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Original Article

Optimal Control of a Vectored Plant Disease Impulse Model

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Abstract - *This paper sets forth a new model for the coupling and simultaneous implementation of integrated pest management in pulsed mode with variable time and fixed dose. In this case, the optimization problem takes the pest population below an economic threshold as a constraint and the control cost as an objective function. Finally, numerical simulations were carried out to evaluate the optimal control model for the pests.*

Keywords - *Economic threshold, Pulse interference, Control parameter enhancing transform, Pulse timing, Released amount.*

1. Introduction

The whitefly stands as a paramount agricultural pest, posing substantial detriment to numerous vital crops worldwide [1]. Cassava is playing a key role in ensuring food security in the midst of famine, owing to its protracted storage capacity in the soil preceding harvest [2]. Nevertheless, agricultural cassava's production faces substantial constraints, notably from the whitefly.

Bokil et al. [3] employed optimal control theory within the framework of the continuous frequency-replanting model to investigate the impacts of both infected plant removal and insecticide application. Their findings underscored the heightened efficacy of a combined approach, surpassing the effectiveness of singular control measures.

Building on the preceding discussion, we constructed a mathematical model involving cassava plants, whiteflies, together with cassava mosaic virus and then integrated pulse interventions such as pesticide application and infected plant removal in Section 2. Optimization problems are subsequently formulated, integrating the economic threshold of pests as a constraint. Moreover, numerical simulations are conducted to explore pest control strategies in section 3. Finally, we provide concluding remarks in section 4.

2. Optimization Problem Creation

2.1. ODE model

Utilizing the frequency-replanting model established in [4], we have devised a new model comprising susceptible plants indicated with S, infected plants indicated with I, non-infective whiteflies indicated with U, and infective whiteflies indicated with W.

$$\begin{cases} \frac{dS}{dt} = b\frac{S}{S+\varepsilon}\left(1-\frac{S+I}{\theta}\right)S - bSW - hS + gI, \\ \frac{dI}{dt} = b\frac{\varepsilon I}{S+\varepsilon I}\left(1-\frac{S+I}{\theta}\right)I + bSW - (g+h)I, \\ \frac{dU}{dt} = a(U+W)\left(1-\frac{(U+W)}{\kappa(S+I)}\right) - gIU - cU, \\ \frac{dW}{dt} = gIU - cW, \end{cases} \tag{1}$$

with initial condition

$$S(0) = S_0, I(0) = I_0, U(0) = U_0, W(0) = W_0. \tag{2}$$

In this context, the overall cassava population is symbolized as $N(t) = S(t) + I(t)$, while the aggregate whitefly population is articulated as $V(t) = U(t) + W(t)$. Table 1 furnishes elucidation on the parameters employed within the designated model.

Table 1. Parameters values of model (1) and (2)

Parameter	Description	Value	Reff
θ	Maximum plant population pressure (per m^2)	10	~
ε	Infected plants selection frequency	0.1	[5]
b	Maximum replanting rate (per day)	0.01	[3]
β	whiteflies to plants inoculation rate (per day per whiteflies)	0.00001	~
h	Plants harvesting rate (per day)	0.003	[3]
g	Infected plants recovery rate (per day)	0.003	[3]
κ	Maximum whiteflies density per host plant	500	[5]
a	Maximum whiteflies birth rate (per day)	0.2	[3]
γ	Plant to whiteflies acquisition rate (per day per plant)	0.001	[3]
c	Whiteflies mortality (per day)	0.16	[5]

In Figure 1, we present the simulated trajectories of the pest population under uncontrolled conditions, with the Economic Threshold (ET) set at a fixed value of 350. Evidently, human intervention is imperative for the efficacious management of pest populations. Herein, we implement a pulsed pattern strategy involving pesticides and the removal of infected plants to counteract pest proliferation.

2.2. Impulse ODE Model

Firstly, we formulated an ODE model incorporating pulse interference, wherein the effects of pulse interference stemming from pesticide application and the removal of infected plants were taken into account. That is

$$\left\{\begin{matrix} \frac{dS}{dt} = b\frac{S}{S+\varepsilon}\left(1-\frac{S+I}{\theta}\right)S - bSW - hS + gI \\ \frac{dI}{dt} = b\frac{\varepsilon I}{S+\varepsilon I}\left(1-\frac{S+I}{\theta}\right)I + bSW - (g+h)I \\ \frac{dU}{dt} = a(U+W)\left(1-\frac{(U+W)}{\kappa(S+I)}\right) - gIU - cU \\ \frac{dW}{dt} = gIU - cW \end{matrix}\right\} 0 \le t \le T, t \ne t_i, \tag{3}$$

and the impulse conditions are

$$\left\{\begin{matrix} U(t_i^{+}) = (1-\eta)U(t_i^{-}) \\ W(t_i^{+}) = (1-\eta)W(t_i^{-}) \\ I(t_i^{+}) = (1-\alpha)I(t_i^{-}) \end{matrix}\right\} t = t_i, i = 1,2,\dots,n-1, \tag{4}$$

and initial condition is (2). where T denotes the terminal time. The time points of impulse, denoted as t_i, adhere to the following criteria:

$$0 \le t_1 \le t_2 \le, \dots, \le t_n = T,$$
$$t_i - t_{i-1} = \tau_i, \sum_{i=1}^{n}\tau_i = T, a_i \le \tau_i \le b_i, i = 1,2,\dots,n-1, \tag{5}$$

where a_i and b_i are given non-negative constants.

Moreover, specific conditions are proposed to regulate the release of pesticides and the removal of infected plants:

$$0 \le \eta \le {c_i}^1, 0 \le \alpha \le {c_i}^2, i = 1,2,\dots,n-1, \tag{6}$$

where ${c_i}^1$ and ${c_i}^2$, are also the given non-negative constants.

2.3. Optimal Control Problem

For convenience, we introduce the following notations:

$$\tau = (\tau_1, \tau_2, \dots, \tau_n)^T.$$

Define a cost function

$$J(\eta,\alpha,\tau) = (n-1)\rho^V A\eta + \sum_{i=1}^{n} \rho^I \alpha I(t_i), \tag{7}$$

Where ρ^V and ρ^I, denote the cost per unit area of pesticide spraying, the labor price of removing unit infected plants, respectively, whereas A refers to the aggregate area of the cropland cultivating cassava, $I(t_i)$ signifies the value of infected plants at t_i

Additionally, postulate that the pest population at time t adheres to the following inequality constraint:

$$V(t) \le \vartheta, \forall t \in [0,T], \tag{8}$$

Where, $ET = \vartheta$ is the Economic Threshold .

Hence, our optimal pest control problem can now be formally framed as:

Problem P

Subject to impulsive dynamic system (3)-(4), with the initial condition (2) and inequality constraint (8), find a combined parameter vector pair (η,α,τ), such that the cost function $J(\eta,\alpha,\tau)$ is minimized.

Note

Based on literatures [6], system (3)-(4) with initial condition (2) has a unique solution. Further, for each pair (η,α,τ) meeting (5) and (6), Problem P exists an optimal solution. Likewise, the subsequent optimization problems all possess optimal solutions, which will not be elaborated further.

Given the inherent uncertainty associated with the states I, characterized by uncertain jump times and magnitudes, the complexity of resolving the issue is notably compounded. To mitigate this challenge, Control Parameter Enhanced Transformation (CPET) [7,8], a time scale transformation technique, is used to transform the system from the time scale (0,T) to (0,1) . The new modified model is

$$\left.\begin{cases} \frac{dS}{dt} = f^i{}_1(x) = \tau_i\left[b\frac{S}{S+\varepsilon}\left(1-\frac{S+I}{\theta}\right)S - bSW - hS + gI\right] \\ \frac{dI}{dt} = f^i{}_2(x) = \tau_i\left[b\frac{\varepsilon I}{S+\varepsilon I}\left(1-\frac{S+I}{\theta}\right)I + bSW - (g+h)I\right] \\ \frac{dU}{dt} = f^i{}_3(x) = \tau_i\left[a(U+W)\left(1-\frac{(U+W)}{\kappa(S+I)}\right) - gIU - cU\right] \\ \frac{dW}{dt} = f^i{}_4(x) = \tau_i[gIU - cW] \end{cases}\right\} 0 \le s \le 1, i = 1, \dots n, \tag{9}$$

and the impulse conditions are

$$\left.\begin{cases} U_i(0) = (1-\eta)U_{i-1}(1) \\ W_i(0) = (1-\eta)W_{i-1}(1) \\ I_i(0) = (1-\alpha)I_{i-1}(1) \end{cases}\right\} i = 1,2,\dots,n, \tag{10}$$

and initial conditions are

$$S_0(0) = S_0, I_0(0) = I_0, U_0(0) = U_0, W_0(0) = W_0. \tag{11}$$

And inequality constraint is is updated to

$$V_i(s) \le \vartheta, i = 1,2,\dots,n, s \in [0,1].$$

As the problem at hand entails an optimization challenge with inequality constraints, the incorporation of a penalty function [9] becomes imperative to render it as a more encompassing constrained optimization problem. Define penalty function:

$$\Delta(\mu) = \sum_{i=1}^{n} \int_0^1 [max\{0, V_i(s) - \vartheta\}]\, ds$$

and $\Delta(\mu) = 0$ holds if and only if the inequality constraint is satisfied. Thus we get the solvable optimization problem

Problem P_1

Subject to impulsive dynamic system (9)-(10), with the initial condition (11), search a combined parameter vector $(\eta, \alpha, \tau, \sigma)$, such that the cost function

$$J_1(\eta, \alpha, \tau, \sigma) = (n-1)\rho^V A\eta + \sum_{i=1}^{n} \rho^I \alpha I(t_i) + \sigma^{-\delta_1} \sum_{i=1}^{n} \int_0^1 [max\{0, V_i(s) - \vartheta\}]^2 ds + \omega\sigma^{\delta_2}$$

is minimized. Here ω is the penalty parameter, $\delta^1$and δ^2 are positive constants satisfying the conditions$0 \le \delta^2 \le \delta^1$with $\delta^1 > 0$and $\delta^2 > 2$.

3. Solve the Optimal Problem

In below, using numerical simulation to solve the optimal problem. First, set the initial values

$$S(0) = 10, I(0) = 5, U(0) = 70, W(0) = 0,$$

and take the terminal time T=150 days. In the cost function, further assume that$A = 100, \rho^V = 1.5, \rho^I = 1$.

Taking initial equal releasing intervals and releasing rates as follows

$$\tau_{i0} = 30, \eta = 0.2, \alpha = 0.3, i = 1,2,3,4,5$$

and $\sigma = 0.1$. Upon the above initial values, we obtain the initial cost function ${J_1}^0 = 2.4831e + 07$. With restrictions

$$0.1 \le \eta \le 1, 0.1 \le \alpha \le 0.6, 10 \le \tau_i \le 60, i = 1, 2, \ldots, 5,$$

the best solutions of optimal problem P_1 are given as follows

$$\tau_1^* = 34.7943, \tau_2^* = 25.4933, \tau_3^* = 32.6026, \tau_4^* = 20.8364, \tau_5^* = 36.2734,$$
$$\eta^* = 0.5934, \alpha^* = 0.2130$$

and $\sigma^* = 0.008$with optimal value${J_1}^* = 358.2375$.

Lastly, Figure 1 illustrates the dynamic behaviour of the pest under uncontrolled, initial control, and optimal control conditions via red, blue, and green lines, respectively. Apparently, pests are controlled below the economic threshold.

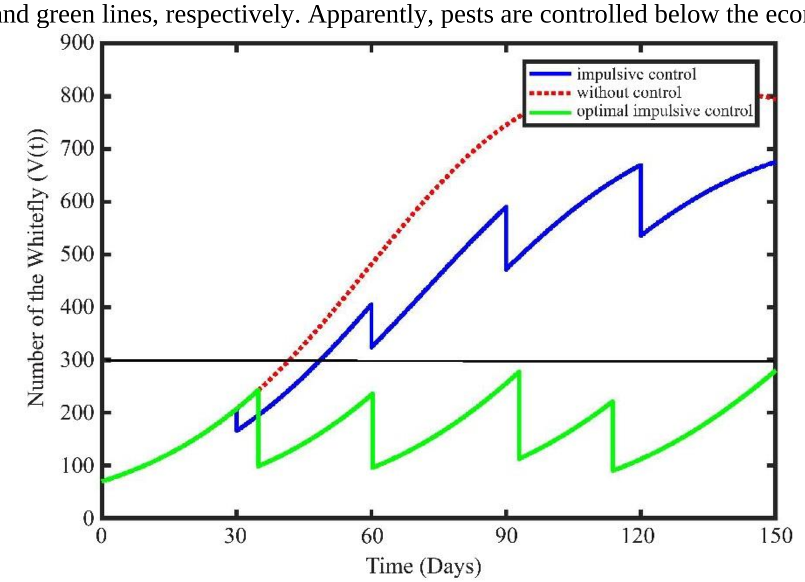

Fig. 1 Simulated trajectory of the whitefly number.

4. Conclusion

The historical entanglement between pests and humanity, spanning millennia, underscores a contemporary challenge necessitating adept resolution by scholars. This study attempts to address this issue by formulating an impulse dynamics model and integrating it with optimal control theory to derive an optimal strategy for pest management. By addressing the optimization problem, the optimal pulse duration and intensity are derived, thereby facilitating proficient pest control below the established economic threshold through the implementation of the optimal strategy.

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