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Original Article

An Improved Class of Mixed Ratio Type Estimators for Population Mean of a Finite Population

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Abstract - *This article explains a class of improved ratio and dual-to-ratio type estimators for estimating finite population mean. We have assumed that the study variable y highly correlates with the auxiliary variable x. The properties of the proposed class of estimators have been obtained theoretically, and the conditions under which the proposed class of estimators is more efficient than the other competing estimators are discussed. The empirical study has been carried out by considering some natural populations in the literature.*

Keywords - *Auxiliary variable, Study variable, Mean square error, Percent relative efficiency.*

1. Introduction

In survey sampling, using auxiliary information plays an important role in increasing the precision of the parameters. The auxiliary information is designed to be used in more efficient estimators. For this, the auxiliary information has been used at several stages, like the pre-selection stage, selection stage, or even post-selection stage, or several simultaneously. Consider a finite population having N distinct and identifiable units and y_i is the value of the study variable for the ith unit of the population($i = 1,2,3 \ldots\ldots N$) with an unknown population mean $\overline{Y}$. Let $\overline{X}$ and $\overline{Y}$ be the population means of the auxiliary and study variable, respectively; S_x^2 and S_y^2 be the population variances of x and y respectively. Let $C_x = \frac{S_x}{\overline{X}}$, $C_y = \frac{S_y}{\overline{Y}}$ be the population coefficient of variation of the auxiliary variable x and y, ρ_{yx} be the correlation coefficient between y and x, respectively. In order to estimate$\overline{Y}$, we select a sample of size n units from the population using simple random sampling without a replacement scheme. Let $\overline{x}$ and $\overline{y}$ be the corresponding sample means of auxiliary and study variable, respectively.

2. A Review of Different Existing Estimators

When no auxiliary information is available, we use a simple mean estimator as

$$t_0 = \overline{y}, \tag{1}$$

and the variance is given as

$$V(t_0) = \overline{Y}^2 \theta C_y^2. \tag{2}$$

Cochran (1940) initially propounded a ratio estimator for the finite population mean when the population mean of the auxiliary variable is available before conducting the survey. The ratio estimator can be defined as

$$t_1 = \frac{\overline{y}}{\overline{x}} \overline{X}, \tag{3}$$

The bias and mean square error is given as

$$B(t_1) = \overline{Y}\theta\left(C_x^2 - \rho C_x C_y\right), \tag{4}$$

$$MSE(t_1) = \overline{Y}^2 \theta\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right). \tag{5}$$

Robson (1957) and Murthy (1964) proposed a product estimator for the population mean, and it is defined as

$$t_2 = \bar{y}\frac{\bar{x}}{\bar{X}}. \tag{6}$$

and the bias and mean square error is given as

$$B(t_2) = \bar{Y}\theta\rho C_y C_x, \tag{7}$$

$$MSE(t_2) = \bar{Y}^2\theta\left(C_y^2 + C_x^2 + 2\rho C_y C_x\right). \tag{8}$$

Using the linear transformation of an auxiliary variable, Srivenkataramana (1980) suggested a ratio estimator for $\bar{Y}$, and the estimator is defined as,

$$t_3 = \bar{y}\frac{\bar{x}^*}{\bar{X}}, \tag{9}$$

Where $\bar{x}^* = (1+g)\bar{X} - g\bar{x}$, $g = \frac{n}{N-n}$, and the bias and mean square error is given as

$$B\ (t_3) = -\bar{Y}\theta g\rho C_y C_x, \tag{10}$$

$$MSE\ (t_3) = \bar{Y}^2\theta\left(C_y^2 + g^2 C_x^2 - g\rho C_y C_x\right). \tag{11}$$

Bandopadhyay (1980) proposed a product estimator for the population mean $\bar{Y}$ and it is defined as

$$t_4 = \bar{y}\frac{\bar{X}}{\bar{x}^*}, \tag{12}$$

and the bias and mean square error is given as

$$B(t_4) = \bar{Y}\theta g[gC_x^2 + \rho C_y C_x], \tag{13}$$

$$MSE(t_4) = \bar{Y}^2\theta\left[C_y^2 + g^2 C_x^2 + g\rho C_y C_x\right]. \tag{14}$$

Bahl and Tuteja (1991) proposed usual exponential-ratio estimators and product estimators for population mean $\bar{Y}$ is given by

$$t_5 = \bar{y}\ exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{15}$$

$$t_6 = \bar{y}\ exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right) \tag{16}$$

and the bias and mean square error are given as

$$B\ (t_5) = \bar{Y}\theta\left(\frac{3}{8}C_x^2 - \frac{1}{2}\rho C_y C_x\right), \tag{17}$$

$$MSE\ (t_5) = \bar{Y}^2\theta\left(C_y^2 + \frac{1}{4}C_x^2 - \rho C_y C_x\right), \tag{18}$$

$$B\ (t_6) = \bar{Y}\theta\left(\frac{\rho C_x C_y}{2} - \frac{C_x^2}{8}\right), \tag{19}$$

$$MSE\ (t_6) = \bar{Y}^2\theta\left(C_y^2 + \frac{1}{4}C_x^2 + \rho C_y C_x\right). \tag{20}$$

The square root type ratio estimator was proposed by Swain (2014), and it is defined as

$$t_7 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{1/2} \tag{21}$$

and the bias and mean square error is given as

$$B\ (t_7) = \bar{Y}\theta\left(\frac{3}{8}C_x^2 - \frac{1}{2}\rho C_y C_x\right), \tag{22}$$

$$MSE(t_7) = \overline{Y}^2\theta\left(C_y^2 + \frac{1}{4}C_x^2 - \rho C_y C_x\right). \tag{23}$$

3. The Proposed Class of Estimators

Motivated by the idea behind the ratio estimator by Cochran (1940), the exponential estimator by Bahl and Tuteja (1991) and the dual to ratio estimator due to Srivenkataramana (1980) and Singh et al. (2016), we proposed an improved class of estimators for estimating finite population mean $\overline{Y}$. The class of estimators can be defined as

$$t = \left[d\overline{y}\left(\frac{a\overline{X}+b}{a\overline{x}+b}\right) + (1-d)\overline{y}\left(\frac{a\overline{x}^*+b}{a\overline{X}+b}\right)\right] exp\left(\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right) \tag{24}$$

Where a and b are known characterizing positive scalars and d are unknown constants to be determined for which the mean square error is minimum. $\bar{x}^* = (1+g)\overline{X} - g\bar{x}$ is an unbiased estimator for the population mean, $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$, $x_i^* = \frac{NX - nx_i}{N-n} = (1+g)\overline{X} - gx_i$, $i = 1,2, \dots N$ and $w = \frac{a\overline{X}}{a\overline{X}+b}$ (say).

To study the large sample properties of the class of estimators, we consider,

$$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}} \Rightarrow \overline{y} = \overline{Y}(1+e_0) \text{ and } e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}} \Rightarrow \overline{x} = \overline{X}(1+e_1) \tag{25}$$

and their expectations are

$$E(e_0) = 0, E(e_1) = 0, E(e_0^2) = \theta C_y^2,\ E(e_1^2) = \theta C_x^2,$$
$$\text{and } E(e_0 e_1) = \theta\rho C_y C_x = \theta K C_x^2, \tag{26}$$

Using (29) in (28) we get

$$t = \overline{Y}(1+e_0)\left[d\left(\frac{a\overline{\mathrm{X}}+b}{a\overline{\mathrm{X}}(1+e_1)+b}\right) + (1-d)\left(\frac{a(1+g)\overline{\mathrm{X}} - g\overline{\mathrm{X}}(1+e_1)+b}{a\overline{\mathrm{X}}+b}\right)\right]$$

Neglecting the higher order of e's and considering up to degree two, we get

$$t = \overline{Y}(1+e_0)[d(1+we_1)^{-1} + (1-d)(1-gwe_1)]\left[1 - \frac{e_1}{2} + \frac{3e_1^2}{8} + \cdots\right]$$

$$= \overline{Y}\left[1 + e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{3e_1^2}{8} - gwe_1 - gwe_0 e_1 + \frac{gwe_1^2}{2} + d\left(gwe_1 + gwe_0 e_1 - \frac{gwe_1^2}{2} - we_1 - we_0 e_1 + \frac{we_1^2}{2} + w^2 e_1^2\right)\right] \therefore$$
$$w = \frac{a\overline{X}}{a\overline{X}+b} \text{(say)}. \tag{27}$$

Using (30) in (29), we get

$$E(t) = \overline{Y} + \overline{Y}\theta\left[\frac{3}{8}C_x^2 - \frac{\rho C_y C_x}{2} - gw\rho C_y C_x + \frac{1}{2}gwC_x^2 + d\left(gw\rho C_y C_x - \frac{1}{2}gwC_x^2 - w\rho C_y C_x + \frac{1}{2}wC_x^2 + w^2 C_x^2\right)\right] \tag{28}$$

and the bias is

$$B(t) = \overline{Y}\theta\left[\frac{3}{8}C_x^2 - \frac{\rho C_y C_x}{2} - gw\rho C_y C_x + \frac{1}{2}gwC_x^2 + d\left(gw\rho C_y C_x - \frac{1}{2}gwC_x^2 - w\rho C_y C_x + \frac{1}{2}wC_x^2 + w^2 C_x^2\right)\right] \tag{29}$$

The class of estimators 't' becomes unbiased up to order $o(\frac{1}{n})$, then $B(t) = 0$, if

$$\Rightarrow d = \frac{\left(\frac{1}{2}+gw\right)\rho C_y - \left(\frac{3}{8}+\frac{gw}{2}\right)C_x}{(g-1)w\rho C_y + \left(w^2 + \frac{w}{2} - \frac{gw}{2}\right)C_x}. \tag{30}$$

The mean square error up to order $o(\frac{1}{n})$ can be written as

$$MSE(t) = \overline{\mathrm{Y}}^2\theta\left[C_y^2 + \frac{1}{4}C_x^2 + g^2w^2C_x^2 + d^2w^2(1-g)^2C_x^2 - \rho C_y C_x - 2gw\rho C_y C_x - 2dw(1-g)\rho C_y C_x + gwC_x^2 + dw(1-g)C_x^2 + 2dgw^2(1-g)C_x^2\right]$$
$$= \theta\overline{Y}^2[C_y^2 + w\{g + d(1-g)\}^2C_x^2 + \frac{1}{4}C_x^2 + w\{g + d(1-g)\}C_x^2 - \rho C_y C_x - 2w\{g + d(1-g)\}\rho C_y C_x]. \tag{31}$$

Differentiating partially of (35) with respect to d and equating to zero, we get

$$\frac{\partial MSE(t)}{\partial d} = 0$$

$$\Rightarrow d = \frac{2\rho C_y - (1+2gw)C_x}{2w(1-g)C_x} = d_{opt}(\text{say}) \tag{32}$$

Substituting the value of d_{opt} in (35), we get the optimum mean square error of the proposed class of estimators as

$$MSE(t)_{opt} = \theta S_y^2\left(1-\rho_{yx}^2\right) \tag{33}$$

The mean square error of the proposed class of estimators is reduced to the mean square error of the linear regression estimators in its optimum class, where ρ_{yx} is the correlation coefficient between the study variable y and the auxiliary variable x, respectively.

4. Particular Cases of the Proposed Class of Estimators

For different values of a, b, the proposed class of estimators is reduced to three new classes of estimators as its particular class.

- For $a = 0$ and $b = 0$, the proposed class of estimators reduces to

$$t = \left[d_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) + (1-d_1)\bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right)\right] exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) = t_{n1}, \tag{34}$$

The bias and mean square error of the new class of estimators of t_{n1} becomes

$$B(t_{n1}) = \bar{Y}\,\theta\left[\frac{3}{8}C_x^2 - \frac{\rho C_y C_x}{2} - g\rho C_y C_x + \frac{1}{2}gC_x^2 + d_1\left((g-1)\rho C_y C_x - \frac{1}{2}gC_x^2 + \frac{3}{2}C_x^2\right)\right], \tag{35}$$

If, $d_1 = \frac{\left(\frac{1}{2}+g\right)\rho C_y C_x - \left(\frac{3}{8}+\frac{g}{2}\right)C_x^2}{(g-1)\rho C_y C_x + \left(\frac{3}{2}-\frac{g}{2}\right)C_x^2}$, then $B(t_{n1}) = 0$.

And the $MSE(t_{n1})$ will become minimal if $d_{1(opt)} = \frac{2\rho C_y - (1+2g)C_x}{2(1-g)C_x}$, then the mean square error of t_{n1} reduces to the variance of the regression estimator up to the first order of approximation,

$$MSE(t_{n1}) = \theta S_y^2\left(1-\rho_{yx}^2\right), \tag{36}$$

- For $a = 1$ and $b = \frac{1}{2}$, then the proposed class of estimators was reduced to

$$t = \left[d_2\bar{y}\left(\frac{2\bar{X}+1}{2\bar{x}+1}\right) + (1-d_2)\bar{y}\left(\frac{2\bar{x}^*+1}{2\bar{X}+1}\right)\right] exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) = t_{n2}. \tag{37}$$

and the bias and mean square error of t_{n2} up to the first order of approximation are given as follows

$$B(t_{n2}) = \bar{Y}\,\theta\left[\frac{3}{8}C_x^2 - \frac{\rho C_y C_x}{2} - \left(\frac{2\bar{X}}{2\bar{X}+1}\right)\left\{g\left(\frac{1}{2}C_x^2 + \rho C_y C_x\right) + d_2\left((g-1)\left\{\rho C_y C_x - \frac{1}{2}C_x^2\right\} + \left(\frac{2\bar{X}}{2\bar{X}+1}\right)C_x^2\right)\right\}\right], \tag{38}$$

If $d_2 = \frac{\left(\frac{1}{2}+\frac{2g\bar{X}}{2\bar{X}+1}\right)\rho C_y C_x - \left(\frac{3}{8}+\frac{g\bar{X}}{2\bar{X}+1}\right)C_x^2}{\left(\frac{2g\bar{X}-2\bar{X}}{2\bar{X}+1}\right)\rho C_y C_x + \left(\frac{\bar{X}}{2\bar{X}+1} + \left(\frac{2\bar{X}}{2\bar{X}+1}\right)^2 - \frac{g\bar{X}}{2\bar{X}+1}\right)C_x^2}$, then $B(t_{n2}) = 0$.

and the MSE(t_{n2}) will be minimum if $d_{2(opt)} = \frac{2\rho C_y - \left(1+2g\left(\frac{2\bar{X}}{2\bar{X}+1}\right)\right)C_x}{2\left(\frac{2\bar{X}}{2\bar{X}+1}\right)(1-g)C_x}$, then the estimator reduces to the variance of the regression estimator as

$$MSE(t_{n2}) = \theta S_y^2\left(1-\rho_{yx}^2\right), \tag{39}$$

- For $a = \frac{1}{2}$ and $b = 1$, the proposed class of estimators reduces to

$$t = \left[d_3\bar{y}\left(\frac{\bar{X}+2}{\bar{x}+2}\right) + (1-d_3)\bar{y}\left(\frac{\bar{x}^*+2}{\bar{X}+2}\right)\right] exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) = t_{n3}, \tag{40}$$

The bias and mean square error of t_{n3} up to the first order of approximation becomes

$$B(t_{n3}) = \bar{Y}\theta\left[\frac{3}{8}C_x^2 - \frac{\rho C_y C_x}{2} + \left(\frac{2\bar{X}}{2\bar{X}+1}\right)\left\{\frac{1}{2}gC_x^2 - g\rho C_y C_x + d_3\left((g-1)\left\{\rho C_y C_x - \frac{1}{2}C_x^2\right\} + \left(\frac{\bar{X}}{\bar{X}+2}\right)C_x^2\right)\right\}\right]. \quad (41)$$

The class of estimators becomes unbiased if $d_3 = \frac{\left(\frac{1}{2}+\frac{g\bar{X}}{\bar{X}+2}\right)\rho C_y C_x - \left(\frac{3}{8}+\frac{g\bar{X}}{2(\bar{X}+2)}\right)C_x^2}{\left(\frac{g\bar{X}-\bar{X}}{\bar{X}+2}\right)\rho C_y C_x + \left(\frac{\bar{X}}{2(\bar{X}+2)}+\left(\frac{\bar{X}}{\bar{X}+2}\right)^2 - \frac{g\bar{X}}{2(\bar{X}+2)}\right)C_x^2}$, then $B(t_{n3}) = 0$ and the $MSE(t_{n3})$ becomes minimum if the optimum value of $d_{3(opt)} = \frac{2\rho C_y - \left(1+2g\left(\frac{\bar{X}}{\bar{X}+2}\right)\right)C_x}{2\left(\frac{\bar{X}}{\bar{X}+2}\right)(1-g)C_x}$, then the mean square error of the estimator of $t(_{n3})$ reduces to the mean square error of the linear regression estimator as its particular case as

$$MSE(t_{n3}) = \theta S_y^2\left(1-\rho_{yx}^2\right). \quad (42)$$

5. Comparison with Different Existing Estimators

Here, we compare the optimum mean square error of the proposed class of estimators t from (37) with different existing estimators proposed by different authors. Again, the Minimum Variance Bound (MVB) is a focus among the estimators of any class. So, we also compare the MVB of the proposed class of estimators t with other estimators and the MVB of other competing classes.

5.1. With Mean Per Unit Estimator

From (2) and (37), the proposed class of estimators t is preferred to mean per unit estimator t_0, if

$$\rho_{yx}^2 > 0, \quad (43)$$

which is always true.

5.2. With Ratio Estimator due to Cochran (1940)

From (5) and (37), the proposed class of estimators 't' is preferred to the ratio estimator t_1, if

$$C_x^2 - 2\rho C_y C_x + \rho_{yx}^2 > 0. \quad (44)$$

5.3. With Product Estimator Murthy (1964)

From (8) and (37), the proposed class of estimators 't' is preferred to the product estimator t_2, if

$$C_x^2 + 2\rho C_y C_x - \rho_{yx}^2 > 0. \quad (45)$$

5.4. With Dual to Ratio Estimator due to Srivenkataramana (1980)

From (16) and (37), the proposed class of estimators 't' is preferred to dual to ratio (1980) estimator t_3, if

$$g^2 C_x^2 - g\rho C_y C_x + \rho_{yx}^2 > 0. \quad (46)$$

5.5. With Dual to Product Estimator due to Bandopadhyaya (1980)

From (19) and (37), the proposed class of estimators 't' is preferred to dual to product (1980) estimator t_4, if

$$g^2 C_x^2 + g\rho C_y C_x + \rho_{yx}^2 > 0. \quad (47)$$

5.6. With Bahl and Tuteja (1991) Estimator

From (22) and (37), (24) and (37), the proposed class of estimators 't' is preferred to Bahl and Tuteja (1991) estimators t_5 and t_6, if

$$\frac{1}{4}C_x^2 - 2\rho C_y C_x + \rho_{yx}^2 > 0, \quad (48)$$

and

$$\frac{1}{4}C_x^2 + 2\rho C_y C_x + \rho_{yx}^2 > 0. \quad (49)$$

5.7. With square root estimator due to Swain (2014)

From (17) and (37), the proposed class of estimators 't' is preferred to the square root estimator t_7, if

$$\frac{1}{4}C_x^2 - 2\rho C_y C_x + \rho_{yx}^2 > 0 \quad (50)$$

6. Empirical Study

In this article, to study the performances of the proposed class of estimators, we have considered six natural populations, which are available in different standard textbooks of survey sampling. In order to study the performance of the proposed class of estimators along with several competing estimators/classes of estimators like simple mean estimator t_0, ratio estimator t_1 due to Cochran (1940), a product estimator t_2 due to Murthy(1964), dual to ratio estimator t_3 due to Srivenkataramana (1980) and the dual-to-product estimator t_4 due to Bandopadhyaya (1980), exponential type ratio and product estimator t_5, t_6 According to Bahl and Tuteja (1991), the square root estimator t_7 due to Swain (2014). The proposed class of estimators is reduced to three new estimators as its particular class due to the different values of a, b, and d, which are described in section 4. We used Python (Jupyter Notebook) software to study their numerical illustrations. Here, we have considered six natural populations; their descriptions and sources are listed in Table 1, and the population characteristics are listed in Table 2. For different values of d's, the new type of exponential estimators obtained the unbiasedness and optimum mean square errors, which are listed in Table 3. The bias and MSE of the different estimators are listed in Tables 4 and 5. Table 6 represents different estimators' Percent Relative Efficiency (PRE).

The PRE's of three new class of estimators t_{n1}, t_{n2} and t_{n3} are compared with the simple mean estimator t_0 is given by

$$PRE(t) = \frac{V(t_0)}{MSE(t)} \times 100$$

Table 1. Sources and description of the populations

P. No.	Source	y	x
1	Murthy (1967), p. 398	Number of absentees	Number of workers
2	Murthy (1967), p. 399	Cultivated area in 1964	Cultivated area in 1961
3	Murthy (1967), p. 178	Area under winter paddy	Data on geographical areas
4	Singh (2003), p. 1111	Amount ($000) of real estate farm loans in different states during 1997	Amount ($000) of non-real estate farm loans in different states during 1997
5	Sukhatme and Sukhatme (1970), p. 916	Acreage of Crops and Grass for 1947	Acreage Under Oats in 1957

Table 2. The values of population parameters

P. No.	N	n	g	$\overline{Y}$	$\overline{X}$	ρ	C_y	C_x
1	43	3	0.075	9.651	79.465	0.661	0.681	0.459
2	34	8	0.308	199.441	747.588	0.904	0.753	0.594
3	108	25	0.301	172.148	460.926	0.785	0.783	0.692
4	50	5	0.111	555.435	878.162	0.804	1.053	1.235
5	35	5	0.094	39.543	164.543	0.838	0.714	0.708

Table 3. For different values of d's

P. No.	For Different Values of d's, the Class of Estimators $'t'$ will be Unbiased			For Different Values of d's, the Mean Square Error of the Class of Estimators $'t'$ will be Optimum		
	d_1	d_2	d_3	d_1	d_2	d_3
1	0.272	0.276	-0.016	0.438	0.441	0.451
2	0.719	0.72	-0.122	0.489	0.49	0.492
3	0.256	0.257	-0.168	0.126	0.126	0.128
4	-0.014	-0.014	-0.204	0.084	0.084	0.084
5	0.116	0.116	-0.05	0.276	0.278	0.281

Table 4. The Bias of different estimators

P. No.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_{n1}	t_{n2}	t_{n3}
1	0.013	0.618	0.618	0.05	0.073	0.23	0.073	0.0	0.0	0.1
2	0.985	7.708	7.708	3.012	1.333	3.014	1.333	0.0	0.0	3.118
3	0.281	2.25	2.25	0.907	0.176	0.809	0.176	0.0	0.0	0.778
4	47.991	104.54	104.54	13.483	4.929	33.204	4.929	0.0	0.0	24.045
5	0.941	5.108	5.108	0.534	0.286	1.798	0.286	0.0	0.0	0.679

Table 5. Mean square error of different estimators

P. No.	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_{n1}	t_{n2}	t_{n3}
1	13.37	7.53	31.38	12.96	13.85	8.93	20.86	8.93	6.5	6.49	6.49
2	2156.90	423.04	6572.50	1810.59	2757.61	954.75	4029.48	954.75	118.50	118.41	118.43
3	558.79	219.83	1769.46	481.67	714.89	280.35	1055.16	280.35	171.74	171.80	171.98
4	61563.86	30154.78	262414.59	56162.50	69052.92	24679.11	140809.02	24679.11	20264.40	20264.79	20265.976
5	242.88	78.10	886.01	226.01	263.98	100.7	504.65	100.7	55.26	55.24	55.18

Table 6. The Percent relative efficiency (PRE) of different estimators

P. No.	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_{n1}	t_{n2}	t_{n3}
1	100	177.538	42.617	103.186	96.526	149.72	64.122	149.72	206	205.833	206.01
2	100	509.854	32.817	119.127	78.216	225.911	53.528	225.911	1820	1820.315	1821.146
3	100	254.185	31.58	116.011	78.165	199.32	52.958	199.32	325	325.247	324.903
4	100	204.16	23.461	109.617	89.155	249.457	43.722	249.457	304	303.797	303.779
5	100	310.967	27.413	107.465	92.007	241.195	48.129	241.195	439	439.631	440.145

7. Conclusion

In this study, an exponential class of ratio cum dual to ratio estimators for population mean $\overline{Y}$ by using the idea due to Cochran (1940), a dual-to-ratio estimator due to Srivenkataramana (1980) along with the exponential estimator proposed by Bahl and Tuteja (1991) is suggested. The proposed class of estimators t reduces to several new estimators as its particular class for different values of a, b and d. The acquainted estimators' bias and mean square error are determined to the first order of approximation. A comparative study is examined to derive the efficiency conditions under which the proposed class of estimators dominates the existing estimators we have considered. Subsequently, an empirical study is carried out to verify the theoretical results using some real populations. On account of theoretical and empirical results, we draw the conclusion as given below: The empirical result in Table 5 shows the Mean Square Error (MSEs) of different estimators alongwith the proposed class of estimators, and it shows that the proposed class of estimators attains the least mean square error for all the considered populations. Table 6 shows the Percent Relative Efficiency (PREs) of the estimators and the proposed class of estimators t_{n1}, t_{n2}, t_{n3} attains the maximum precision for all the population over the contemporary estimators. Thus, the proposed class of estimators justifies its worthiness and can be highly recommended for the researchers.

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