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Original Article

Onset of Instability in Hadley-Prats Flow in an Anisotropic Porous Media with Viscous Dissipation

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Abstract *- The stability of a Hadley-type flow in a horizontal porous medium under an inclined temperature gradient is investigated when a fundamental horizontal mass flow (Prats flow) is present. Consequently, the Hadley-prats flow is the name given to the basic flow. We consider anisotropic thermal diffusivity and weak vertical heterogeneity in permeability. It is believed that viscous dissipation has a significant impact. The Rayleigh number associated with the vertical thermal gradient Ra is one of the eigenvalues. The other parameters include the thermal diffusivity* (ξ)*, the Gebhart number* (Ge) *for viscous dissipation, the horizontal Rayleigh number (Rah) for the horizontal through flow, and the parameter* λ *for changes in permeability. A linear stability analysis is executed, wherein the governing equations are numerically solved to determine the critical Rayleigh and wave numbers. The discussion pertains to longitudinal and transverse rolls. Longitudinal rolls are generally considered the most effective means of characterizing instability in most scenarios.*

Keywords *- Porous medium, Thermal diffusivity, Anisotropy, Permeability.*

1. Introduction

The study of convection in porous media has attracted significant attention due to its broad applications, including geothermal energy, sedimentary basin diagenesis, oil reservoir modeling, and thermal insulation. Traditional research has primarily focused on convective flows in isotropic porous layers, often emphasizing vertical or horizontal temperature gradients. However, actual porous media commonly exhibit anisotropy in thermal diffusivity and permeability, leading to complex flow behaviors that are not adequately represented in isotropic models. While inclined temperature gradients and throughflows have been explored, limited research has addressed the impact of viscous dissipation in anisotropic media despite its potential to affect flow stability substantially. Previous studies have investigated the linear stability conditions for convection in a horizontal porous layer, starting with Horton and Rogers [1] and later Lapwood [2], who examined the presence of a vertical thermal gradient within horizontal boundaries. Prats [3] extended this work by considering horizontal throughflow, noting that the Rayleigh number remains unchanged if the basic flow is zero. Recent research on inclined temperature gradients in horizontal porous media has focused on convective flow stability. A vertical temperature gradient generates the Hadley flow, as Weber [4] explored under perfectly conducting boundaries, where longitudinal rolls are the preferred mode, and the critical Rayleigh number is higher than in the classical Bénard problem. Nield [5] further examined convection from an inclined temperature gradient in a horizontal porous layer and conducted a linear stability analysis. Kaloni and Qiao [6] later determined the critical Rayleigh number through a nonlinear energy stability analysis. Additional studies by Nield et al. [7] investigated the effects of inclined thermal and solutal gradients in a horizontal porous layer. Manole et al. [8] examined horizontal mass flow's impact, identifying destabilizing effects due to net mass flux. Subsequent studies have explored convective flow under inclined thermal gradients and vertical throughflow, with findings indicating that vertical flow raises the critical Rayleigh number. Anisotropy, internal heat sources, and variable gravity are among the factors influencing flow in such media. Despite its importance, viscous dissipation has often been neglected in modeling efforts. Nield et al. [12] recently highlighted the role of viscous dissipation, with Barletta et al. [13] suggesting that dissipation can induce instability under specific conditions. Further, Barletta et al. [14] analyzed instability onset in Darcy-Forchheimer flow involving viscous dissipation, finding that the critical Rayleigh number for longitudinal rolls remains unaffected by the drag coefficient, though it varies for transverse rolls. In a related study, Barletta et al. [15] examined how vertical throughflow and dissipation impact convective instability, revealing that energy dissipation can stabilize or destabilize flow based on throughflow direction. Barletta and Nield analyzed inclined temperature gradients in horizontal throughflow [16], with Barletta and Celli examining dissipation effects in a local thermal equilibrium model [17]. Roy and Murthy [18] and Dubey and Murray [19] investigated convective instability and inclined thermal and solutal gradients, considering horizontal throughflow's effects. Convection in anisotropic porous media, commonly found in natural and industrial systems, poses unique challenges. In such media, fluid movement and transport properties vary with flow direction, as observed in materials like rock, sand, wood, sponge, and filter beds. Anisotropy, often due to grain orientation and geometry, affects fluid properties and flow, though it is challenging to characterize qualitatively [20].

Anisotropic porous media influence geophysical and energy-related processes, from sedimentary basin diagenesis to geothermal systems. Historical studies have furthered understanding in this field, such as Green and Freehill's [21] examination of marginal stability in porous layers with variable permeability and thermometric conductivity. Castinel and Combarnous [22] and Epherre [23] extended these analyses to cases with anisotropic thermal diffusivity. Research by McKibbin and Tyvand [25] bridged gaps between convection in layered and anisotropic porous media. This article investigates the linear stability of thermal convection with viscous dissipation in horizontal porous media, introducing thermal diffusivity anisotropy and slight vertical permeability variation. Prior studies have explored weakly heterogeneous porous media. Barletta and Nield [26] examined vertical heterogeneity's effect on Hadley flow stability, and Barletta and Rees [28] studied Hadley flow in an inclined porous layer with a varying temperature gradient along the horizontal axis. Further, Phan et al. [29] and Kamalika [30] analyzed the stability and flow behavior in porous media with anisotropic properties, considering the impact of viscous dissipation within the medium.

2. Mathematical Formulation

Several key assumptions are made to simplify the analysis:

- Darcy Flow Regime: The Darcy flow model assumes a low Reynolds number, where inertial effects are minimal. This is suitable for porous media with small pore sizes or for slow flows.
- Oberbeck-Boussinesq Approximation
- Local Thermal Equilibrium: It is assumed that the temperature of the solid and fluid phases in the porous medium is equal, a valid approximation in highly conductive porous media.
- Anisotropic Permeability and Thermal Diffusivity: Permeability and thermal diffusivity vary with direction but are assumed to remain constant within each direction. This simplification facilitates the application of the linear stability analysis without introducing further spatial dependencies in these properties.
- Anisotropic thermal diffusivity

3. Governing Equations

Consider a horizontal porous width layer H. For fluid flow in porous media, the Darcy model is used. The porous medium's solid and fluid are in local thermal equilibrium. The linear approximation of Oberbeck-Boussinesq is used, yielding the continuity equation, momentum balance equation, and energy balance equation.

$$\bar{\nabla} \cdot \bar{V} = 0, \tag{1}$$

$$\frac{\mu}{K(z)}\bar{V} = -\bar{\nabla}\bar{P} - \rho_0 \beta g(\bar{T} - \bar{T}_0)\hat{K}, \tag{2}$$

$$M\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{V} \cdot \bar{\nabla}\bar{T} = k_h \bar{\nabla}_1^2 \bar{T} + k_v \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + \frac{\mu}{(c_p \rho_0)_f K(z)} \bar{V} \cdot \bar{V}. \tag{3}$$

Boundary conditions

$$\bar{T} = \bar{T}_0 + \bar{\Delta}\bar{T} - \bar{\eta}\bar{x} at z = 0;$$

$$\bar{T} = \bar{T}_0 - \bar{\eta}\bar{x}, at z = H,$$

where a "bar" is employed as a symbol to denote dimensional quantities. The vector $v = (u, v, w)$ represents the velocity of an object. g denotes the gravitational acceleration, μ stands for the dynamic viscosity, p represents the pressure, ρ is the density of the fluid, c represents the specific heat, T_0 is the reference temperature, and $\bar{\eta}$ denotes the uniform horizontal temperature gradient. K(z) is a function that depends on z.

4. Non-Dimensionalization

Next, the governing equations Equations 1-3 are nondimensionalized using the following non-dimensional variables:

$$(x, y, z) = \frac{1}{H}(\bar{x}, \bar{y}, \bar{z}),\ t = \frac{K_v}{MH^2}\bar{t}, \quad V = \frac{H}{K_v}\bar{V},\ P = \frac{\langle K \rangle}{\mu K_r}\bar{P},\ T = \frac{(\bar{T} - \bar{T}_0)}{\Delta\bar{T}}, K(z) = \frac{\bar{K}(\bar{z})}{K_0},$$

The above scaling leads to the following non-dimensional parameters:

$$Pe = \frac{u_0 H}{k_v}, \quad Rz = \frac{\rho_0 g\beta\langle K\rangle H\bar{\Delta}\bar{T}}{\mu k_v}, \; Rx = \frac{\rho_0 g\beta\langle K\rangle H^2\bar{\eta}}{\mu k_v}, \; \xi = \frac{k_h}{k_v}.$$

Where $'Pe'$ is the horizontal peclet number, Rz, Rx are vertical and horizontal Rayleigh Numbers, and ξ is the horizontal and vertical thermal diffusivity ratio. The non-dimensional governing equations are

$$\nabla \cdot V = 0 \tag{4}$$

$$\frac{V}{K(z)} = \nabla P - R2T\hat{K} \tag{5}$$

$$\frac{\partial T}{\partial t} + V \cdot \nabla T = \xi \nabla_1^2 T + \frac{\partial^2 T}{\partial z^2} + \frac{Ge}{12(z)Rz} V \cdot V \tag{6}$$

subjected to the boundary condition

$$W = 0T = 1 - \frac{Rx}{Rz} xatZ = 0$$

$$W = 0T = -\frac{Rx}{Rz} xatZ = 1$$

Taking curl in Eqn. 5.

$$\nabla \times \left(\frac{V}{K(z)}\right) = Rz\nabla \times (T\hat{K})$$

Rz represents the vertical thermal Rayleigh number. In contrast, Rx represents the horizontal Rayleigh number associated with the horizontal thermal gradient: The Gebhart number, Ge, quantifies the impact of viscous dissipation. The average thermal conductivity and permeability in the vertical direction, $\langle k\rangle$ and $\langle K\rangle$, respectively, are defined for $\bar{z}\epsilon[0, H]$. Additionally, α represents the average thermal diffusivity and is calculated as the ratio of $\langle k\rangle$ to ρc_f. Furthermore, based on the definition of k(z), it can be inferred that

$$\int_0^1 K(z)dz = 1$$

5. Basic State Solution

A throughflow in the horizontal direction and the inclined temperature gradient also induce a basic flow. Let us assume that the basic flow is purely horizontal, such that $V_B = (u_B, 0,0)$ and $\frac{dT_B}{dz} = -\frac{Rx}{Rz}$. Then, the basic velocity profiles are given by

$$u_B(z) = (Pe + Rx(z - d))K(z),$$

$$where \qquad d = \int_0^1 zK(z)dz,$$

and

$$\tilde{T}_B(z) = -\int_0^z \left(\int_0^\psi K(z)\left(\tilde{u}_B \frac{Rx}{Rz} + \frac{Ge}{Rz}\tilde{u}_B^2\right)dz\right)d\psi + C_1 z + C_2$$

using the boundary condition

$$\tilde{T}_B(0) = 1, \tilde{T}_B(1) = 0, C_2 = 1 \text{and}$$

$$C_1 = -1 + \int_0^1 \left(\int_0^\psi K(z)\left(\tilde{u}_B \frac{Rx}{Rz} + \frac{Ge}{Rz}\tilde{u}_B^2\right)dz\right)d\psi.$$

6. Linear Perturbation

Small, infinitesimal disturbances perturb the basic flow as

$$V = V_B + \epsilon U$$

$$T = T_B + \epsilon\theta,$$

Where ϵ is a small perturbation parameter and $U = (U, V, W)$ is the disturbance velocity, and θ represents the disturbance temperature field. For linear stability analysis, the second and higher-order terms of ϵ are discarded. The linearized governing equations are then written as,

$$\nabla \cdot U = 0, \tag{7}$$

$$(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) = -RzK(z)\frac{d\theta}{dx}, \tag{8}$$

$$\frac{\partial \theta}{\partial t} + V_B \cdot \nabla\theta + U \cdot \nabla T_B = \xi\nabla_1^2\theta + \frac{\partial^2\theta}{\partial z^2} + \frac{2Ge}{K(z)Rz}V_B u \tag{9}$$

gives the boundary conditions

$$W = \theta = 0atz = 0,1 \tag{10}$$

In the following two-dimensional plane, a wave having an axis either in the direction of the basic flow (x-direction) or perpendicular to the basic flow will be discussed.

7. Transverse Rolls

The basic flow is in the horizontal (x) direction. Let's look at the transverse rolls with axes parallel to the main flow. Therefore

$$U = U(x, z, t)V = 0W = W(x, z, t) \tag{11}$$

Now, the problem can be treated as a two-dimensional problem. Thus, the stream function-temperature formulation introduces a stream function ψ. Then,

$$U = \frac{\partial\psi}{\partial z}, W = -\frac{\partial\psi}{\partial x} \tag{12}$$

On account of Equations.11 and 12, Equations. 7 and 8 are identically satisfied, while 9 and 10 are rewritten as

$$\nabla^2\psi + RzK(z)\frac{\partial\theta}{\partial x} = 0 \tag{13}$$

$$\frac{\partial\theta}{\partial t} + V_B\frac{\partial\theta}{\partial x} - \frac{Rx}{Rz}\frac{\partial\psi}{\partial z} - \frac{\partial\psi}{\partial x}\frac{\partial T_B}{\partial z} = \xi\nabla_1^2\theta + \frac{\partial^2\theta}{\partial z^2} + \frac{2Ge}{K(z)Rz}u_B\frac{\partial\psi}{\partial z} \tag{14}$$

The solutions of Equations. 13 and 14 are sought to have the form of a plane wave

$$\psi(x, z, t) = R\{if(z)e^{i(ax-\sigma t)}\} \tag{15}$$

$$\theta(x, z, t) = R\{h(z)e^{i(ax-\sigma t)}\} \tag{16}$$

Where R stands for the real part of a complex function, 'a' is real wave number $\sigma = \sigma_r + i\sigma_i$ is a complex exponential coefficient,

Whenever $\sigma_i > 0$, we have an exponentially growing disturbance that means instability, while $\sigma_i < 0$ implies an exponentially damped disturbance. i.e. stability. We will be interested in the threshold condition of marginal stability so that hereafter we will set $\sigma_i = 0$. Then Equation. 13 implies

$$f''(z) - a^2 f(z) + aRzK(z)h(z) = 0, \tag{17}$$

$$h''(z) - \xi a^2 h(z) + i(\Gamma - au_B)h(z) + 2i\frac{Ge}{K(z)Rz}(\frac{K(z)Rx}{2Ge} + u_B)f - a\frac{dT_B}{dz}f(z) = 0,$$

Let

$$\gamma = \sigma_r - aPe$$

$$f(z) = u_B - Pe$$

$$h'' - \xi a^2 h(z) + i[Y - aF(z)]h(z) + 2i\frac{Ge}{K(z)Rz}\left[\frac{K(z)Rx}{2Ge} + Pe + f(z)\right]f'(z) - a\frac{dT_B}{dz}f = 0 \quad (18)$$

8. Longitudinal Rolls

The axes of the longitudinal rolls are parallel to the basic flow in the x direction. Thus, the streamlines lie in planes where $x =$ constant and the velocity and temperature fields become independent of the x −coordinate. Therefore,

$$U = 0, V = V(y, z, t), W = W(y, z, t), \theta = \theta(y, z, t) \; (19)$$

As the problem can now be treated as a two-dimensional problem, stream function-temperature formulation is used again. Then

$$V = \frac{\partial\psi}{\partial z} W = -\frac{\partial\psi}{\partial y} \qquad (20)$$

Following the same argument as in the case of transverse rolls, the stream function-temperature formulation is given by

$$\nabla^2\psi + RzK(z)\frac{\partial\theta}{\partial y} = 0$$

$$\frac{\partial\theta}{\partial t} - \frac{\partial\psi}{\partial y}\frac{dT_B}{dz} = \xi\nabla_1^2\theta + \frac{\partial^2\theta}{\partial z^2}$$

The solution in the form of plane waves is assumed as

$$\psi(y, z, t) = R\{if(z)e^{i(ay - \sigma t)}\}$$

$$\theta = R\{h(z)e^{i}(ay - \sigma t)\} \; (21)$$

For longitudinal rolls, $\sigma = 0$ is considered to make the problem self-adjoint. The final set of governing equations and boundary conditions for transverse rolls are given by

$$f'' - a^2 f + RzK(z)ah = 0$$

$$h'' - a^2\xi h - af\frac{dT_B}{dz} = 0$$

Boundary condition

$$f = h = 0 \quad at \quad z = 0\,,1$$

9. Results And Discussion

Table 1. Longitudinal rolls: comparison between the results obtained by Kaloni and Qiao and present result for $Pe = 0, Ge = 0$

Ra_h	Kaloni and Qiao		Present results	
	Ra_cr	**a_cr**	**Ra_cr**	**a_cr**
0	39.4784	3.1416	39.4784	3.1416
10	42.0076	3.1418	42.0076	3.1418
20	49.5486	3.1457	49.5486	3.1457
30	61.9567	3.1634	61.9566	3.1633
40	78.9664	3.2152	78.9664	3.2153
50	100.117	3.3445	100.117	3.3445
60	124.473	3.6721	124.473	3.6721
70	149.186	4.6712	149.186	4.6712
80	164.371	6.5312	164.346	6.5332
90	160.999	7.7312	160.999	7.7312

Table 2. Transverse rolls: comparison between the results obtained by Barletta and Nield with the present result for $Rah = 10, Ge = 0.1$

Pe	Barletta and Nield		Present results	
	Ra	**a**	**Ra**	**a**
0	43.9314	3.0975	43.9314	3.0974
10	42.3337	3.2229	42.3337	3.2229
20	34.3776	3.7027	34.3776	3.7027

30	16.4844	4.4085	16.4844	4.4085
-30	34.5316	3.5465	34.5317	3.5465
-20	38.4432	3.3602	38.4432	3.3601
-10	42.0463	3.1793	42.0463	3.1793

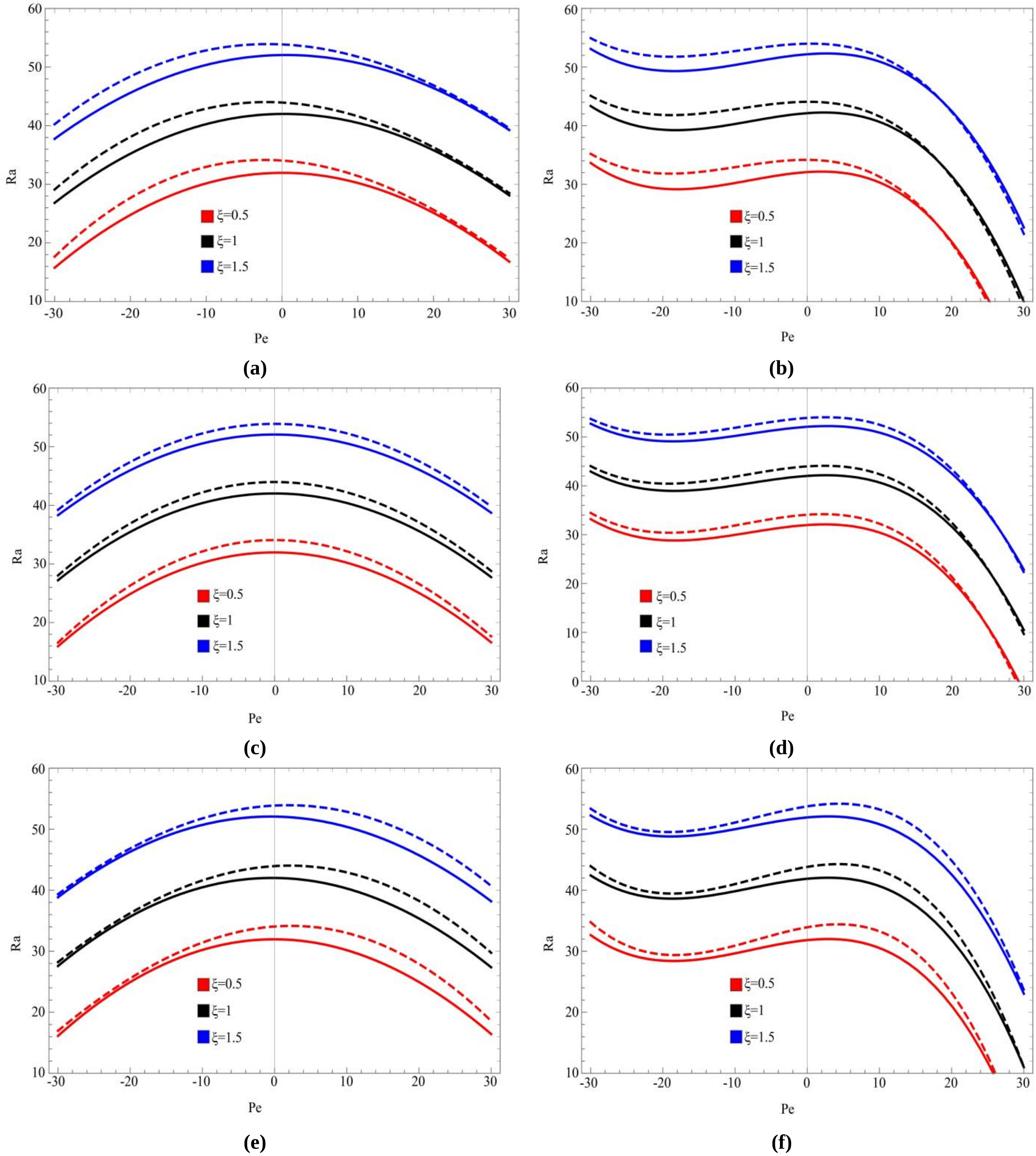
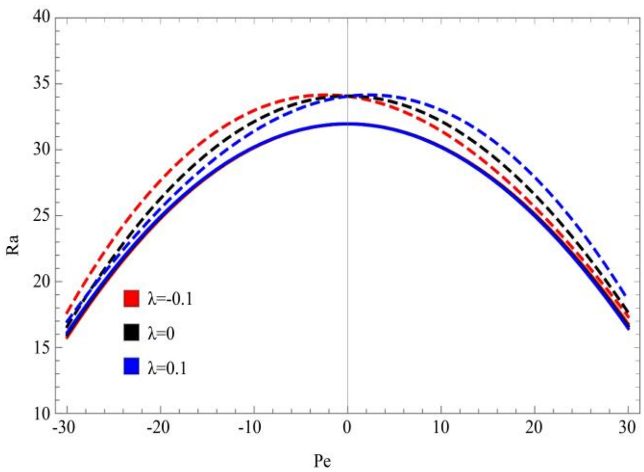


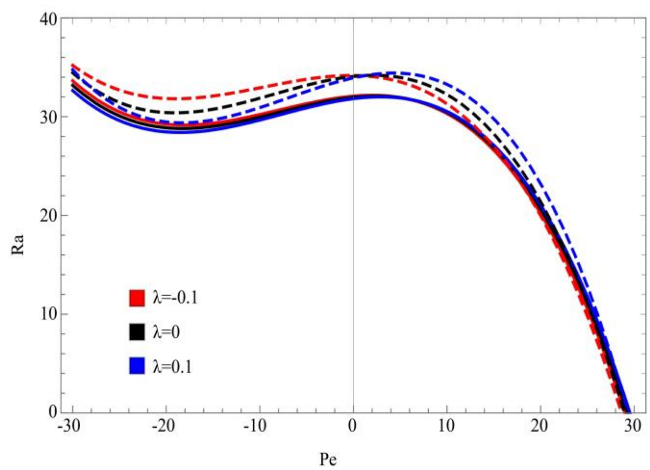
Fig. 1 Ra vs Pe for different values of ξ when Rah=10,(a)Ge=0,$\lambda = -0.1$, (b) Ge =0.2, $\lambda = -0.1$, (c) Ge =0, $\lambda = 0$, (d) Ge = 0.2,$\lambda = 0$ (e) Ge = 0, $\lambda = 0.1$ (f) Ge = 0.2, $\lambda = 0.1$

Figure 1 depicts the variation in the critical Rayleigh number (Ra) for both positive and negative Peclet (Pe) numbers, with several values of the Gebhart number (Ge) assessed. When Ge = 0, no viscous dissipation is present, whereas Ge = 0.2 represents a significant viscous dissipation effect. The system is characterized by a horizontal temperature gradient and Rah = 10, considering both longitudinal and transverse rolls. As Barletta and Nield [27] report, the Ra versus Pe curves are symmetric with respect to the sign of the Peclet number when Ge = 0 and $\lambda = 0$. In Figures 1(a) and 1(b), with $\gamma = -0.1$ and assuming a non zero ξ, it is observed that an increase in ξ leads to a rise in the critical Rayleigh number for both longitudinal and transverse rolls. This suggests that ξ contributes to the stabilization of both types of motion. Transverse rolls show greater stability than longitudinal rolls when $Pe \neq 20$.

Figure 1(b) further explores the role of viscous dissipation, showing that the variation in the vertical Rayleigh number due to a non-zero Ge is smaller than when Ge = 0. In the presence of viscous dissipation, the influence of anisotropy is notably reduced. Barletta et al. have also reported similar phenomena. When Pe < 15, longitudinal rolls are less stable than transverse rolls, whereas when Pe > 15, the reverse holds. For Ge = 0.2, the stabilizing effect of the anisotropic parameter ξ remains unchanged, and only longitudinal rolls are considered for instability onset. Figures 1(c) and 1(d) examine the effect of permeability in conductivity with $\lambda = 0$. When Ge = 0, as opposed to the case where $\lambda = -0.1$ and $\xi \neq 0$, the critical Rayleigh number changes more significantly for both types of rolls. Figures 1(a) and 1(b) show that the trends for longitudinal and transverse rolls are consistent. Increasing permeability in the upward direction stabilizes both roll types. Figure 1(d) indicates that permeability also reduces the critical Rayleigh number in the presence of viscous dissipation. Additionally, Figures 1(c) and 1(d) reveal that when Pe < 0, the difference between the critical Rayleigh numbers of longitudinal and transverse rolls decreases, whereas when Pe > 0, this difference increases. Figures 1(e) and 1(f) demonstrate that the effects of permeability and anisotropy are most pronounced when both λ and ξ are non-zero. As observed in Figures 1(c) and 1(d), Figures 1(e) and 1(f) exhibit similar trends, with the critical Rayleigh number increasing as permeability in the upward direction rises.



(a)



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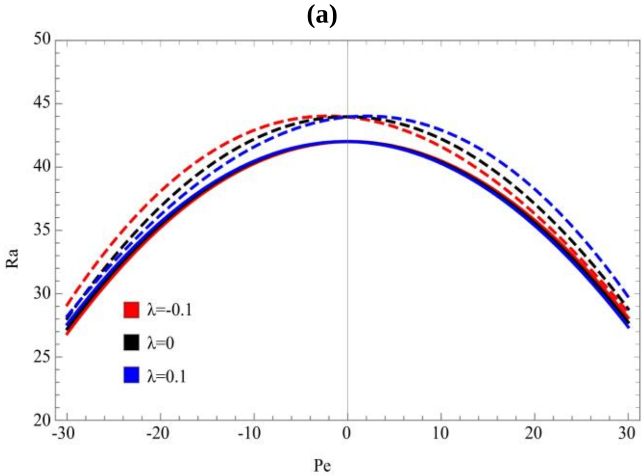

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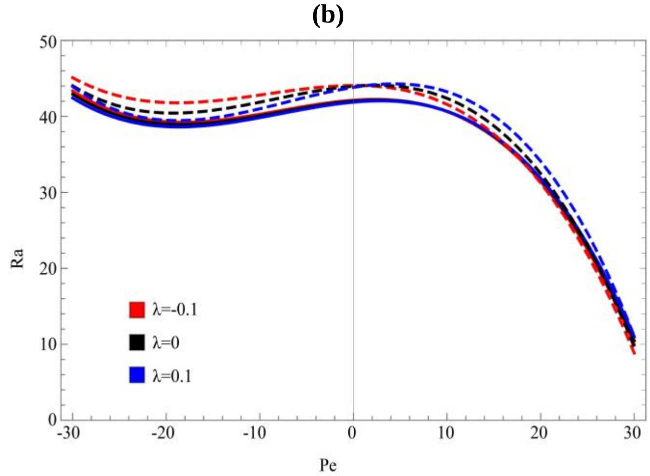

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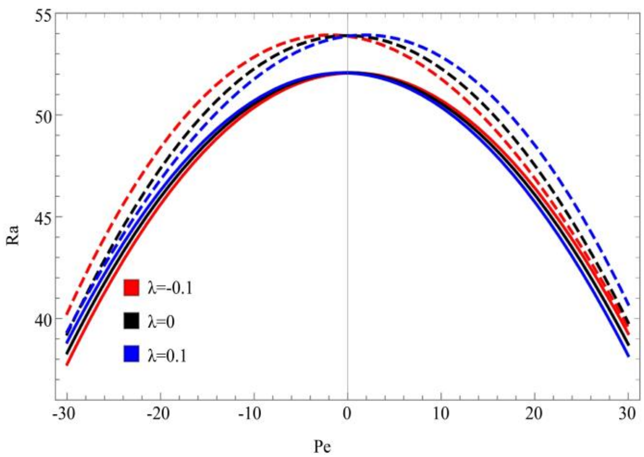
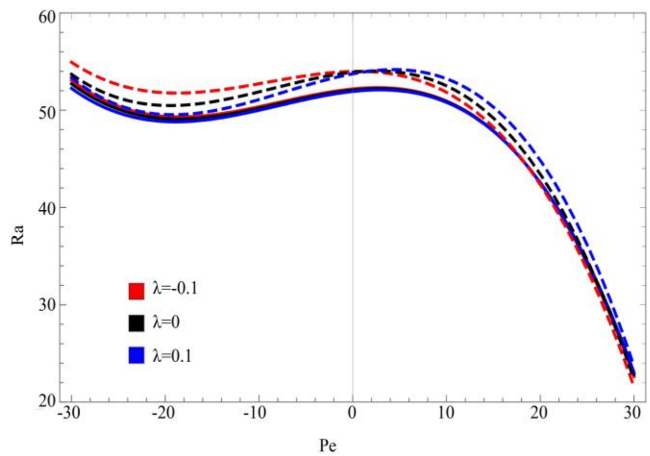


(e)

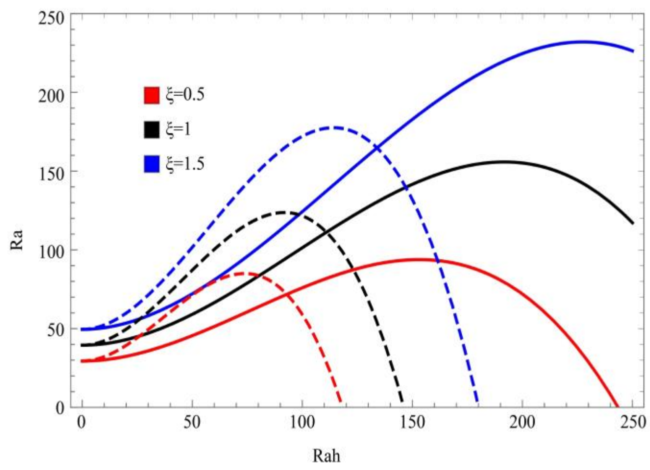


(f)

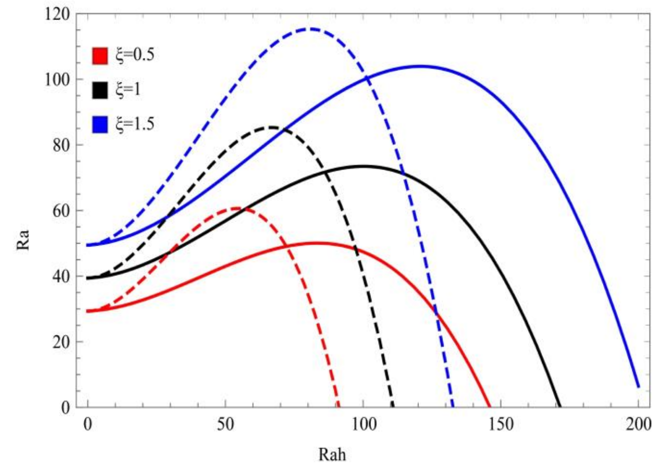
Fig. 2 Ra vs Pe when Rah = 10,(a)Ge=0, $\xi = 0.5$ (b) Ge=0.2,$\xi = 0.5$, (c) Ge =0, $\xi = 1$ (d) Ge = 0.2, $\xi = 1$(e) Ge = 0, $\xi = 1.5$ (f) Ge = 0.2, $\xi = 1.5$

The variance of Rayleigh number (Ra) for both positive and negative Peclet numbers (Pe) is illustrated in Figure 2. The analysis includes various Gebhart numbers, where Ge = 0 signifies no viscous dissipation, and Ge = 0.2 emphasizes the role of dissipation. A value of Rah = 10 represents a horizontal temperature gradient. Both transverse and longitudinal rolls are considered in the study. According to Barletta and Nield, when Ge = 0 and $\gamma \neq 0$, the Ra vs. Pe curves exhibit symmetry around the sign of Pe. However, anisotropy causes the perfect symmetry seen in homogeneous porous media to break down, even if the shape of the curve for Ge = 0 remains the same for both media types. In Figures 2(a) and 2(b), $\gamma = 0$ is interpreted as non-zero. When Pe < 0, the critical Rayleigh number for longitudinal rolls decreases, while Ra for transverse rolls increases as permeability decreases in the upward direction ($\gamma < 0$). This indicates that transverse rolls are stabilized when $\gamma < 0$, while longitudinal rolls become destabilized for Pe < 0. Conversely, with increased permeability in the upward direction ($\gamma > 0$), Ra for longitudinal rolls rises, and Ra for transverse rolls decreases when Pe < 0. For Pe > 0, as permeability increases in the vertical direction ($\gamma > 0$), longitudinal rolls destabilize, while transverse rolls stabilize. If $\gamma < 0$ for Pe > 0, longitudinal rolls become more stable, and transverse rolls become more unstable. Previous work by Nield and Kuznetsov showed that $\gamma > 0$ destabilizes the flow, and a decrease in permeability in the upward direction stabilizes it. In contrast, this study demonstrates that increasing permeability in the upward direction ($\gamma > 0$) can stabilize transverse rolls under throughflow conditions (Pe > 0), while longitudinal rolls stabilize when throughflow is opposite (Pe < 0).

A reduction in permeability in the upward direction can destabilize the transverse rolls when Pe > 0 and the longitudinal rolls when Pe < 0 due to the differing propagation directions of the rolls. Figure 3(b) considers the impact of viscous dissipation. For non-zero Ge, the effect of permeability heterogeneity is most prominent on transverse rolls when Pe < 0, with this influence diminishing as Pe becomes positive. The impact of γ on longitudinal rolls is minimal for Ge = 0.2 across both positive and negative Pe values. Heterogeneity's stabilizing and destabilizing effects are unchanged from the case of Ge = 0. Longitudinal rolls are favored for the onset of instability, except under strong positive throughflow conditions and $\gamma < 0$. Figures 2(c) and 2(d) focus on homogeneous thermal diffusivity with $\gamma \neq 0$. When Ge = 0, a significant shift in the critical Rayleigh number is noted for both longitudinal and transverse rolls. For Pe > 0, an increase in thermal diffusivity in the horizontal direction stabilizes both types of rolls, while for Pe < 0, it destabilizes them. A higher vertical temperature difference is needed to initiate convection when thermal diffusivity increases, as heat is transferred more quickly, stabilizing the flow and increasing the vertical Rayleigh number. In contrast, lower thermal diffusivity stabilizes the flow for Pe < 0 and destabilizes it for Pe > 0. As shown in Figure 2(d), when viscous dissipation is present, the effect of anisotropic thermal diffusivity is diminished. The critical Rayleigh numbers of transverse and longitudinal rolls show a decreasing difference as Pe increases, with longitudinal rolls being the preferred mode of instability. Finally, the influence of anisotropic properties ($\xi \neq 1$) is more pronounced, as seen in Figures 2(e) and 2(f), where a significantly larger change in Ra occurs for transverse rolls compared to longitudinal rolls. For higher Pe values and non-zero Ge, transverse rolls become the most unstable when γ is negative. This section examines the variation of the crucial vertical Rayleigh number with Ra_h in vertical throughflow for different ξ values. As shown in Figure 3(a), the vertical Rayleigh number R_a initially increases with Ra_h, then decreases when Ge=0, indicating no viscous dissipation. The figure further reveals that when $\xi \geq 1$, the longitudinal roll exhibits greater stability compared to the transverse roll, while the transverse roll is more stable when ξ=0.5. Figure 3(b) takes into account the effect of viscous dissipation, showing a reduction in the vertical Rayleigh number criticality. The trends observed in Figure 3(b) mirror those in Figure 3(a). Additionally, it is apparent that, for all ξ values, the transverse roll is more stable than the longitudinal roll.

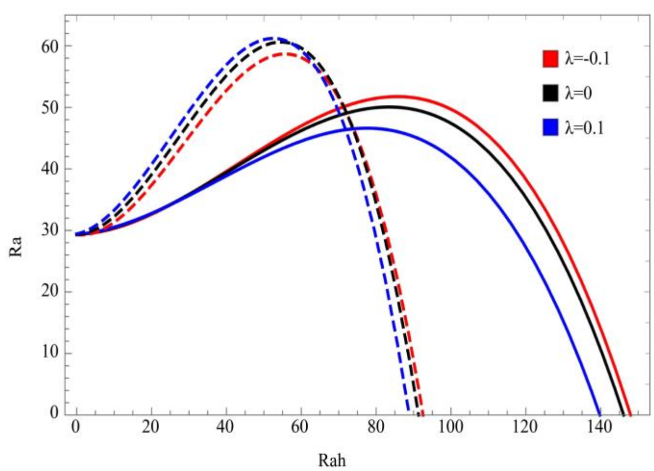


(a)

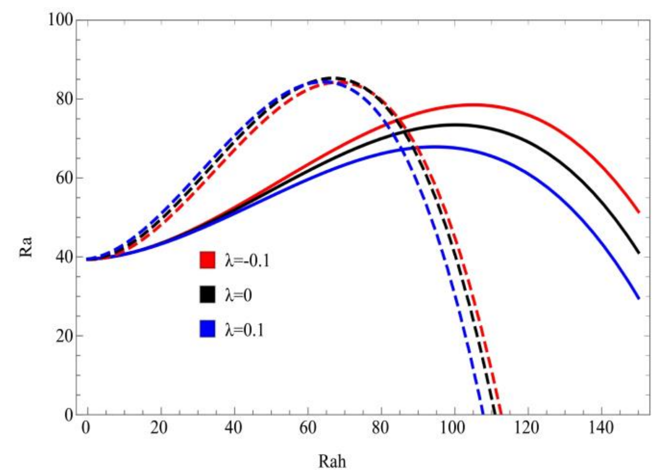


(b)

Fig. 3 Ra vs Rah, for different values of ξ, when Pe = 10, (a) Ge = 0, (b) Ge = 0.2



(a)



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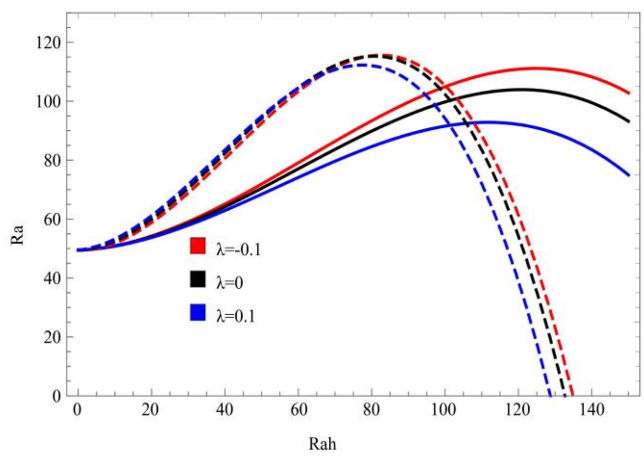

(c)

Fig. 4 Ra vs Rah for different values of λ when Pe=10, Ge=0.2,$(a)\xi = 0.5$, (b) $\xi = 1$and (c) $\xi = 1.5$

The variations between the horizontal thermal Rayleigh number, Ra_h, and the vertical thermal Rayleigh number, Ra, for different values of the permeability parameter λ, with Ge=0.2 and Pe=10, are presented in Figures 4(a), 4(b), and 4(c). In Figure 4(a) for ξ=0.5, the critical Rayleigh number for the longitudinal roll decreases within the range of Ra_h. In contrast, for the transverse rolls, the vertical critical Rayleigh number increases when Ra_h reaches 70 after Ra begins to decrease. This indicates that the transverse roll exhibits greater stability than the longitudinal roll, as shown in Figure 4(a). Figures 4(a), 4(b) (with $\xi=1$), and 4(c) (with $\xi=1.5$) display similar trends. Additionally, it is evident that as ξ increases, the critical values of the vertical Rayleigh number also rise. This behavior can be attributed to the increased thermal diffusivity along the horizontal axis, which reduces the rate of heat diffusion within the fluid, thereby promoting a more stable and less mobile state. Consequently, this demonstrates that the system is stabilized by ξ. All the graphs have been generated using Mathematica software.

10. Conclusion

The impact of anisotropy on Hadley flow within a horizontal porous layer is examined, taking into account substantial viscous dissipation and uniform horizontal mass flow. A constant level of anisotropy is assumed, with the Rayleigh number linked to the vertical temperature gradient serving as the eigenvalue. The analysis involves small-amplitude perturbations and a linear stability approach. The critical Rayleigh number (Ra) is determined, and its variation with other parameters such as Pe, Ge, λ, and Rah is explored. The following conclusions are drawn:

- An increase in horizontal thermal diffusivity stabilizes both transverse and longitudinal rolls across the entire Pe range when throughflow is present.

- A rise in permeability acts as a destabilizing factor for transverse rolls and a stabilizing factor for longitudinal rolls when Pe < 0 in the presence of throughflow.

- For Pe > 0, an increase in permeability serves as a stabilizing factor for transverse rolls but destabilizes longitudinal rolls.

- Permeability heterogeneity has a more significant impact on transverse rolls than on longitudinal rolls, while anisotropic thermal diffusivity affects both types of rolls similarly when uniform basic mass flow is considered in the horizontal direction.

- Numerical simulations show that varying ξ does not significantly influence the results in the presence of viscous dissipation, indicating that heterogeneity and anisotropy are not prominent when Ge exceeds zero, and strong throughflow is present.

- The effects of heterogeneity and anisotropy in thermal diffusivity become more pronounced when both are considered simultaneously.

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