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Original Article

A Deterministic Inventory Model for Deteriorating Products with Replenishment Rate Dependent on Inventory Level

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Abstract - *In this study, we present a deterministic stock model for overseeing deteriorating items, where the replenishment rate is progressively reliant upon the stock level. The model integrates the complexities associated with holding, ordering, and deterioration costs, providing a comprehensive framework for optimizing inventory control in supply chains dealing with perishable or time-sensitive goods. By incorporating a finite replenishment rate that varies with the current stock level, the model reflects realistic replenishment scenarios and helps minimize total costs by determining the optimal cycle length. Sensitivity analysis is directed to assess the effect of varieties in key boundaries, for example, deterioration rates, holding costs, and demand rates on the complete stock expense. The findings offer valuable insights into the cost drivers in inventory systems with deteriorating items, enabling decision-makers to formulate more effective inventory policies that balance replenishment frequency, storage, and deterioration losses. This model is particularly applicable to industries like pharmaceuticals, food, and chemicals, where product shelf life and replenishment dynamics play a crucial role in operational efficiency.*

Keywords - *Deterministic inventory model, Deteriorating products, Replenishment rate, Inventory level dependency, Total cost optimization, Cycle length, Holding cost, Ordering cost, Deterioration cost, Sensitivity analysis, Cost minimization.*

1. Introduction

Effective inventory management is crucial for businesses dealing with deteriorating products such as perishable goods, pharmaceuticals, and chemicals, where the shelf life significantly impacts operational decisions. Traditional inventory models often overlook the complexities associated with the deterioration of items and the need for dynamic replenishment strategies. This study presents a deterministic stock model explicitly intended for weakening items, where the recharging rate relies on the ongoing stock level. By determining the ideal cycle length for replenishment, the model aims to minimize the total cost, which includes costs associated with holding, ordering, and deterioration. By integrating the effects of varying replenishment rates based on inventory levels, the model provides a more realistic approach to managing inventory in environments where deterioration and replenishment dynamics are critical. Responsiveness examination is utilized to inspect the impact of key boundaries, for example, crumbling rates, request rates, and cost factors, on the absolute expense, and this offers important bits of knowledge for creating savvy stock approaches.

This research provides a robust framework that can be applied to various industries, guiding inventory managers in balancing replenishment frequency, storage costs, and product deterioration to enhance overall supply chain efficiency. Ahmed et al. (2013) introduced stock models with a slope-type request rate, halfway multiplying, and general disintegration rate. Their model makes it possible to take a more realistic approach to inventory management in a dynamic environment when demand for an item rises over time and unmet demand is partially backlogged. A monetary request amount model for products with incline type interest, deficiencies, and a three-boundary Weibull dispersion weakening was introduced by Sanni and Chukwu (2013). The Weibull distribution helps represent various deterioration patterns, providing a robust framework for managing inventory for items that deteriorate according to more complex patterns. Duari and Chakraborti (2014) proposed a Monetary Request Amount (EOQ) model for disintegrating things in a solitary distribution center framework, considering cost subordinate interest and considering deficiencies. This model addresses inventory management by factoring in the deterioration of items over time, thereby optimizing order levels to minimize costs associated with holding, ordering, and shortages. Zhao and Wang (2015) investigated estimating and retail administration choices in conditions with fluffy vulnerability, giving a model that obliges the intricacies of certifiable stock, and it is not effectively quantifiable to cost choices where vulnerabilities. Their work is particularly relevant for businesses operating in uncertain and fluctuating market conditions. Srivastava and Singh (2017) fostered a deterministic stock model for things with straight interest, variable crumbling, and incomplete multiplying. This model provides a far-reaching system to address stock issues where the request is direct, and things break down at a variable rate, featuring the compromises between holding decaying things and the expense of multiplying unsatisfied interest. Singh et al. (2018) proposed an EOQ stock model for decaying things with time-subordinate disintegration rates, incline-type request rates, and deficiencies.

This model is especially valuable for overseeing inventories where the decay pace of things isn't steady over the long run, and request follows an incline design, further confounded by possible deficiencies. Shi et al. (2019) provided valuable insights into inventory management, where demand increases over time, and payment terms can be negotiated, making it highly relevant for businesses that deal with deteriorating goods and fluctuating demand. Uthayakumar and Karuppasamy (2019) focused on the unique challenges faced in healthcare inventory management, where demand can vary significantly over time, and items are prone to deterioration, thus requiring specialized approaches to inventory control. Shaikh et al. (2020) figured out a stock model for disintegrating things with a conservation office, incline type interest, and exchange credit. Their model coordinates the idea of protection innovation to decrease weakening rates, giving adaptability in installment terms through exchange credit, a significant part of present-day stock administration rehearses. An inventory model for deteriorating items with ramp-type demand and a permissible delay in payment was created by Sharma and Kaushik (2021). When it comes to managing inventory levels for products that deteriorate over time, this model emphasizes the significance of incorporating credit policies and varying demand rates, particularly for businesses aiming to maximize cash flow and inventory levels. Supakar and Mahato (2022) presented a Financial Creation Amount (EPQ) model with time-proportionate disintegration and incline-type interest under various installment plans with fluffy vulnerabilities. Their research extends traditional inventory models by incorporating fuzzy logic to handle uncertainties in demand and deterioration rates, making it more adaptable to real-world scenarios. Pinto and Gil (2022) inspected the utilization of fake safe frameworks in cutting-edge fabricating, adding to the field by introducing an original way to deal with taking care of assembling intricacies through naturally propelled calculations.

This approach can advance different assembling parts, including stock control and creation planning. Hossain et al. (2024) introduced a benefit-cost proportion expansion approach for an assembling stock model with stock-subordinate creation rates and stock-and-cost subordinate interest rates. Their model introduces a multi-faceted approach to inventory management by considering how stock levels and pricing influence both production and demand, offering a more holistic view of inventory optimization in manufacturing settings.

1.1. Key Assumptions

- Demand Rate (D): Constant or a function of inventory level.
- Replenishment Rate (R): Finite and dependent on the inventory level.
- Deterioration Rate (θ): Constant fraction of inventory deteriorates over time.
- Lead Time: Zero or constant.
- Shortages: Not allowed or partially backlogged.

1.2. Notation

$I(t)$: Inventory level at time t.
D: Constant demand rate or $D(I)$, a function of inventory level.
$R(I)$: Replenishment rate as a function of inventory level.
θ: Constant deterioration rate.
C_h: Holding cost per unit per unit time.
C_d: Deterioration cost per unit.
C_O: Ordering cost per order.

1.3. Inventory Level

The rate of progress of stock after some time can be depicted by a differential condition that records for request, recharging, and decay:

$$\frac{dI}{dt} = R\big(I(t)\big) - DI(t) - \theta I(t) \tag{1.1}$$

Here, $R\big(I(t)\big)$ represents the replenishment rate, $DI(t)$ is the demand rate which might be dependent on $I(t)$, and $\theta I(t)$ represents the deterioration of items.

$D(t) = D$ (constant demand rate)

$$\frac{dI}{dt} = R\big(I(t)\big) - D - \theta I(t) \tag{1.2}$$

1.4. Replenishment Rate

The replenishment rate can be modeled as a function of inventory level. One common form is:

$$R\big(I(t)\big) = \alpha I(t) \tag{1.3}$$

where α is a proportional constant determining the replenishment rate based on the current inventory level.

1.5. Cost Function

The total cost function generally consists of the sum of holding cost, ordering cost, and deterioration cost:

$$\text{Holding Cost: } \mathrm{TC_h} = \int_0^T \mathrm{C_h}\, \mathrm{I(t)dt} \tag{1.4}$$

$$\text{Ordering Cost: } \mathrm{TC_O} = \frac{\mathrm{C_O}}{\mathrm{T}} \tag{1.5}$$

where T is the cycle length.

$$\text{Deterioration Cost: } \mathrm{TC_d} = \int_0^T \mathrm{C_d}\, \mathrm{I(t)dt} \tag{1.6}$$

The goal is to limit the all-out cost, TC, which is the amount of holding, ordering and deterioration costs:

$$\mathrm{TC} = \mathrm{TC_h} + \mathrm{TC_O} + \mathrm{TC_d} \qquad 1.7)$$

1.6. Solution of the inventory level

$$\frac{dI}{dt} = \alpha I(t) - D - \theta I(t) \tag{1.8}$$

$$\frac{dI}{dt} - (\alpha - \theta)I(t) = -D \tag{1.9}$$

The integrating factor is given by:

$$IF = e^{\int -(\alpha-\theta)dt} = e^{-(\alpha-\theta)t} \tag{1.10}$$

Multiply the Entire Equation by the Integrating Factor:

$$\mathrm{e}^{-(\alpha-\theta)t}\frac{\mathrm{dI}}{\mathrm{dt}} - (\alpha-\theta)\mathrm{e}^{-(\alpha-\theta)t}\mathrm{I(t)} = -\mathrm{De}^{-(\alpha-\theta)t} \qquad \frac{\mathrm{d}}{\mathrm{dt}}\left[\mathrm{e}^{-(\alpha-\theta)t}\mathrm{I(t)}\right] = -\mathrm{De}^{-(\alpha-\theta)t}$$

Integrate Both Sides:

$$\mathrm{e}^{-(\alpha-\theta)t}\mathrm{I(t)} = \frac{\mathrm{D}}{\alpha-\theta}\mathrm{e}^{-(\alpha-\theta)t} + \mathrm{C}$$

where C is the constant of integration.

$$\mathrm{I(t)} = \frac{\mathrm{D}}{\alpha-\theta} + \mathrm{Ce}^{(\alpha-\theta)t} \tag{1.11}$$

We assume that at $t = 0$, the inventory level is $I(0) = I_0$, we can find the constant C:

$$I(0) = \frac{D}{\alpha-\theta} + Ce^{0} \Rightarrow I_0 = \frac{D}{\alpha-\theta} + C \Rightarrow C = I_0 - \frac{D}{\alpha-\theta} \quad (1.12)$$

Substituting C back into the general solution (1.11):

$$I(t) = \frac{D}{\alpha-\theta} + \left(I_0 - \frac{D}{\alpha-\theta}\right) e^{(\alpha-\theta)t} \quad (1.13)$$

The first term, $\frac{D}{\alpha-\theta}$, represents the steady-state inventory level when $t \longrightarrow \infty$, assuming $\alpha > \theta$.

The second term, $\left(I_0 - \frac{D}{\alpha-\theta}\right) e^{(\alpha-\theta)t}$, represents the transient behavior of the inventory level, which decays or grows depending on the sign of $(\alpha - \theta)$.

2. Solution of Total Cost

The all-out cost capability, TC, is the amount of holding cost, ordering cost, and deterioration cost:

$TC = TC_h + TC_O + TC_d$

$$TC_h = \int_0^T C_h \left[\frac{D}{\alpha-\theta} + \left(I_0 - \frac{D}{\alpha-\theta}\right) e^{(\alpha-\theta)t}\right] dt$$

$$= C_h \left[\frac{D}{\alpha-\theta} T + \left(I_0 - \frac{D}{\alpha-\theta}\right) \frac{e^{(\alpha-\theta)T} - 1}{\alpha-\theta}\right]$$

The ordering cost per cycle is a proper expense caused each time a request is made. Assuming the stock cycle length is T, the quantity of orders per unit of time is $\frac{1}{T}$. In this manner, the ordering cost per unit of time is:

$TC_O = \frac{C_O}{T}$

The deterioration cost over a cycle of length T is given by:

$$TC_d = \int_0^T \theta C_d \, I(t) dt = \int_0^T C_d \left[\frac{D}{\alpha-\theta} + \left(I_0 - \frac{D}{\alpha-\theta}\right) e^{(\alpha-\theta)t}\right] dt$$

$$= C_d \theta \left[\frac{D}{\alpha-\theta} T + \left(I_0 - \frac{D}{\alpha-\theta}\right) \frac{e^{(\alpha-\theta)T} - 1}{\alpha-\theta}\right]$$

Combining the three components, the total cost per cycle is:

$$TC = \left[\frac{D}{\alpha - \theta} T + \left(I_0 - \frac{D}{\alpha - \theta}\right) \frac{e^{(\alpha-\theta)T} - 1}{\alpha - \theta}\right] (C_h + C_d\theta) + \frac{C_O}{T}$$

To find the ideal cycle length T that limits the complete expense, separate TC concerning T and settle for T:

$$\frac{d(TC)}{dt} = 0$$

$$\frac{d}{dt}\left[\left[\frac{D}{\alpha - \theta} T + \left(I_0 - \frac{D}{\alpha - \theta}\right) \frac{e^{(\alpha-\theta)T} - 1}{\alpha - \theta}\right] (C_h + C_d\theta) + \frac{C_O}{T}\right] = 0$$

$$\left[\frac{D}{\alpha - \theta} + \left(I_0 - \frac{D}{\alpha - \theta}\right) e^{(\alpha-\theta)T}\right] (C_h + C_d\theta) - \frac{C_O}{T^2} = 0$$

$$\left(I_0 - \frac{D}{\alpha - \theta}\right) e^{(\alpha-\theta)T} = \frac{\frac{C_O}{T^2} - (C_h + C_d\theta) \frac{D}{\alpha - \theta}}{C_h + C_d\theta}$$

$$e^{(\alpha-\theta)T} = \frac{\frac{C_O}{T^2} - (C_h + C_d\theta) \frac{D}{\alpha - \theta}}{(C_h + C_d\theta)\left(I_0 - \frac{D}{\alpha - \theta}\right)}$$

Solving for T requires taking the natural logarithm:

$$T = \frac{1}{(\alpha - \theta)} \ln \left(\frac{\frac{C_O}{T^2} - (C_h + C_d\theta) \frac{D}{\alpha - \theta}}{(C_h + C_d\theta)\left(I_0 - \frac{D}{\alpha - \theta}\right)} \right)$$

This is a transcendental equation in T, which means it cannot be solved explicitly in closed form. Instead, the solution for T must be found numerically.

2.1. Sensitivity Analysis

Consider a business managing a perishable item, such as a pharmaceutical product, with the following inventory management parameters:

- Replenishment Rate (α): 0.1 (replenishment rate proportionality constant)
- Deterioration Rate (θ): 0.05 (constant deterioration rate)
- Holding Cost (C_h): $2 per unit per unit of time
- Deterioration Cost (C_d): $1 per unit
- Ordering Cost (C_O): $50 per order
- Demand Rate (D): 10 units per unit of time
- Initial Inventory Level (I_0): 100 units

The cycle length T is considered in the range from 0.1 to 5. We vary each parameter by $+10\%$ and -10% to assess their impact on the total cost. The new total cost is computed for each scenario, and the percentage change in total cost compared to the base scenario is calculated. Table (1) presents the results of a sensitivity analysis conducted to evaluate how changes in key parameters affect the total cost in a deterministic inventory model for deteriorating products. The table displays variations of $+10\%$ and 10% for each parameter, including the replenishment rate (α), deterioration rate (θ), holding cost (C_h), deterioration cost (C_d), ordering cost (C_O), demand rate (D), and initial inventory level (I_0). For each variation, the table provides the base total cost, the new total cost after the variation, and the percentage change in the total cost. From the analysis, it is evident that the holding cost (C_h), ordering cost (C_O), and initial inventory level (I_0) have the most significant impact on the total cost, with variations leading to changes of up to ±10% in total cost. In contrast, parameters such as the replenishment rate (α) and deterioration rate (θ) show a much smaller impact, with changes in total cost around ±0.1%.

This sensitivity analysis provides valuable insights into which parameters are the most critical for managing and optimizing inventory costs in systems with deteriorating items, allowing decision-makers to focus on these factors to enhance cost efficiency. The graph (1) is a 3D surface plot showing the relationship between total cost (TC), replenishment rate constant (α), and cycle length (T) in a given system, possibly related to inventory or production management.

The x-axis represents the replenishment rate constant (α), ranging from 0.0 to 0.2, while the y-axis represents the cycle length (T), ranging from 0 to 5. The z-axis shows the total cost (T) , which ranges from 0 to over 1500. The surface plot reveals that as the cycle length (T) increases, the total cost (TC) tends to increase linearly. Similarly, as the replenishment rate constant (α) increases, there is a less steep increase in total cost (TC). The lowest costs are observed at lower values of both α and T, indicating a region where optimal replenishment rates and shorter cycle lengths minimize total costs. This graph can be useful for understanding how different combinations of α and T affect the total cost and for finding an optimal balance between these parameters.

Table 1. Sensitivity analysis of total cost for a deterministic inventory model

Parameter	Variation	Base Total Cost	New Total Cost	Change in Total Cost (%)
α	10%	201.2236808	201.4741892	0.124492505
α	-10%	201.2236808	200.9740011	-0.124080665
θ	10%	201.2236808	201.3423424	0.058969999
θ	-10%	201.2236808	201.1046163	-0.059170177
C_h	10%	201.2236808	210.8852036	4.801384594
C_h	-10%	201.2236808	191.099765	-5.031175109
C_d	10%	201.2236808	201.4675903	0.121213159
C_d	-10%	201.2236808	200.9797712	-0.121213159
C_o	10%	201.2236808	210.977458	4.847231318

C_o	-10%	201.2236808	190.9561738	-5.102534109
D	10%	201.2236808	200.9715025	-0.125322341
D	-10%	201.2236808	201.475859	0.125322341
Io	10%	201.2236808	211.3446719	5.029721687
Io	-10%	201.2236808	190.5657992	-5.296534417

3. Results and Discussion

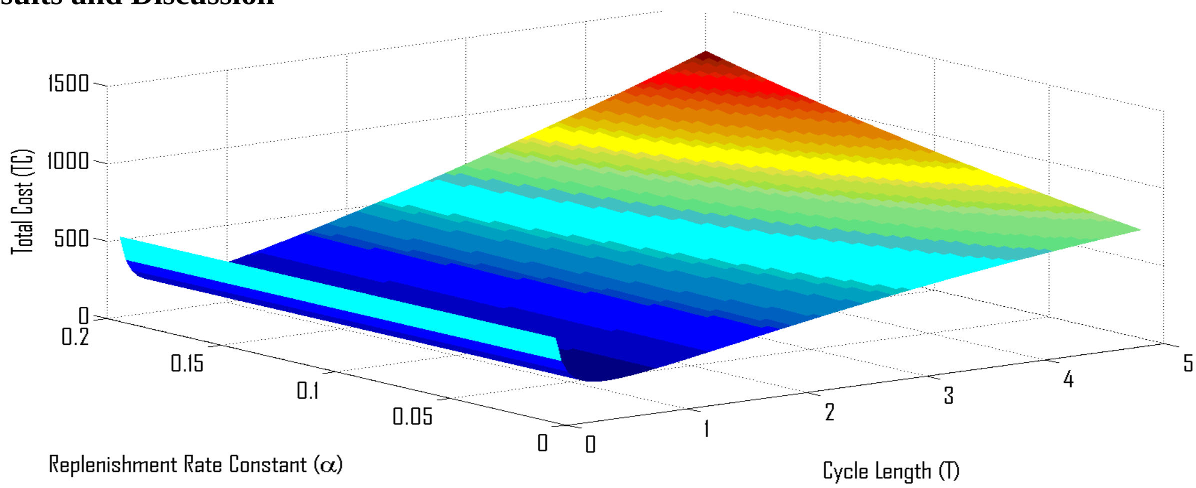

Fig. 1 3D Surface plot of total cost as a function of T and α

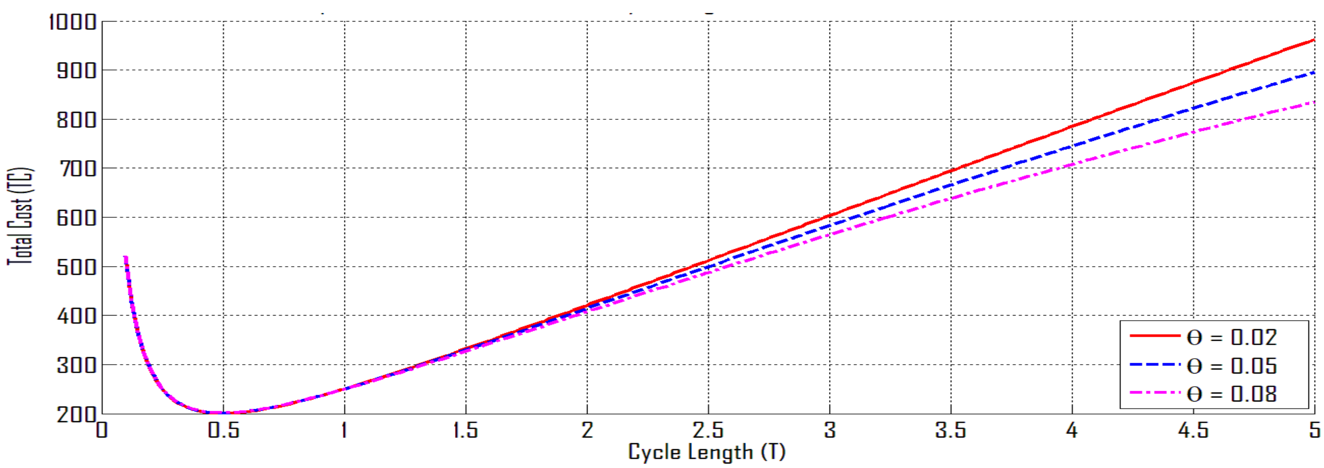

Fig. 2 2D Plot of total cost vs. cycle length (T) for different deterioration rates ()

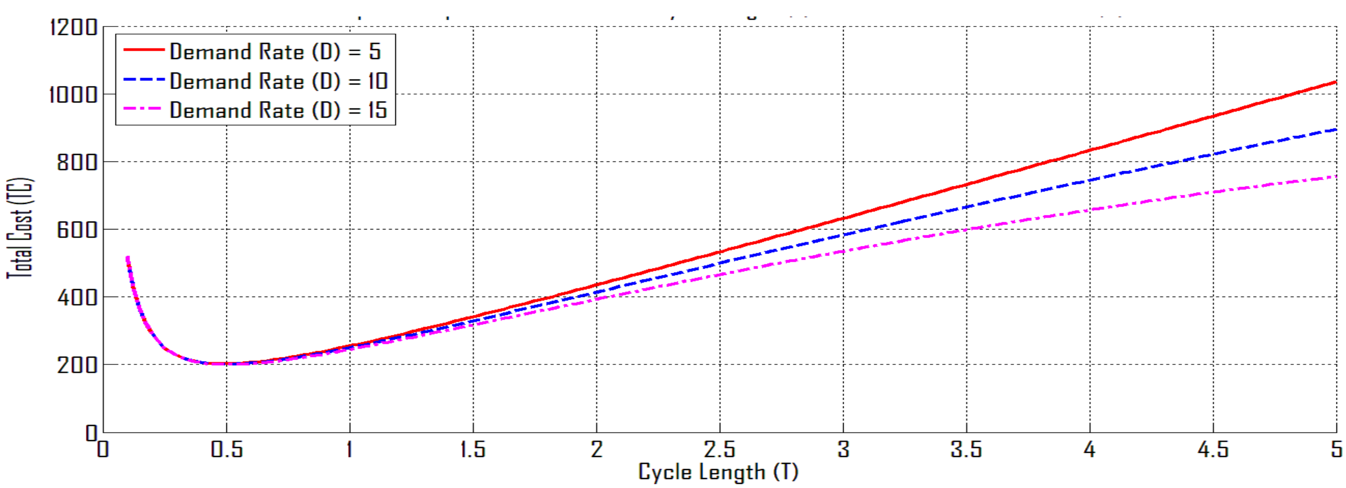

Fig. 3 2D Plot of total cost vs. cycle length (T) for different demand rates (D)

The graph (2) is a 2D plot delineating the connection between complete expense (TC) and cycle length (T) for various crumbling rates (θ) in a stock or creation framework. The x-pivot addresses the cycle length (T), going from 0 to 5, while the y-hub addresses the complete expense (TC), going from 200 to 1000. Three different lines are plotted, each representing a different deterioration rate: $\theta = 0.02$ (red solid line), $\theta = 0.05$ (blue dashed line), and $\theta = 0.08$ (magenta dotted line). According to the plot, the total cost initially decreases for all deterioration rates as cycle length increases, reaching a minimum before beginning to rise. The rate of increase in total cost becomes more pronounced at higher cycle lengths, and the total cost is consistently higher for greater deterioration rates. This graph indicates that lower deterioration rates (θ) result in lower total costs, particularly as the cycle length (T) increases. The plot suggests the importance of optimizing cycle length to minimize total costs, considering the impact of deterioration rates.

The graph (3) introduced is a 2D plot outlining the connection between the all-out cost (TC) and the cycle length (T) for various interest rates (D) in a stock or production network, the executives were setting. The total cost is plotted on the vertical axis, while the cycle length is on the horizontal axis. Three curves are depicted, each representing a different demand rate: $D = 5$ (red solid line), $D = 10$ (blue dashed line), and $D = 15$ (purple dotted line). As the cycle length increases, the total cost initially decreases, reaching a minimum point, and then starts to increase again. This pattern shows an ideal cycle length that limits the all-out cost for each request rate. The bends show that as the interest rate expands, the base all-out cost additionally increments, and the cycle length at which this base happens is marginally unique for each request rate. This proposes that the stock strategy should be changed in light of the interest rate to accomplish cost proficiency.

4. Concluding Remarks

This study presents a deterministic stock model custom-made for deteriorating product items, where the replenishment rate is unpredictably connected to the stock level. By incorporating this dynamic replenishment mechanism, the model effectively captures the complexities of real-world inventory systems where deterioration and replenishment rates are critical factors. The analysis shows that optimizing the replenishment cycle length can significantly reduce total inventory costs by balancing holding, ordering, and deterioration costs. Sensitivity analysis further highlights the impact of key parameters such as deterioration rate, demand rate, and cost components, providing valuable insights for decision-makers to fine-tune inventory policies. The findings underscore the importance of adopting more adaptive and responsive inventory management strategies, particularly in industries dealing with perishable or time-sensitive products. Overall, this model offers a comprehensive framework for developing cost-effective and efficient inventory control practices that respond dynamically to changes in inventory levels, thereby enhancing supply chain performance and profitability. Future research could explore extending the model to stochastic environments, incorporating more complex demand patterns, and considering multi-echelon supply chains to broaden its applicability and robustness.

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