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Original Article

Effect of Treatment and Awareness on HIV/AIDS Epidemic Model

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Abstract *- Here an HIV/AIDS epidemical model has been taken with awareness and treatment. The disease spreads in a variable size population through a horizontal transmission that is by contact. The equilibrium points of the reckon model are found and their stability is investigated. The basic reproductive numbers of the proposed model have been calculated using the next-generation matrix technique, which plays an important role in discussing the model. When the basic reproductive number R_0, its value is less than one disease-free equilibrium E_0 is locally asymptotically stable and unstable while $R_0 > 1$. If $R_0 > 1$, then the endemic equilibrium point E^* is asymptotically stable, which is calculated using the Jacobian matrix. Also, discuss the effect of treatment and awareness with the help of sensitivity analysis. Finally, find out some sensitive parameters that are more reliable to decrease disease transmission. The considered model has been solved numerically and the model's numerical solution justifies the analytical outcomes.*

Keywords *- HIV/AIDS Epidemic, Basic Reproductive Number, Local Stability, Aware- ness, Treatment, and Numerical Outcomes.*

1. Introduction

Today the most deadly infectious disease is HIV/AIDS. All the humankind of the world has already faced it. A little reflection of history shows that the American organization Centers for Disease Control (CDC) recognized the first Acquired Immunodeficiency Syndrome (AIDS) case among homosexual men in 1981 [1]. In the next year, AIDS spread rapidly to all the countries. Most probably in 1986, the (ICT) International Committee on Taxonomy of Viruses gave a separate name to the AIDS virus Human Immunodeficiency Virus or HIV [2, 3]. In India, firstly the primary known case of HIV/AIDS among female sex workers in the city of Chennai was in the year 1986. Next year i.e. 1987, 135 new cases have been reported. In 1992, India first initiated to organize a National AIDS Control Programme. Further NACO (National AIDS Control Organization) was reposed and completed all over India through more than 35 different HIV/AIDS prevention and control societies [4-6]. According to the WHO database around 39 million people all over the world were living with HIV at the closing of 2022. In 2022, all over the world 6 million people died due to HIV. Within a very short period, Human immunodeficiency virus (HIV) infection and acquired immunodeficiency syndrome (AIDS) has become a significant infectious disease and which spread amongst various Nations. In India, 86.27 percent of cases of infection are due to sexual transmission which indicates that heterosexual contact is the predominant mode of transmission of (HIV). So many researchers have been working in the field of epidemiology and they are trying to work out how can reduce the trans- mission rate of such types of infectious diseases. The treatment (antiretroviral therapy) process cannot remove the HIV from the infected person but it can give some resistance or slow down the disease transmission process.

Routine access to the treatment process (antiretroviral therapy) is expensive and it is not sufficiently available in most countries of the World. Mostly, HIV is spread by unprotected sexual contact and vertical transmission [7]. The basic modes of horizontal transmission of HIV are sexual contact without protection, injection needles sharing, transfusion of blood, plasma-derived blood products, transplantation of organs, and artificial insemination Collazos et al. [8-9]. From the initial models of May and Anderson [10-11] various refinements have been added to modeling frameworks and researchers have addressed specific issues [12-20]. Formulated the mathematical model and defined its parameters. Found the basic reproduction number called the threshold quantity R_0. With the assistance of numerical tools, the model is solved which justifies the analytical outcomes.

2. Mathematical Model

In this section, consider an HIV/AIDS epidemic model with treatment and awareness. The total population is divided into five subclasses of the relevant population i, e, the susceptible population $S(t)$, undiagnosed HIV-infected people who were infected via sexual intercourse $U(t)$, diagnosed HIV-infected people $D(t)$, people diagnosed with AIDS $A(t)$ and the treatment class $R(t)$ respectively at time t. Let Λ be the influx rate of the susceptible class, α be the contact rate between undiagnosed HIV infective and susceptible, α_1 be the contact rate between diagnosed HIV infective and susceptible, μ is mortality rate of the adults class, d is mortality rate of AIDS class, θ is the rate at which undiagnosed HIV infected class are diagnosed through random tracking, β is the rate at which HIV infective class develop AIDS. δ is the awareness rate of susceptible, η is the rate at which the diagnosed HIV infective class develops AIDS, γ is the treatment rate of the diagnosed HIV infective class and γ_1 is the treatment rate of the AIDS class.

With these assumptions the mathematical model is

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \alpha US - \alpha_1 DS - (\mu + \delta)S, \\
\frac{dU}{dt} &= \alpha US + \alpha_1 DS - (\mu + \beta + \theta)U, \\
\frac{dD}{dt} &= \theta U - (\mu + \gamma + \eta)D, \qquad (1) \\
\frac{dA}{dt} &= \beta U + \eta D - (\mu + \gamma_1 + d)A, \\
\frac{dR}{dt} &= \gamma D + \gamma_1 A + \delta S - \mu R.
\end{aligned}$$

Since A and D population classes do not appear in the above three equations, so consider the following sub-system

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \alpha US - \alpha_1 DS - (\mu + \delta)S, \\
\frac{dU}{dt} &= \alpha US + \alpha_1 DS - (\mu + \beta + \theta)U, \qquad (2) \\
\frac{dD}{dt} &= \theta U - (\mu + \gamma + \eta)D.
\end{aligned}$$

3. Basic Properties of the Model

3.1. Invariant Region

Theorem 3.1.1 All the solutions of the system (1) are contained, in the feasible region Φ

Proof: Suppose $\Phi = (S, U, D, A, R) \in \mathbf{R}^5$ is any solution of my reckon systems (1) with the positive initial conditions. Now adding all the equation of the system (1), we get

$$\begin{aligned}
\frac{dS}{dt} + \frac{dU}{dt} + \frac{dD}{dt} + \frac{dA}{dt} + \frac{dR}{dt} &= \Lambda - \mu(S + U + D + A + R) - dA \\
&\leq \Lambda - \mu(S + U + D + A + R)
\end{aligned}$$

Hence, $\lim_{t\to\infty} sup(S + U + D + A + R) \leq \frac{\Lambda}{\mu}$

Hence all the solutions of the system (1) is positively invariant in the ascertain region

$\Phi = \{(S, U, D, A, R)$: S+U+D+A+R$\leq \frac{\Lambda}{\mu}, S \geq 0, U \geq 0, D \geq 0, A \geq 0, R \geq 0\}$.

3.2. Positivity of the Solutions

Whereas the model conveys the human population therefore needs to show that all the state variables remain positive at all times.

Theorem 3.2.1 Let $\Phi = \{(S, U, D, A, R)$*: S+U+D+A+R* $\leq \frac{\Lambda}{\mu}, S \geq 0, U \geq 0, D \geq 0, A \geq 0, R \geq 0\}$ *then all solutions S(t), U (t), D(t) ,A(t) and R(t) of the system (1) are non-negative for all t ≥0.*

Proof: From the first equation of the system (1), we get

$$\frac{dS}{dt} = \Lambda - \alpha US - \alpha_1 DS - (\mu + \delta)S > -\{\alpha U + \alpha_1 D + (\mu + \delta)\}S$$

$$S(t) \geq S(0)e^{-\int\{\alpha U(\xi) + \alpha_1 D(\xi) + (\mu+\delta)\}d\xi} \geq 0$$

Similarly, it can show that, $U(t) \geq \frac{U(0)}{e^{(\mu+\beta+\theta)t}}, D(t) \geq \frac{D(0)}{e^{(\mu+\gamma+\eta)t}}, A(t) \geq \frac{A(0)}{e^{(\mu+\gamma_1+d)t}}$ *and* $R(t) \geq \frac{R(0)}{e^{\mu t}}$

4. Equilibrium points, the Basic reproduction number, and stability analysis

The system (2) possesses the following equilibria:

(i) The disease free equilibrium $\boldsymbol{E_0} = \left(\frac{\Lambda}{\mu+\delta}, \mathbf{0}, \mathbf{0}\right)$,

(ii) The endemic equilibrium $\boldsymbol{E^*} = (\boldsymbol{S^*}, \boldsymbol{U^*}, \boldsymbol{D^*})$ ***with***

$$\boldsymbol{S^*} = \frac{(\mu+\beta+\theta)(\mu+\gamma+\eta)}{\alpha_1\theta + \alpha(\mu+\gamma+\eta)}, \quad \boldsymbol{U^*} = \frac{(\mu+\delta)(\mu+\gamma+\eta)(R_0 - 1)}{\alpha_1\theta + \alpha(\mu+\gamma+\eta)} \ \ \textbf{and} \ \ \boldsymbol{D^*} = \frac{\theta U^*}{(\mu+\gamma+\eta)}$$

It is obvious that when $R_0 > 1$, there exists a unique positive stable equilibrium exists. On the other hand if $R_0 < 1$, there is no possibility to exist any positive equilibrium. The stability analysis of both E_0 and E^* is represented with the assistance of basic reproductive number R_0 which is calculated by using the next-generation matrix technique. The infection terms, the noninfectious terms defined by the singular matrix f and v.

F = Jacobiann of 'f' at disease-free equilibrium point=$\begin{pmatrix} \frac{\Lambda\alpha}{\mu+\delta} & \frac{\Lambda\alpha_1}{\mu+\delta} \\ 0 & 0 \end{pmatrix}$

V = Jacobiann of 'v' at the disease-free equilibrium point=$\begin{pmatrix} \boldsymbol{\mu+\beta+\theta} & \mathbf{0} \\ \boldsymbol{-\theta} & \boldsymbol{\mu+\gamma+\eta} \end{pmatrix}$

$$R_0 = \rho FV^{-1} = \frac{\Lambda\alpha}{(\mu+\delta)(\mu+\beta+\theta)} + \frac{\Lambda\alpha_1\theta}{(\mu+\delta)(\mu+\beta+\theta)(\mu+\gamma+\eta)}$$

4.1 Stability Analysis

In this current portion, discuss the stability of both the without-disease and with-disease-related equilibrium points. The local stability of without disease-related equilibrium point $E_0 = \left(\frac{\Lambda}{\mu+\delta}, 0, 0\right)$ is discuss by the following theorem

Theorem 4.1.1 The disease-free equilibrium $E_0 = \left(\frac{\Lambda}{\mu+\delta}, 0, 0\right)$ *is locally asymptotically stable if,* R_0*<1 and unstable if,* R_0*>1.*

Proof: The variational matrix of the system (2) around the disease-free equilibrium point E_0 is given by

$$J_0 = \begin{pmatrix} -(\mu+\delta) & -\dfrac{\Lambda\alpha}{\mu+\delta} & -\dfrac{\Lambda\alpha_1}{\mu+\delta} \\ 0 & \dfrac{\Lambda\alpha}{\mu+\delta} - (\mu+\beta+\theta) & \dfrac{\Lambda\alpha_1}{\mu+\delta} \\ 0 & \theta & -(\mu+\gamma+\eta) \end{pmatrix}$$

The one eigenvalue of the characteristics equation of the Jacobian matrix J_0 is $h_1 = -(\mu+\delta)$ and the other eigenvalues of the following equation

$$\lambda^2 + a_1\lambda + a_2 = 0, where$$

$$a_1 = (\mu+\beta+\theta)(\mu+\gamma+\eta) - \frac{\Lambda\alpha}{\mu+\delta},\ \ a_2 = (\mu+\beta+\theta)(\mu+\gamma+\eta)(1-R_0)$$

If $R_0 < 1$ and $2\mu + \beta + \theta + \gamma + \eta > \frac{\Lambda\alpha}{\mu+\delta}$ then it is clear that all the eigenvalues of the Jacobian matrix J_0 are negative. So the disease-free equilibrium is locally asymptotically stable when $R_0 < 1$ and if $R_0 > 1$, then $a_2 < 0$ and the above quadratic equation has one positive eigenvalue, so a saddle unstable equilibrium exists.

Theorem 4.1.2 The disease related equilibrium point E^* is locally asymptotically stable while $R_0 > 1$.

Proof: The Jacobian matrix J^* around the endemic equilibrium point E^* is given by,

$$J^* = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \text{ where}$$

$a_{11} = -\alpha U^* - \alpha_1 D^* - (\mu+\delta)$, $a_{12} = -\alpha S^*$, $a_{13} = -\alpha_1 S^*$, $a_{21} = \alpha U^* + \alpha_1 D^*$, $a_{22} = \alpha S^* - (\mu+\beta+\theta)$, $a_{23} = \alpha_1 S^*$, $a_{31} = 0$, $a_{32} = \theta$, $a_{33} = -(\mu+\gamma+\eta)$.

The characteristics equation of J^* is

$$(h^*)^3 + \rho_1 (h^*)^2 \rho_2 h^* + \rho_3 = 0, \qquad where$$

$$\rho_1 = -a_{11} - a_{22} - a_{33} > 0$$

$$\rho_2 = a_{11}a_{22} + a_{11}a_{33} + a_{33}a_{22} - a_{23}a_{32} - a_{12}a_{21}$$

$$\rho_3 = a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} - a_{11}a_{23}a_{33} - a_{13}a_{21}a_{32} > 0 \ and\ \rho_1\rho_2 - \rho_3 > 0$$

According to the Routh-Hurwitz Criterion if all $\rho_i > 0, i = 1, 2, 3, \rho_1 > 0, \rho_3 > 0\ and\ \rho_1\rho_2 > \rho_3$ then all roots of the above characteristic equation has negative real parts. Therefore the theorem is proof.

5. Sensitivity Analysis

This section tries to discover some sensible parameters of the model (2) which prevent disease transmission. The table shows the sensible values of all the parameters on R_0. In epidemiology, the disease is either spread or not it depends on the value of the basic reproduction number. Therefore, it is essential to find out some sensible parameters. First the sensible values of all the parameters with the help of the formula [21, 22].

$$S_{R_0}^{x_j} = \frac{\partial R_0}{\partial x_j} \times \frac{x_j}{R_0}$$

From Table 1 the positive values of $S_{R_0}^{x_j}$ indicate that while increasing (decrease) the value of the parameter which is engaged with R_0, then the value of R_0 will also increase (decrease). On the other hand, the negative values of $S_{R_0}^{x_j}$ indicate that while increasing (decrease) the corresponding values of the parameter then the value of R_0 will also decrease (increase). From the analysis, the most sensible manageable parameter is treatment better than awareness.

Table 1. Sensible values of all parameters on R_0

Parameter x_j	Λ	Α	α_1	Μ	Δ	Β	Θ	γ	η
Value	1.4	0.0001	0.0003	0.01	0.001	0.03	0.02	0.002	0.001
$S_{R_0}^{x_j}$	1	0.18	0.82	-1.71	-0.09	-0.50	0.49	-0.13	0.63

6. Numerical Simulations

Now present the numerical simulations of the proposed epidemic model with awareness and treatment to illustrate the analytical results. First, suppose that the total population is constant and there is no migration. Figure 1 shows the disease-free equilibrium point E_0(81.8182, 0, 0) is locally asymptotically stable when R_0=0.7657<1, $2\mu + \beta + \theta + \gamma + \eta\ (0.073) > \frac{\Lambda\alpha}{\mu+\delta}$ (0.0081) with eigenvalues -0.0110, -0.0619, -0.0030 for Λ=0.9 and all other parameters values which is given in the Table 2 and unstable whileR_0>1. Figure 2 shows the endemic equilibrium point E*(106.493, 3.7443, 5.7604) is asymptotically stable locally with eigenvalues -0.0624, -0.0105, -0.0025 for all parameters values which are given in the Table 2. Figure 3 displays the sensitivity analysis of all the parameter values of the system (2) on R_0. Finally, Figure 4(a) susceptible class and diagnosed HIV infected class.

Table 2. A set of parametric values

Parameter	Definition	Default Value
Λ	Influx rate of susceptible class	1.4
Α	Contact rate between undiagnosed HIV infected and susceptible class	0.0001
α_1	Contact rate between diagnosed HIV infected and susceptible class	0.0003
Μ	Mortality rate of adults class	0.01
Δ	Awareness rate of susceptible	0.001
Β	HIV infective class develop AIDS	0.03
Θ	Rate at which undiagnosed HIV infected class are diagnosed through random tracking	0.02
Γ	Treatment rate of the diagnosed HIV infective class	0.002
Η	Rate at which the diagnosed HIV infective class develops AIDS	0.001

7. Conclusion

In this work, consider and analyze an HIV/AIDS epidemic model with awareness and treatment. Here also suppose that the disease is spread only by sexual contact without any protection and horizontal transmission. By exploring the model, invent a expression denote as basic reproduction number R_0. It is observe and ensured that when R_0<1 the disease dies out and when R_0>1 the disease persists and it will became endemic. The model has consists two positive equilibria namely, $E_0 = \left(\frac{\Lambda}{\mu+\delta}, 0, 0\right)$ the without disease and $E^* = (S^*, U^*, D^*)$ the disease related equilibrium. It is proves that the without disease related equilibrium point E_0 is locally asymptotically stable if R_0<1 and $2\mu + \beta + \theta + \gamma + \eta > \frac{\Lambda\alpha}{\mu+\delta}$ and unstable if R_0>1. The endemic equilibrium E^*, which exists only when R_0>1 is asymptotically stable. If the awareness rate and treatment rate are in a certain range then the value of R_0 will decrease and the system has no endemic equilibria. Therefore, treatment and awareness is an effective way to obstruct the disease transmission rate. It has also been mentioned that if the HIV infection is suppressed at may early stage by effectively treating the infective, the progression to AIDS can be slowed down and the life span of HIV HIV-infected population can be increased. Finally, the numerical analysis values ensure that if can increase awareness to avoid unprotected sexual contact between susceptible and diagnosed HIV-infected class, then the transmission rate of it can kept under grip.

References

[1] Centers for Disease Control(CDC), " *Pneumocystis Pneumonia - Los Angeles*, " Morbidity and Mortality Weekly Report (MMWR), vol. 30, pp. 250-252, 1981. [Google Scholar] [Publisher Link]

[2] Centers for Disease Control(CDC), "*Update on Acquired Immune Deficiency Syndrome (AIDS) - United States*," Morbidity and Mortality Weekly Report (MMWR), vol. 31, pp. 507-514, 1982. [Google Scholar] [Publisher Link]

[3] John Coffin et al., "Human Immunodeficiency Viruses," *Science*, vol. 232, no. 4751, p. 697, 1986. [CrossRef] [Google Scholar] [Publisher Link]

[4] Annual Report 2013-14, The Department of AIDS Control, Ministry of Health and Family Welfare, Government of India. [Online]. Available. https://naco.gov.in/sites/default/files/NACO_English%202013-14.pdf

[5] Annual Report 2008-2009, National AIDS Control Organization, NACO, Ministry of Health, Government of India. [Online]. Available. https://naco.gov.in/sites/default/files/Annual_Report_NACO_2008-09.pdf

[6] UNAIDS, HIV Estimates with Uncertainty Bounds, 1990- 2012, Report on the Global AIDS Epidemi 2013. [Online]. Available. https://www.unaids.org/sites/default/files/media_asset/UNAIDS_Global_Report_2013_en_1.pdf

[7] R.O. Simwa, and G.P. Pokhariyal, "A Dynamical Model for Stage-Specific HIV Incidences with Application to Sub-Saharan Africa," *Applied Mathematics and Computation*, vol. 146, no. 1, pp. 93-104, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[8] Centers for Disease Control and Prevention, "*HIV Transmission through Transfusion-Missouri and Colorado*," Morbidity and Mortality Weekly Report (MMWR), vol. 15, pp. 1335-1339, 2010. [Google Scholar] [Publisher Link]

[9] J. Collazos, V. Asensi, and J.A. Carton, "Association of HIV Transmission Categories with Socio Demographic, Viroimmunological and Clinical Parameters of HIV- Infected Patients, *Epidemiology and Infection*, vol. 138, no. 7, pp. 1016-1024, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[10] R.M. Anderson et al., "A Preliminary Study of the Transmission Dynamics of the Human Immunodeficiency Virus(HIV), the Causative agent of AIDS," *Mathematical Medicine and Biology: A Journal of the IMA*, vol. 3, no. 4, pp. 229-263, 1986. [CrossRef] [Google Scholar] [Publisher Link]

[11] R.M. Anderson, "The Role of Mathematical Models in the Study of HIV Transmission and the Epidemiology of AIDS," *Journal of Acquired Immune Deficiency Syndromes*, vol. 1, no. 3, pp. 241-256, 1988. [Google Scholar] [Publisher Link]

[12] H. de Arazoza, and R. Lounes, "A Nonlinear Model for Sexually Transmitted Diseases with Contact Tracing," *Mathematical Medicine and Biology: A Journal of the IMA*, vol. 19, no. 3, pp. 221–234, 2002. [CrossRef] [Google Scholar] [Publisher Link]

[13] Brandy Rapatski et al., "Mathematical Epidemiology of HIV/AIDS in Cuba during the Period 1986-2000," *Mathematical Biosciences and Engineering*, vol. 3, no. 3, pp. 545-556, 2006. [CrossRef] [Google Scholar] [Publisher Link]

[14] O. Diekmann, J.A.P. Heesterbeek, and J.A.J. Metz, "On the Definition and the Computation of the Basic Reproduction Ratio R0 in Models for Infectious Diseases in Heterogeneous Populations," *Journal of Mathematical Biology*, vol. 28, no. 4, pp. 365-382, 1990. [CrossRef] [Google Scholar] [Publisher Link]

[15] P. van den Driessche, and James Watmough, "Reproduction Numbers and sub-threshold Epidemic Equilibria for Compartmental Models of Disease Transmission," *Mathematical Biosciences*, vol. 180, no. 1-2, pp. 29-48, 2002. [CrossRef] [Google Scholar] [Publisher Link]

[16] Stavros Busenberg, Kenneth Cooke, and Ying-Hen Hsieh, "A Model for HIV in Asia," *Mathematical Biosciences*, vol. 128, no. 1-2, pp. 185-210, 1995. [CrossRef] [Google Scholar] [Publisher Link]

[17] Klaus Dietz, "On the Transmission Dynamics of HIV," *Mathematical Biosciences*, vol. 90, no. 1-2, pp. 397-414, 1988. [CrossRef] [Google Scholar] [Publisher Link]

[18] Ying-Hen Hsieh, and Chien Hsun Chen, "Modeling the Social Dynamics of a Sex Industry: Its Implications for the Spread of HIV/AIDS," *Bulletin of Mathematical Biology*, vol. 66, pp. 143-166, 2004. [CrossRef] [Google Scholar] [Publisher Link]

[19] Issa Shabani, Estomih S. Massawe, and Oluwole Daniel Makinde, "Modelling the Effect of Screening on the Spread of HIV Infection in a Homogeneous Population with Infective Immigrants," *Scientific Researchs and Essays(SRE)*, vol. 6, no. 20, pp. 4397–4405, 2011. [CrossRef] [Google Scholar] [Publisher Link]

[20] Jack H. Hale, *Theory of Functional Differential Equations*, 2nd ed., Springer Verlag, New York, 1977. [CrossRef] [Publisher Link]

[21] Ali Raza et al., "Modeling the Effect of Delay Strategy on Transmission Dynamics of HIV/AIDS Disease, *Advances in Difference Equations*, vol. 2020, no. 663, pp. 1-13, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[22] Sudip Samanta, "Effects of Awareness Program and Delay in the Epidemic Outbreak," *Mathematical Methods in the Applied Sciences*, vol. 40, no. 5, pp. 1679-1695, 2016. [CrossRef] [Google Scholar] [Publisher Link]

Appendix

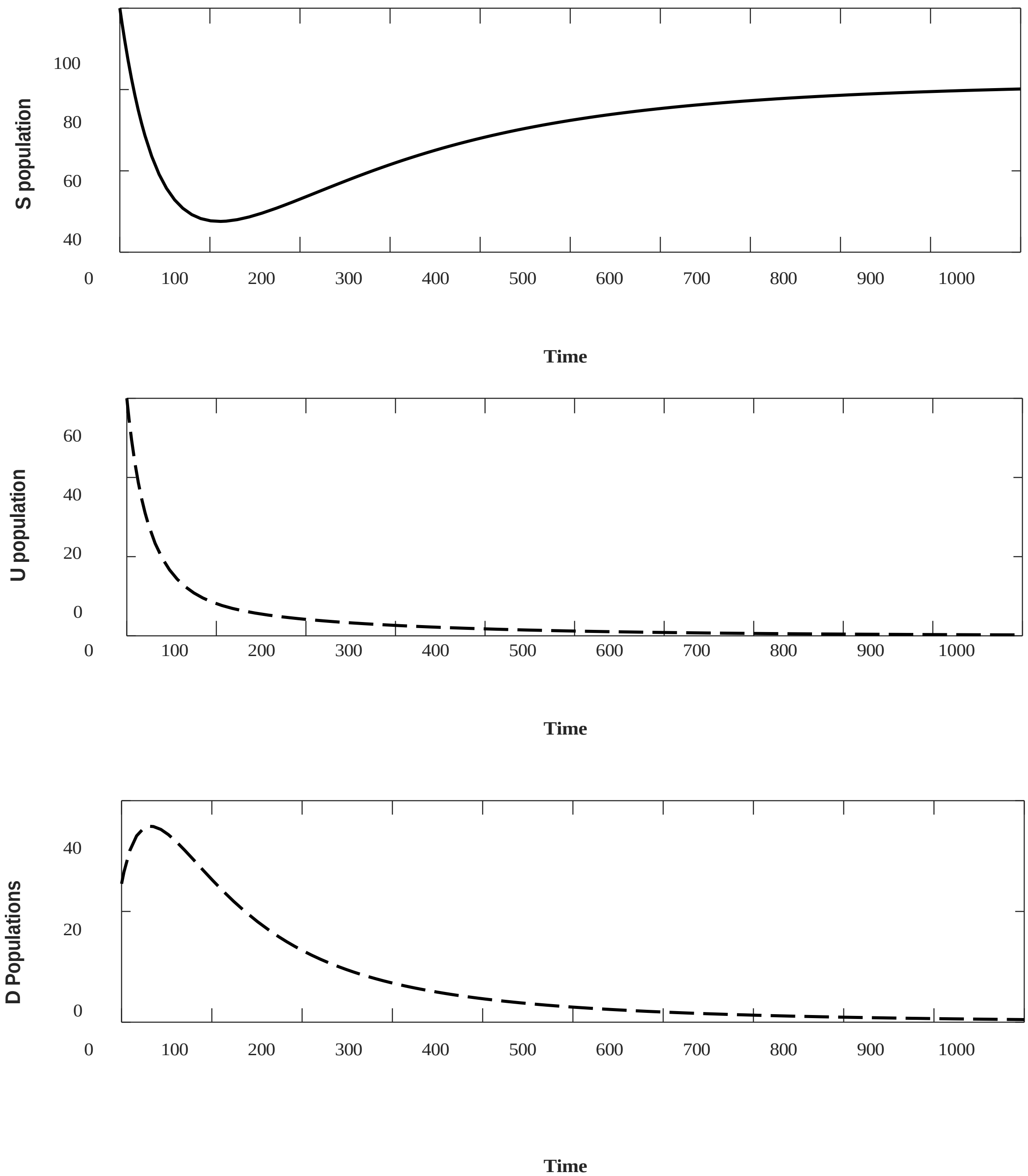

Fig. 1 E_0 **is asymptotically stable for** $\Lambda = 0.9$ **and the other value is given in Table 2 while** $R_0 < 1$.

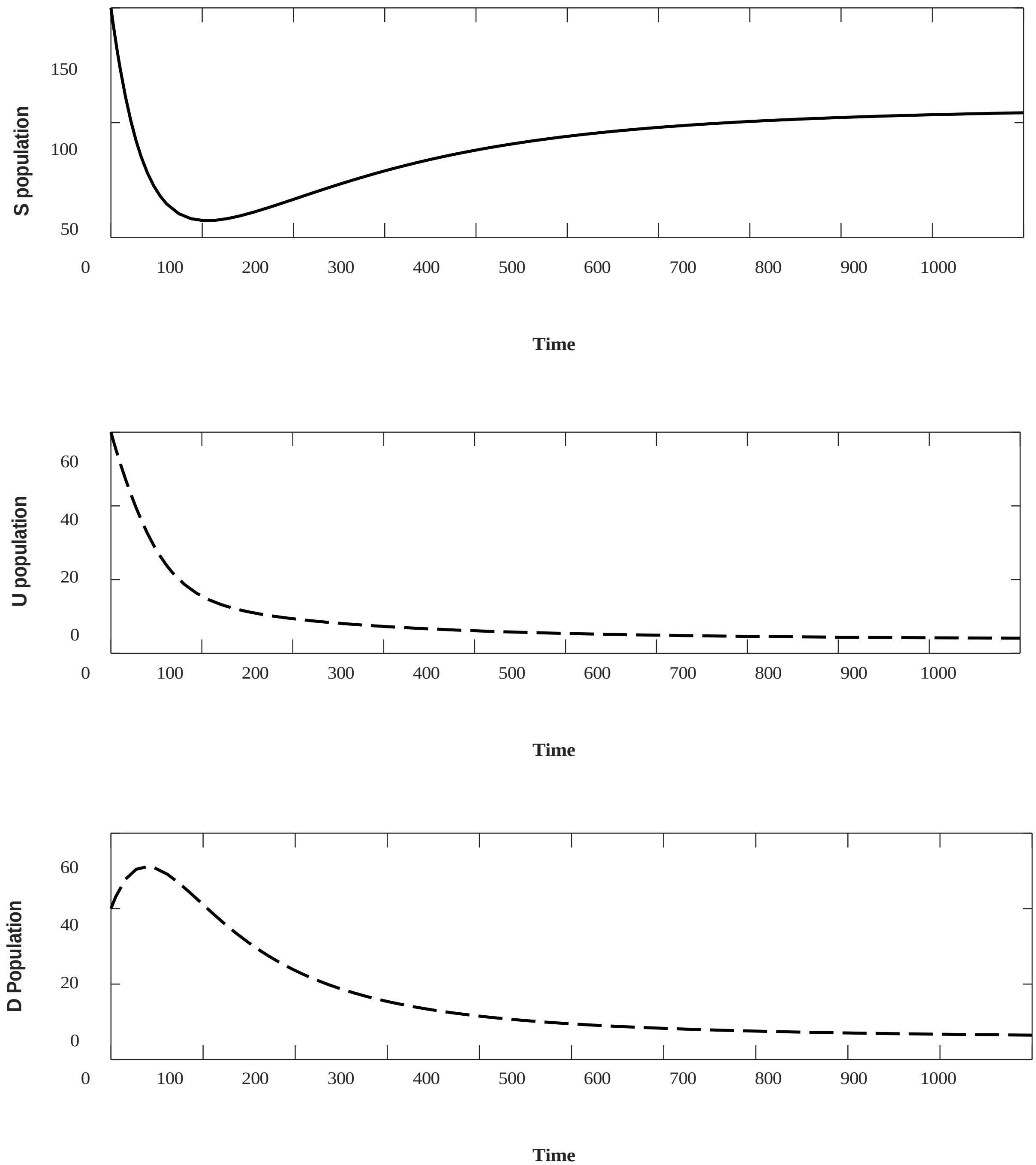

Fig. 2 The equilibrium point E^* is locally asymptotically stable for all parameter values is taken from Table 2 while $R_0 > 1$.

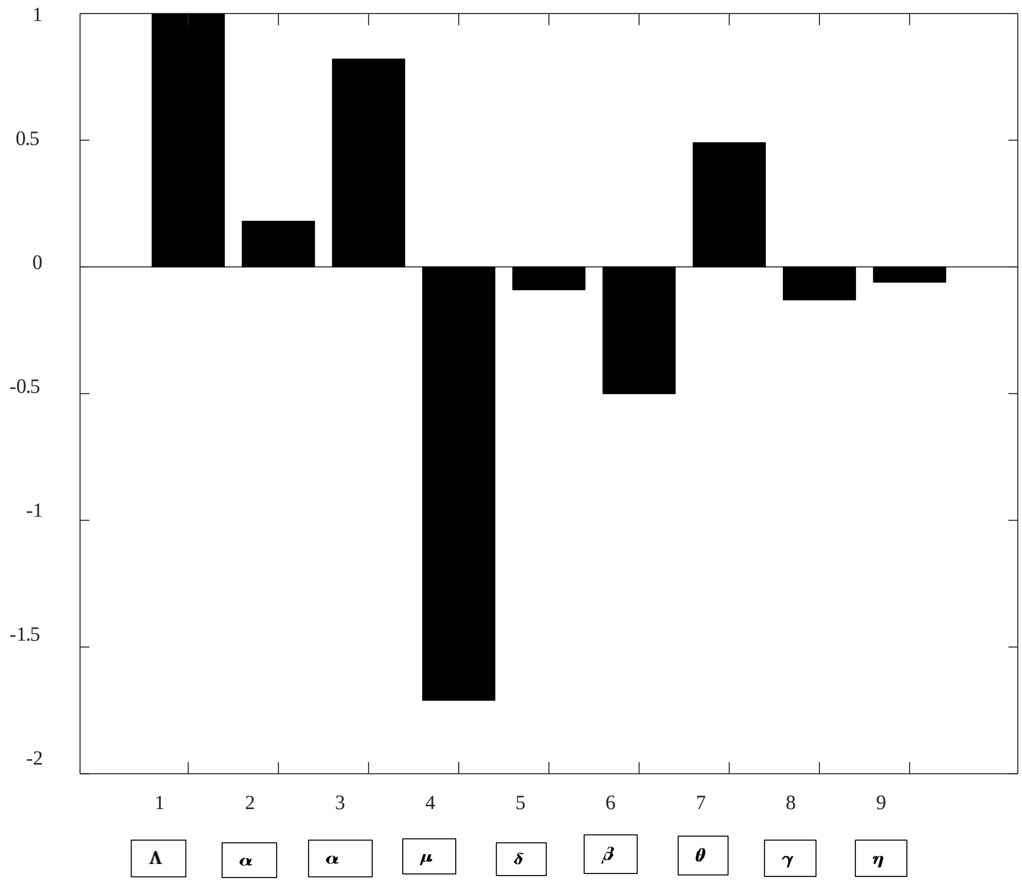

Fig. 3 Figure shows that the sensitivity analysis of all the parameters values of the system (2) on R_0.

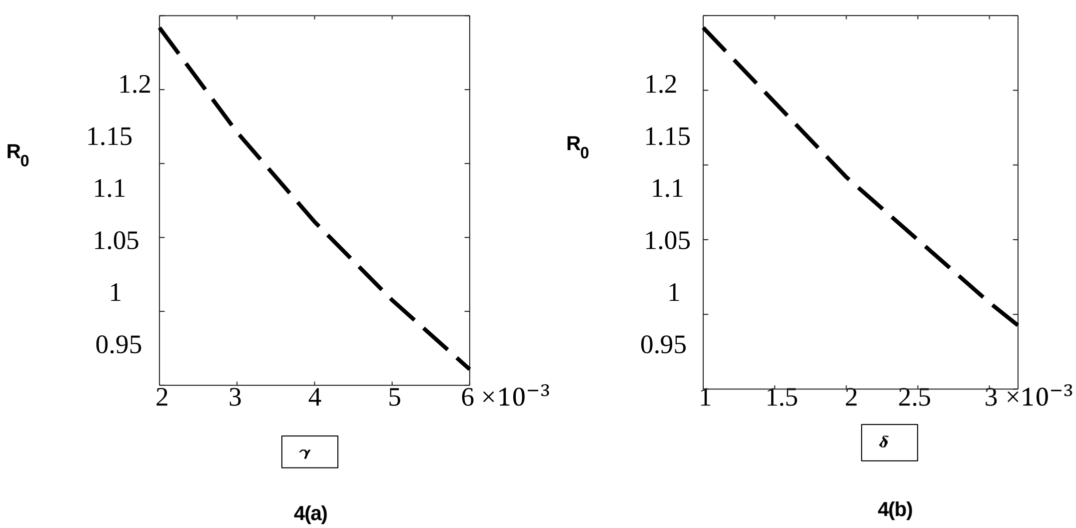

Fig. 4 Figure 4(a) shows that if the value of γ increases then R_0 its value decreases. Figure 4(b) exposes that if the value of δ increases, then R_0 its value will decrease.