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Original Article

Heinz Quarter Mean Labeling of Some Special Graphs

S. Latha[1], S.S. Sandhya[2]

[1,2]*Department of Mathematics, Sree Ayyappa College for Women, Tamil Nadu, India.*

[1]*Corresponding Author : lathasubash123@gmail.com*

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Abstract *- A graph $G = (V, E)$ having a number of vertices as p and a number of edges as q can be called a Heinz – Quarter Mean graph if the vertices $x \in V$ can be labeled with distinct labels $f(x)$ from $1, 2, 3, \ldots \ldots, q+1$. Here, each edge is labeled with* $f(e=uv)=\left\lfloor \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rfloor$ *or* $\left\lceil \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rceil$*, then the resulting edge labels are distinct. Here, f is called Heinz – Quarter mean labeling of G. In this section, we investigate the Heinz Quarter Mean Labeling of some special named graphs.*

Keywords *- Heinz Quarter Mean, Zagreb indices, Grotzch, Bidiakis, Fritch, etc.*

1. Introduction

Graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Graph labeling is one of the most interesting problems in graph theory, and they serve as useful models for a broad range of applications such as coding theory, X-ray crystallography, circuit designs, etc. Graph labeling was first introduced in the late 1960s and has been motivated by practical problems. In the intervening years variety of graph labeling techniques have been studied, and the subject is growing exponentially. For more details, one may refer to the survey article (1) by J. A. Gallian. The graceful labeling methods were introduced by Rosa in 1967. Terms and notations not defined here are used in the sense of Harary. In Bhatia introduced the Heinz mean $f(u,v;\ y) = \frac{f(u)^y f(v)^{1-y} + f(u)^{1-y} f(v)^y}{2}$, for $0 \le y \le \frac{1}{2}$.

In 2003, Somasundaram and Ponraj [2] introduced the notion of mean labellings of graphs. S.S.Sandhya and S. Latha introduced a new labeling pattern namely Heinz Quarter mean labeling of Graph. In this paper, we investigate the Heinz Quarter Mean Labeling of Grotzch, Bidiakis, Fritch,etc.

2. Main Results

Definition 2.1. A graph $G = (V, E)$ having the number of vertices as p and number of edges as q can be called as Heinz Quarter Mean graph if the vertices $x \in V$ can be labeled with distinct labels $f(x)$ from $1, 2, 3, \ldots \ldots, q+1$ and each edge $e = uv$ is labeled with $f(e=uv) = \left\lfloor \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rceil$, then the resulting edge labels are distinct. Here, f is called Heinz Quarter, mean labeling of G. This Graph is said to be the Heinz Quarter Mean Graph.

Definition 2.2. Grotzch Graph is a Triangle free graph with 11 vertices and 20 edges having chromatic number 4 and crossing number 5 and named by German mathematician Hebert Grotzch.

Definition 2.3. Bidiakis Cube is a cubic Hamiltonian, triangle-free,polyhedral, planar Graph with 12 vertices and 18 edges.

Definition 2.4. David's Star Graph is a graph with 12 vertices and 18 edges in which 6 vertices have degree 4, and all the other vertices have degree 2.

Definition 2.5. Fritch Graph is a Planar graph with 9 vertices and 21 edges that tangles the Kempe Chain in the Kempe Algorithm.

Definition 2.6. The Herschel graph is a Bipartite undirected graph with 11 vertices and 16 edges.

Theorem 2.7. Grotzch Graph is a Heinz Quarter Mean graph.

Proof. Let G be a Grotzch graph with 11 vertices and 20 edges. Let us label the vertices of G. $v_1, v_2, \dots, v_{11}$ and edge set is $\{v_1v_2, v_1v_3, v_2v_4, v_3v_5, v_4v_5, v_1v_7, v_1v_8, v_6v_2, v_6v_3, v_2v_9, v_9v_5, v_5v_8, v_3v_{10}, v_{10}v_4,$ $v_4v_7, v_6v_{11}, v_7v_{11}, v_8v_{11}, v_9v_{11}, v_{10}v_{11}\}$ as shown in the figure.

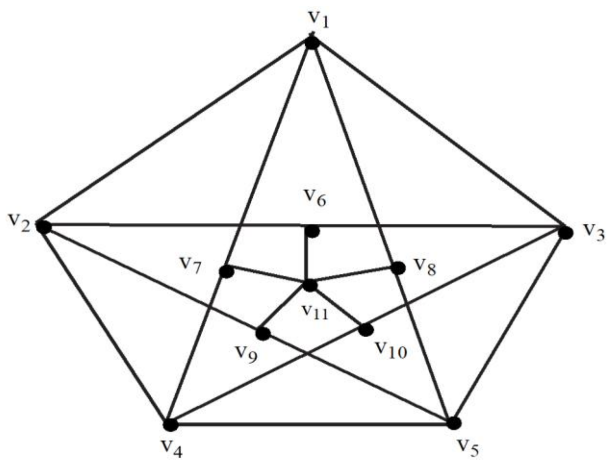

Fig. 2.1 Grotzch graph

Let the vertex set of G be $V = V_1 \cup V_2 \cup V_3 \cup \dots \cup V_n$ Where $V_i = \{v_1^i, v_2^i, v_3^i, \dots, v_{11}^i / 1 \le i \le n\}$

A function defined by $f: V(G) \to \{1,2,3, \dots \dots, q+1\}$ by

$f(v_1^i) = 21i - 7; 1 \le i \le n$ $\quad f(v_2^i) = 21i - 3; 1 \le i \le n$

$f(v_3^i) = 21i - 8; 1 \le i \le n$ $\quad f(v_4^i) = 21i - 6; 1 \le i \le n$

$f(v_5^i) = 21i; 1 \le i \le n$ $\quad f(v_6^i) = 21i - 9; 1 \le i \le n$

$f(v_7^i) = 21i - 18; 1 \le i \le n$ $\quad f(v_8^i) = 21i - 16; 1 \le i \le n$

$f(v_9^i) = 21i - 2; 1 \le i \le n$ $\quad f(v_{10}^i) = 21i - 15; 1 \le i \le n$

$f(v_{11}^i) = 21i - 20; 1 \le i \le n$

From the Heinz Quarter mean labeling pattern, the edge labels are all distinct. Hence, G is a Heinz Quarter mean Graph.

Example 2.8. The labeling pattern of the Grotzch graph is shown in Figure 2.2

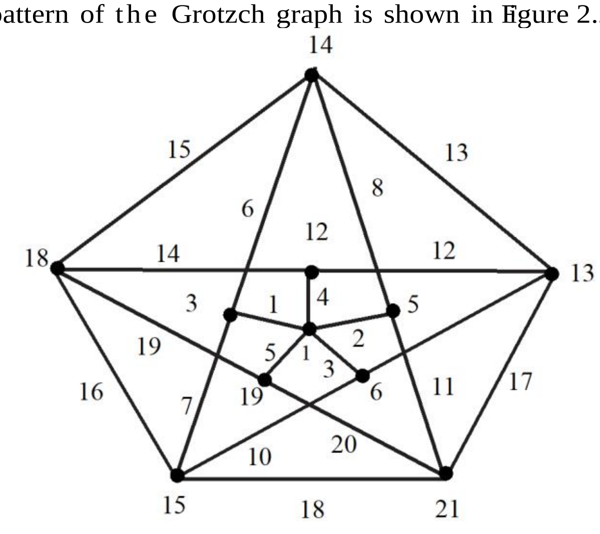

Fig. 2.2 Grotzch graph

Theorem 2.9. Bidiakis Cube graph is a Heinz Quarter Mean graph.

Proof. Let G be a Bidiakis Cube graph with 12 vertices and 18 edges.Let us label the vertices of G as $v_1, v_2, \ldots, v_{12}$ and whose edge set is$\{v_1v_2, v_1v_3, v_2v_4, v_3v_5, v_3v_4, v_1v_7, v_7v_8, v_7v_{10}, v_5v_6, v_2v_9, v_{10}v_{11}, v_5v_{10}, v_9v_{12}, v_4v_6, v_6v_{12}, v_7v_{10}, v_8v_{11}, v_{11}v_{12}\}$ as shown in the figure.

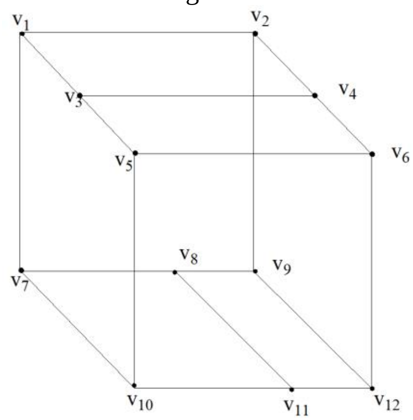

Fig. 2.3 Bidiakis cube graph

Let the vertex set of G be $V = V_1 \cup V_2 \cup V_3 \cup \ldots . \cup V_n$ Where $V_i = \{v_1^i, v_2^i, v_3^i, \ldots, v_{12}^i / 1 \leq i \leq n\}$
A function defined by $f: V(G) \rightarrow \{1,2,3, \ldots \ldots, q+1\}$ by

$f(v_1^i) = 19i - 14; 1 \leq i \leq n$
$f(v_2^i) = 19i - 9; 1 \leq i \leq n$
$f(v_3^i) = 19i - 18; 1 \leq i \leq n$
$f(v_4^i) = 19i - 16; 1 \leq i \leq n$
$f(v_5^i) = 19i - 11; 1 \leq i \leq n$
$f(v_6^i) = 19i - 13; 1 \leq i \leq n$
$f(v_7^i) = 19i - 1; 1 \leq i \leq n$
$f(v_8^i) = 19i - 3; 1 \leq i \leq n$
$f(v_9^i) = 19i - 7; 1 \leq i \leq n$
$f(v_{10}^i) = 19i - 2; 1 \leq i \leq n$
$f(v_{11}^i) = 19i - 4; 1 \leq i \leq n$
$f(v_{12}^i) = 19i - 6; 1 \leq i \leq n$

From the Heinz Quarter mean labeling pattern, the edge labels are all distinct.

Hence, G is a Heinz Quarter mean Graph.

Example 2.10. The labeling pattern of the Bidiakis graph is shown in thefigure

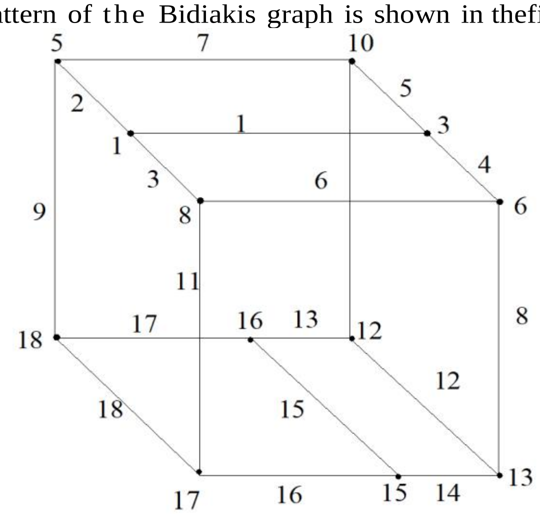

Fig. 2.4 Bidiakis cube graph

Theorem 2.11. David's Star graph is a Heinz Quarter Mean graph.

Proof. Let G be a David's Star graph with 12 vertices and 18 edges.Let us label the vertices of G as $v_1, v_2, \ldots, v_{12}$ and whose edge set is

$\{v_1v_4, v_1v_5, v_2v_4, v_3v_5, v_4v_5, v_5v_7, v_3v_7, v_7v_{10}, v_2v_6, v_7v_{11}, v_{10}v_{11}, v_6v_9, v_9v_{12}, v_4v_6, v_6v_8, v_9v_{10}, v_8v_9, v_{10}v_{12}\}$ as shown in the figure.

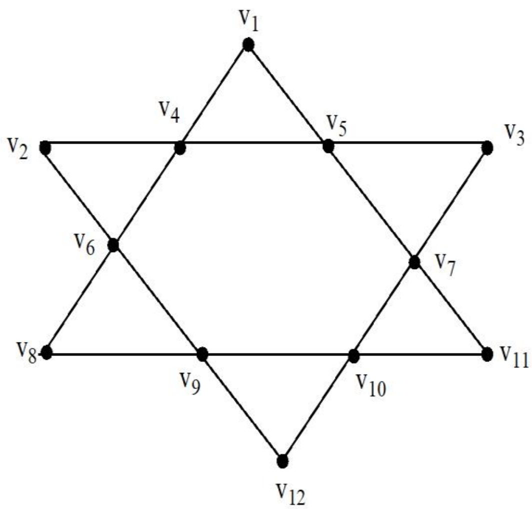

Fig. 2.5. David's star graph

Let the vertex set of G be $V = V_1 \cup V_2 \cup V_3 \cup \ldots .\cup V_n$ Where $V_i = \{v_1^i, v_2^i, v_3^i, \ldots, v_{12}^i / 1 \leq i \leq n\}$

A function defined by $f: V(G) \rightarrow \{1,2,3, \ldots \ldots, q + 1\}$ by

$f(v_1^i) = 19i - 19; 1 \leq i \leq n$

$f(v_2^i) = 19i - 13; 1 \leq i \leq n$

$f(v_3^i) = 19i - 11; 1 \leq i \leq n$

$f(v_4^i) = 19i - 16; 1 \leq i \leq n$

$f(v_5^i) = 19i - 5; 1 \leq i \leq n$

$f(v_6^i) = 19i - 8; 1 \leq i \leq n$

$f(v_7^i) = 19i - 7; 1 \leq i \leq n$

$f(v_8^i) = 19i - 10; 1 \leq i \leq n$

$f(v_9^i) = 19i - 6; 1 \leq i \leq n$

$f(v_{10}^i) = 19i - 3; 1 \leq i \leq n$

$f(v_{11}^i) = 19i - 1; 1 \leq i \leq n$

$f(v_{12}^i) = 19i ; 1 \leq i \leq n$

From the Heinz Quarter mean labeling pattern, the edge labels are all distinct.

Hence, G is a Heinz Quarter mean Graph.

Example 2.12. The labeling pattern of David's Star graph is shown in thefigure

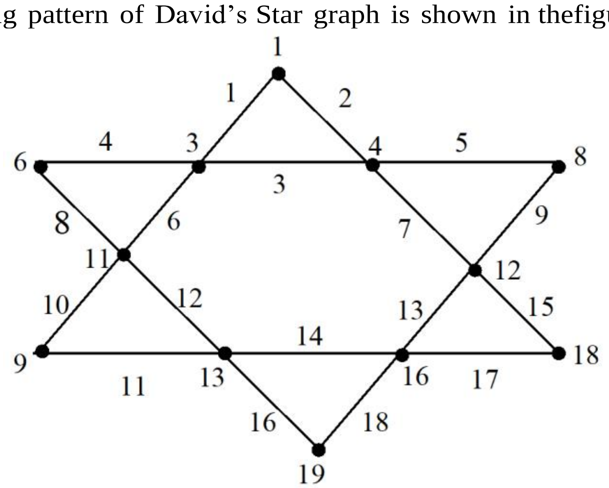

Fig. 2.6. David's star graph

Theorem 2.13. Fritch Graph is a Heinz Quarter Mean Graph.
Proof. Let G be a Fritch graph with 9 vertices and 21 edges.Let us label the vertices of G as $v_1, v_2, \dots, v_9$ and whose edge set is
$\{v_1v_4, v_1v_5, v_2v_4, v_3v_5, v_4v_5, v_5v_7, v_3v_7, v_7v_{10}, v_2v_6, v_7v_{11}, v_{10}v_{11}, v_6v_9, v_9v_{12},$
$v_4v_6, v_6v_8, v_9v_{10}, v_8v_9, v_{10}v_{12}\}$ as shown in the figure.

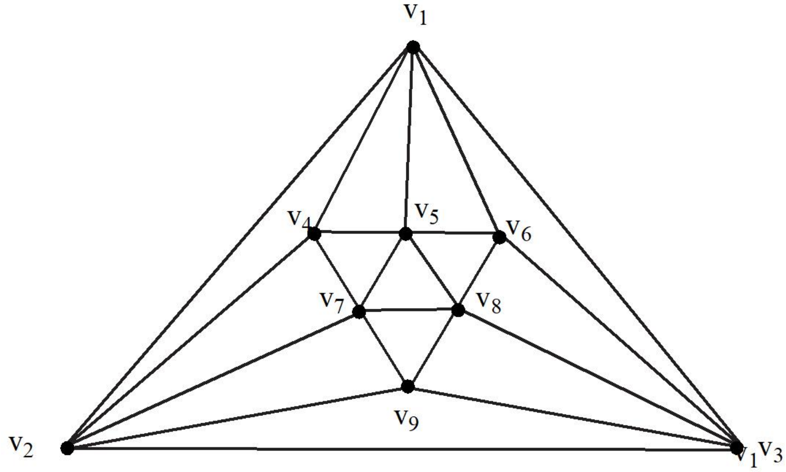

Fig. 2.7. Fritch graph

Let the vertex set of G be $V = V_1 \cup V_2 \cup V_3 \cup \dots . \cup V_n$ Where $V_i = \{v_1^i, v_2^i, v_3^i, \dots, v_9^i / 1 \leq i \leq n\}$
A function defined by $f: V(G) \to \{1,2,3, \dots \dots, q + 1\}$ by

$f(v_1^i) = 22i - 19; 1 \leq i \leq n$
$f(v_2^i) = 22i - 12; 1 \leq i \leq n$
$f(v_3^i) = 22i; 1 \leq i \leq n$
$f(v_4^i) = 22i - 21; 1 \leq i \leq n$
$f(v_5^i) = 22i - 17; 1 \leq i \leq n$
$f(v_6^i) = 22i - 11; 1 \leq i \leq n$
$f(v_7^i) = 22i - 3; 1 \leq i \leq n$
$f(v_8^i) = 22i - 7; 1 \leq i \leq n$
$f(v_9^i) = 22i - 2; 1 \leq i \leq n$

From the Heinz Quarter mean labeling pattern, the edge labels are all distinct.

Hence, G is a Heinz Quarter mean Graph.

Example 2.14. The labeling pattern of the Fritch graph is shown in thefigure

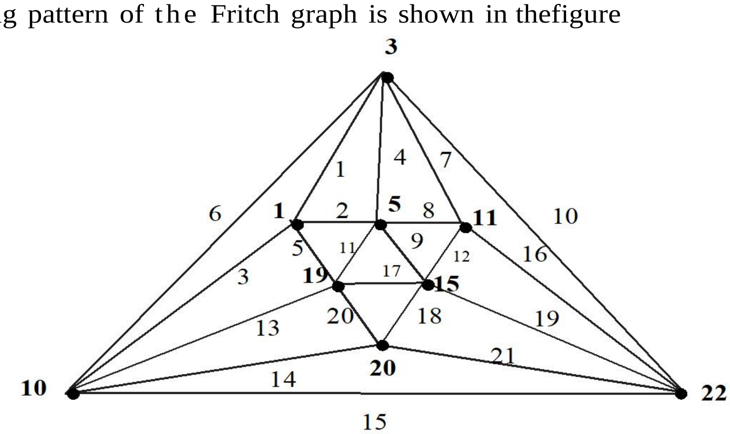

Fig. 2.8 Fritch graph

Theorem 2.15. Herschel Graph is a Heinz Quarter Mean Graph.

Proof. Let G be a Herschel graph with 11 vertices and 18 edges.

Let us label the vertices of G as $v_1, v_2, \dots, v_{11}$ and whose edge set

is $\{v_1v_4, v_1v_8, v_9v_{11},, v_1v_3, v_3v_6, v_4v_5, v_5v_9, v_6v_9, v_3v_7, v_7v_{10}, v_7v_8, v_2v_6, v_4v_{11}, v_{10}v_{11}, v_2v_5, v_3v_6, v_8v_{11}\}$ as shown in the figure.

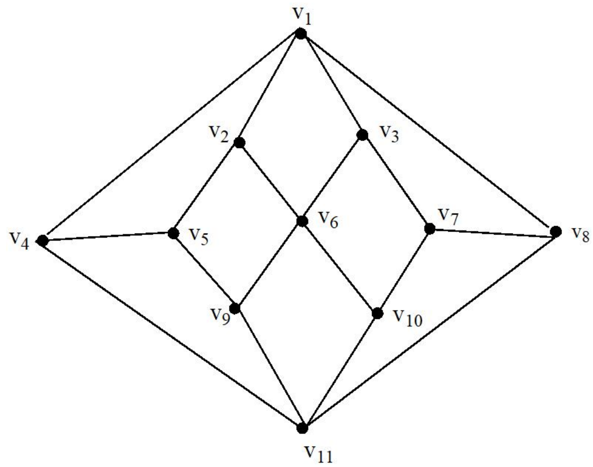

Fig. 2.9 Herschel graph

Let the vertex set of G be $V = V_1 \cup V_2 \cup V_3 \cup \dots . \cup V_n$ Where $V_i = \{v_1^i, v_2^i, v_3^i, \dots, v_{11}^i / 1 \leq i \leq n\}$
A function defined by $f: V(G) \rightarrow \{1,2,3, \dots \dots, q + 1\}$ by

$f(v_1^i) = 19i - 12; 1 \leq i \leq n$

$f(v_2^i) = 19i - 13; 1 \leq i \leq n$

$f(v_3^i) = 19i - 6; 1 \leq i \leq n$

$f(v_4^i) = 19i - 18; 1 \leq i \leq n$

$f(v_5^i) = 19i - 16; 1 \leq i \leq n$

$f(v_6^i) = 19i - 5; 1 \leq i \leq n$

$f(v_7^i) = 19i - 1; 1 \leq i \leq n$

$f(v_8^i) = 19i; 1 \leq i \leq n$

$f(v_9^i) = 19i - 15; 1 \leq i \leq n$

$f(v_{10}^i) = 19i - 3; 1 \leq i \leq n$

$f(v_{11}^i) = 19i - 9; 1 \leq i \leq n$

From the Heinz Quarter mean labeling pattern, the edge labels are all distinct.

Hence, G is a Heinz Quarter mean Graph.

Example 2.16. The labeling pattern of the Herschel graph is shown in thefigure

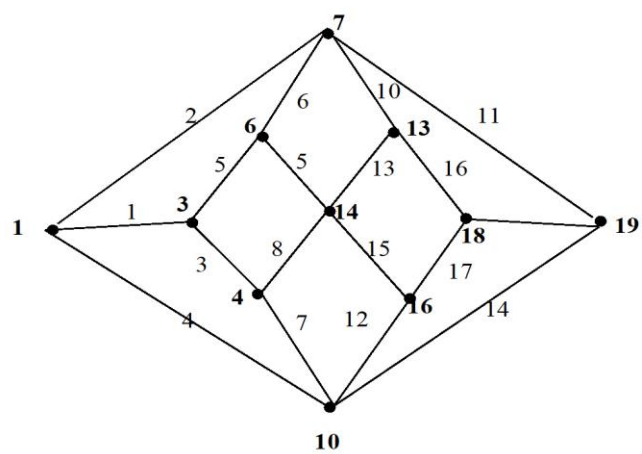

Fig. 2.10 Herschel graph

References

[1] Frank Harary, *Graph Theory*, Narosa Publishing House, New Delhi, 2001.

[2] John Adrian Bondy, and U.S.R. Murty, *Graph Theory with Applications*, American Elsevier Publishing Company, pp. 1-264, 1976. [Google Scholar] [Publisher Link]

[3] S. Somasundaram, and R. Ponraj, “Some Results on Mean Graphs,” Pure and Applied Mathematika Science, vol. 58, pp. 29-35, 2003.

[4] S. Somasundaram, and R. Ponraj, “On Mean Graphs of Order < 5,” Journal of Decision Mathematical Science, vol. 9, pp. 47-58, 2004.

[5] S.S. Sandhya, and S. Somasundaram, “Harmonic Mean Labeling for Some Special Graphs,” *International Journal of Mathematics Research*, vol. 5, no. 1, pp. 55-64, 2013. [Google Scholar] [Publisher Link]

[6] S.S. Sandhya, and H. Aswathy, “Arbitrary Super Subdivision of Root Square Mean Labeling of Graphs,” *International Journal of Computer Science*, vol. 8, no. 1, pp. 2421-2427, 2020. [Publisher Link]

[7] S.S. Sandhya, and S. Latha, “Zagreb Indices of Some Harmonic Mean Graphs,” *Journal of Fundamental & Comparative Research*, vol. 8, no. 2, 2022.

[8] S. Latha, and S.S. Sandhya, “Heinz Quarter Mean Labeling of Graphs,” *Ratio Mathematica*, vol. 51, pp. 1-10, 2024. [Google Scholar] [Publisher Link]