities also be given from equations (52)-(55) by putting z=1

Prob [the server is idle in non-empty queue]= $I_q(1)$

$$= \frac{C_{[1]}(1-\bar{I}(\lambda))(p\rho - q\theta)}{\bar{I}(\lambda)(1+\rho(1-p))} \tag{60}$$

Prob [the server is busy]= $P_q(1)$

$$= \frac{\lambda C_{[1]}\mu_1}{(1+\rho(1-p))} \tag{61}$$

Prob [the server is on repair]= $Q_q(1)$

$$= \frac{\alpha\lambda C_{[1]}\mu_1 r_1}{(1+\rho(1-p))} \tag{62}$$

Prob [the server is on vacation] = $V_q(1)$

$$= \frac{\theta\lambda C_{[1]} v_1}{(1+\rho(1-p))} \tag{63}$$

The necessary and sufficient condition for stability condition is given by the following

$$p\rho < C_{[1]}[q\theta(1-\bar{I}(\lambda))] + [1 - C_{[1]}[1-\bar{I}(\lambda)]] \tag{64}$$

The expected number of customers in the orbit

$$E[N_0] = \frac{(1-q\theta)(1-\bar{I}(\lambda))p\rho C_{[1]}}{C_{[1]}[q\theta(1-\bar{I}(\lambda))]+[1-C_{[1]}[1-\bar{I}(\lambda)]]-p\rho} + \frac{p[\lambda C_{[1]}]^2[\mu_2(1+\alpha r_1)^2 + \alpha\mu_1 r_2]}{2[C_{[1]}[q\theta(1-\bar{I}(\lambda))]+[1-C_{[1]}[1-\bar{I}(\lambda)]]-p\rho]}$$

$$+ \frac{p\theta[\lambda C_{[1]}]^2[v_2 + 2v_1\mu_1(1+\alpha r_1)]}{2[C_{[1]}[q\theta(1-\bar{I}(\lambda))]+[1-C_{[1]}[1-\bar{I}(\lambda)]]-p\rho]} + \frac{(1-q\theta)C_{[1]}(1-\bar{I}(\lambda))+p\rho}{[1-C_{[1]}(1-\bar{I}(\lambda))-p\rho]} C_{[R]} \tag{65}$$

where $C_{[R]} = \dfrac{C_{[2]}}{2C_{[1]}}$ is the residual batch size.

After finding the expected number of units in the orbit,we can obtain the related performance measures viz mean number of units in the system, mean waiting time in the queue and mean waiting time in the system by using Little's formula

$$E[N_s] = E[N_0] + \rho \tag{66}$$

$$E[W_s] = \frac{E[N_s]}{\rho\lambda C_{[1]}} \tag{67}$$

$$E[W_0] = \frac{E[N_0]}{\rho\lambda C_{[1]}} \tag{68}$$

V RELIABILITY INDICES

Let $A_v(t)$ be the system availability at time 't' i.e the probability that the server is either working for a customer or in an idle period such that the steady state availability of the server is given by

$$A_v = Lt_{t\to\infty} A_v(t) \tag{69}$$

$$A_v = P_{00} + Lt_{z\to 1} P_q(1) = 1 - \frac{\lambda C_{[1]}[\alpha\mu_1 r_1 + \theta v_1]}{1+\rho(1-p)} \tag{70}$$

The steady state failure frequency of the server

$$F = \alpha P_q(1) = \frac{\alpha\lambda C_{[1]}\mu_1}{1+\rho(1-p)} \tag{71}$$

VI. CONCLUSION

In this paper, we have obtained the probability generating function of various states of the system in transient state and also discussed the steady state solution with performance measures of the system and the reliability indices like availability of the server and failure frequency of the server. The prescribed model can be modeled in the design of computer networks. As a future work we can try to incorporate the effect of balking/reneging on this service system.

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