International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V70I5P105

Volume 70 Issue 5, 28-34, May 2024

Original Article

Enhancing Stochastic Stability in Semi-Markov Jump Systems via Dynamic Event-Triggering Mechanisms: A Control Perspective

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Received: 01 March 2024 Revised: 30 April 2024 Accepted: 18 May 2024 Published: 30 May 2024

Abstract *- This study explores achieving stochastic stability in Semi-Markov jump systems under dynamic event-triggering mechanisms. Addressing critical challenges in system control, it investigates the feasibility of stability dynamic triggers, offering insights into enhancing system performance and reliability.*

Keywords *- Switched nonlinear systems, Dynamic event triggering mechanism, H_∞ performance, Semi-Markov jump systems, Stochastic stability.*

1. Introduction

As a specialized form of stochastic hybrid system, Markov jump systems (MJSs) find extensive applications across various domains, including manufacturing, neural networks, robotics, and epidemiology.[1-5] Nonetheless, their practical utility is limited due to the exponential distribution governing the sojourn time at each state of a Markov process.

This stringent requirement significantly constrains the applicability of MJSs, leading to inherent conservatism in many obtained outcomes. To overcome these constraints, Semi-Markov jump systems (SMJSs) have been proposed.[6]-[8]

Numerous significant works have emerged for the dynamic analysis of SMJSs, spanning stability and stabilization, [9]-[12] event-triggered schemes,[13]-[16] passivity[17], and H∞ performance analysis.[18]-[19] Additionally, remarkable progress has been made in the advancement of dynamic event-triggering mechanisms. Dynamic event-triggered mechanisms (DETM) can ensure trigger intervals surpassing those achieved by static mechanisms, thereby enhancing system performance.[20]

Generally, due to the existence of SMJSs and DETM, investigating such systems poses a challenge. However, there have been no relevant theoretical results available, which underscores the significance of our study.

2. Problem Statement

2.1. Consider a Nonlinear Hybrid Time-Delay Switching System with Perturbations as follows:

$$\begin{cases}\dot{x}(t) = A(r_t)x(t) + C(r_t)f(x(t), r_t) + B(r_t)u(t) + E(r_t)\omega(t), \\ y(t) = D(r_t)x(t), \end{cases} \tag{1}$$

where $x(t)$ is the state variable; $u(t)$ is the control input; $y(t)$ is the measurement output; $f(x(t),\ x(t-d(t))$ is a nonlinear perturbation; $A_{r_t}, B_{r_t}, E_{r_t}, C_{r_t}, D_{r_t}$ are constant matrices with the appropriate dimension. The continuous-time semi-Markov chain r_t takes values of $\mathcal{N}_1 = \{1,2,\ldots,\mathcal{N}\}$, subject to the transition probability matrix as follows:

$$P[r(t+h) = j \mid r(t) = i] = \begin{cases}\lambda_{ij}(h)h + o(h), i \neq j \\ 1 + \lambda_{ii}(h)h + o(h), otherwise\end{cases} \tag{2}$$

where h represents the sojourn time. $\pi_p q(h)$ denotes the transition rate from mode p at time t to mode q at time $t+h$, and $\pi_{pp}(h) = -\sum_{q\in\mathcal{N}, q\neq p} \pi_{pq}(h)$.

The nonlinear perturbation satisfies the following assumption:

$$f_{r_t}^{\mathrm{T}}(t, x(t)) f_{r_t}(t, x(t)) \leq x^{\mathrm{T}}(t) H_{r_t} x(t), \tag{3}$$

where H_{r_t} is symmetrical positive definite matrices. Suppose the last trigger success moment is r_k, and the next trigger success moment is $r_k + 1$. The threshold error:$e(t) = x(t) - x(t_{mh})$. Construct the following controller:

$$u(t) = K_i x(r_k),\ t \in [r_k, r_{k+1}). \tag{4}$$

Consider the following dynamic event-triggered mechanism:

$$r_{k+1} = \inf\left\{t | t > r_k \wedge \eta(t) + \xi\left(x^T(t)\Xi_{\sigma(t)}x(t) - e^T(t)\Xi_{\sigma(t)}e(t)\right) \leq 0\right\}, \tag{5}$$

Where $\Xi_\sigma(t) > 0$ is a positive weight matrix of the triggered condition to be designed, $\xi \geq 0$. $\eta(t)$ is the internal dynamic variable satisfying

$$\dot{\eta}(t) = -\delta\eta(t) + x^T(t)\Xi_{\sigma(t)}x(t) - e_\tau^T(t)\Xi_{\sigma(t)}e_\tau(t), \tag{6}$$

Where $\delta > 0$, the initial condition $\eta(0) \geq 0$.

Considering the controller (4) and dynamic event-triggered mechanism (5), (1)can be described as follows:

$$\begin{cases}\dot{x}(t) = \left(A(r_t) + B(r_t)K(r_t)\right)x(t) + C(r_t)f(x(t), r_t) - B(r_t)K(r_t)e(t) + E(r_t)\omega(t), \\ y(t) = D(r_t)x(t), \end{cases} \tag{7}$$

Definition 2.1. For $r_0 \in \mathcal{N}$,the system under $\omega(t) = 0$ is called stochastic stability, if the following conditionis satisfled:

$$\lim_{t\to\infty} \mathbb{E}\left\{\int_0^t \| x(s) \|^2\, ds \mid (x_0, r_0)\right\} \leq \infty. \tag{8}$$

lemma 2.1. The system (7) is said to be H_∞ performance if the following inequality holds for any $t_f \geq 0$ and all $\omega(t) \in L_2[0,\infty)$:

$$\mathcal{E}\left\{\int_0^{t_f} - y^T(s)y(s) + \gamma^2\omega^T(s)\omega(s)ds\right\} \geq 0, \tag{9}$$

3. Stochastic Stability and H_∞ Performance Analysis

Theorem3.1. Given positive scalars δ, the nonlinear asynchronous switched system (7) under dynamicon $event-triggered\ \ mechanism\ (5)\ and\ the\ control\ action\ \ (4)\ is\ stochastic\ stability\ if\ there\ exist\ symmetrical\ positive$ defnite matrices $P_{r_t}, H_{r_t}, \Xi_{r_t} with proper\ dimensions, such\ that$

$$\Psi_p = \begin{bmatrix} \Psi_{11} & -P_pB_pK_p & 0 & P_pC_p \\ * & -\Xi_p & 0 & 0 \\ * & * & -\delta I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{10}$$

$$\Psi_{11} = A_p^TP_p + P_pA_p + \Xi_p + P_pB_pK_p + K_p^TB_p^TP_p + H_{1p} + \sum_{q\in\mathcal{N}'} \pi_{pq}(h)P_q.$$

Proof: Assume rt = p, define the Lyapunov function as follows:

$$U_p = x^T(t)P_px(t) + \eta(t), \tag{11}$$

Taking the time derivative of U_p along the state trajectory of (11), when ω (t) = 0 have

$$\mathcal{L}U(x(t), r_t = p) = x^T(t)P_p\left(A_p + B_pK_px(t) + C_pf^{\mathrm{T}}(x(t), r_t)\right) - B_pK_pe(t))x(t)$$
$$+\left(A_p + B_pK_px(t) + C_pf^{\mathrm{T}}(x(t), r_t)\right) - B_pK_pe(t))x(t)^TP_px(t) + x^T(t)\left(\sum_{q\in\mathcal{N}'} \pi_{pq}(h)P_q\right)x(t) + \dot{\eta}(t).$$

According to (3), get

$$\begin{aligned} \mathcal{L}U(x(t), r_t = p) \quad & \le x^T(t)P_p\big(A_p + B_pK_px(t) + C_pf^{\mathrm{T}}(x(t), r_t)) - B_pK_pe(t) + H_p\big) \\ & + \left(A_p + B_pK_px(t) + C_pf^{\mathrm{T}}(x(t), r_t)\right) - B_pK_pe(t))x(t)^TP_px(t) \\ & + x^T(t)\big(\textstyle\sum_{q\in\mathcal{N}} \pi_{pq}(h)P_q\big)x(t) - f^{\mathrm{T}}(x(t), r_t))f(xt, r_t) - \delta\eta(t) + x^T(t)\Xi_{\sigma(t)}x(t) \\ & - e^T(t)\Xi_{\sigma(t)}e(t). \end{aligned} \tag{12}$$

Combined with (10) and (12), get

$$\mathcal{L}U(x_t, r_t = p) \le \zeta^T(t)\Gamma^p\zeta(t) < 0,$$

where $\zeta(t) = col\{x(t),\ e(t),\ \eta(t),\ f(x(t), r_t = p)\}$. Thus $\mathcal{L}U(x_t, r_t = p) \le -\epsilon \parallel x(t) \parallel^2$. By Dynkin's formula and the well-known Gronwell-Bellman lemma, obtain:

$$\mathcal{E}\left\{\int_0^t \parallel x(s) \parallel^2 ds\right\} \le \frac{1}{\epsilon}U\big(x_{t_0}, r_0\big) - \frac{\varepsilon}{\epsilon}\mathcal{E}\{\parallel x(s) \parallel^2\} \le \frac{1}{\epsilon}U\big(x_{t_0}, r_0\big) < \infty.$$

Thereby, by Definition 2.1, The system (7) under ω (t) = 0 is called SS.

Theorem3.2. Given positive scalars $\delta, \gamma,$ the nonlinear asynchronous switched system (7) under the dynamic event-triggered mechanism (5) and the control action (4) is H_∞, if there exist symmetrical positive definite matrices $P_{r_t}, H_{r_t}, \Xi_{r_t}$ with proper dimensions, such that

$$\Phi_p = \begin{bmatrix} \Phi_{11} & -P_pB_pK_p & 0 & P_pC_p & P_pE_p \\ * & -\Xi_p & 0 & 0 & 0 \\ * & * & -\delta I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{13}$$

$$where, \Phi_{11} = \Psi_{11} + D_p^TD_p.$$

Proof: Define $\Delta(t) = -y^T(t)y(t) + \gamma^2\omega^T(t)\omega(t)$. Take the same Lyapunov function as Theorem 3.1. When $\omega(t) \neq 0$,

$$\mathcal{L}U(x(t), r_t = p) - \Delta(t) = \zeta_1^T(t)\Upsilon_p\zeta_1(t) \leq 0,$$

where, $\zeta_1(t) = col\{\zeta(t),\ \omega(t)\}$.

By Dynkin's formula, for any $t \geq 0$, obtain:

$$\mathcal{E}\{U(x_t, r_t = p)\} \leq \mathcal{E}\{U(x_{t_0}, r_0)\} + \mathcal{E}\left\{\int_0^t \Delta(s)ds\right\}.$$

Recalling(11), have

$$U(x(t), r_t = p) \geq x^T(t)P_p x(t) \geq 0.$$

Thus, for $\forall t_f \geq 0$,

$$\mathcal{E}\left\{\int_0^{t_f} \Delta(s)ds\right\} \geq \mathcal{E}\left\{\int_0^t \Delta(s)ds\right\} \geq x^T(t)px(t) \geq 0.$$

Thereby, by Definition 2.1, The system (7) is called H_∞.

4. Controller Design

Theorem 4.1. Given positive scalars δ, the nonlinear asynchronous switched system (7) under dynamicon $event-triggered\ mechanism$ (5) $and\ the\ control\ action$ (4) $is\ SS\ and\ H_\infty$, $if\ there\ exist\ symmetrical\ positive$defnite matrices $P_{r_t}, H_{r_t}, \Xi_{r_t}$ $and\ L_{rt}$ $with\ proper\ dimensions, such\ that$

$$\Delta_{p}=\begin{bmatrix} \Theta_p & Z_{1p}^T & Y_p^T & Z_{2p}^T & Y_p^T \\ * & -M_p & 0 & 0 & 0 \\ * & * & -M_p & 0 & 0 \\ * & * & * & -M_p & 0 \\ * & * & * & * & -M_p \end{bmatrix} < 0, \qquad (14)$$

Where, $\Theta_p = \begin{bmatrix} \Theta_{11} & -B_pL_p & 0 & P_pC_p & P_pE_p \\ * & -\Xi_p & 0 & 0 & 0 \\ * & * & -\delta I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$,

$$\Theta_{11} = A_p^TP_p + P_pA_p + \Xi_p + P_pB_pK_p + K_p^TB_p^TP_p + H_{1p} + \sum_{q\in\mathcal{N}'} \pi_{pq}P_q + D_p^TD_p.$$

Proof: Due to $P_PB_PK_P$,in inequality (12) ,we cannot resort to LMI to solve it. Let $M_PK_P = L_P$. Since M_P is a positive definite matrix, there exists M_p^{-1}. Defining $X_{1p} = N_p^{-1}Z_{1p} = N_p^{-1}[0, L_p, 0, 0, 0]$, $X_{2p} = N_p^{-1}Z_{2p} = N_p^{-1}[L_p + L_p^T, 0, 0, 0]$, $Y_p = [B_p^TP_p - N_p^TB_p^T, 0, 0, 0]$, According to (14), one can obtain

$$\begin{aligned}&\Theta_p + X_{1p}^TY_p + Y_p^TX_{1p} + X_{2p}^TY_p + Y_p^TX_{2p} \\ &\leq \Theta_p + X_{1p}^TN_pX_{1p} + Y_p^TN_p^{-1}Y_p + X_{2p}^TN_pX_{2p} + Y_p^TN_p^{-1}Y_p = \Delta_p\end{aligned}$$

Based on Schur complement and (14), get $\Delta_p < 0$. This completes the proof.

5. Simulation Example

5.1. Consider the Switched System (12) with Two Subsystems in the following Matrices

$$A_1=\begin{bmatrix}0 & 1\\ -4.9050 & -0.1\end{bmatrix}, A_2=\begin{bmatrix}0 & 1\\ -4.9050 & -0.8\end{bmatrix}, B_1=\begin{bmatrix}0\\ 1\end{bmatrix}, B_2=\begin{bmatrix}0\\ 0.8\end{bmatrix}, D_1=\begin{bmatrix}0.2 & 0\\ 0 & 0.1\end{bmatrix}, E_1=\begin{bmatrix}0.02 & 0\\ 0 & 0.01\end{bmatrix},$$

$$C_1=\begin{bmatrix}0.1 & 0.01\\ 0 & 0.3\end{bmatrix}, D_2=D_1, E_2=E_1, C_2=C_1, \delta=0.2,\ \gamma=0.1, H_1=H_2=0.01I.$$

Solving the matrix inequalities in Theorem 4.1 gives rise to

$$P_1=\begin{bmatrix}0.6751 & -0.0533\\ -0.0533 & 3.4856\end{bmatrix},\ P_2=\begin{bmatrix}0.6417 & -0.1503\\ -0.1503 & 2.8056\end{bmatrix},\ X_1=\begin{bmatrix}0.0508 & 0.0002\\ 0.0002 & 0.0633\end{bmatrix},\ X_2=\begin{bmatrix}0.0564\\ -0.0001\end{bmatrix}$$
$$L_1=[0.0019 \quad -0.0495],\ L_2=[-0.0028 \quad -0.0571],\ K_1=\begin{bmatrix}0.0006\\ -0.0144\end{bmatrix},\ K_2=\begin{bmatrix}-0.0010\\ -0.0200\end{bmatrix},$$
$$N_1=3.4489,\ N_2=2.8517$$

$$\mathcal{E}\{\pi_{pq}(h)\}=\begin{bmatrix}-1.7725 & 1.7725\\ 2.7082 & -2.7082\end{bmatrix}$$

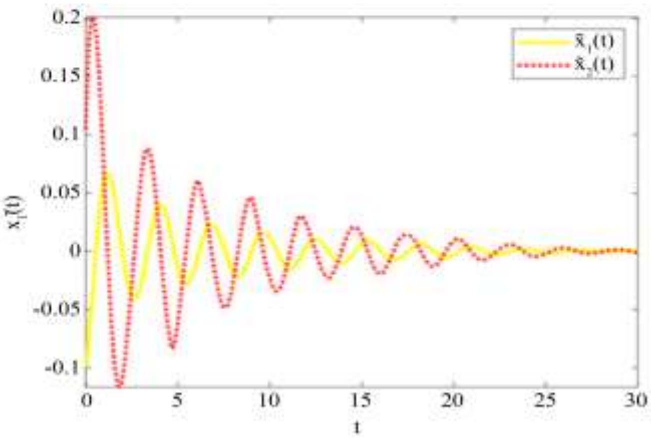

Fig. 1 The state trajectories of the system (7)

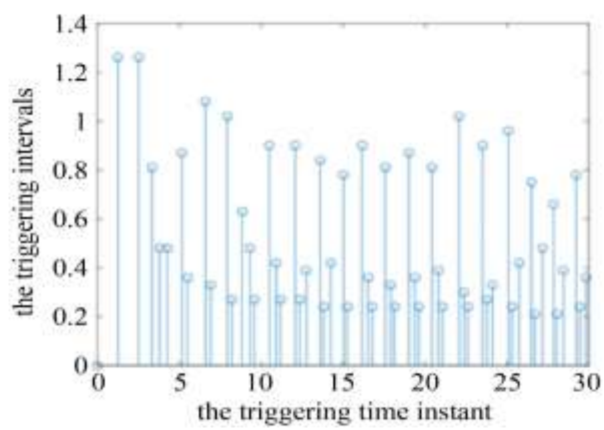

Fig. 2 The triggering intervals

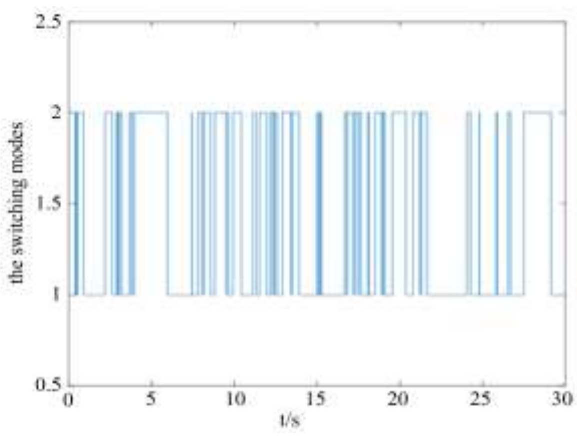

Fig. 3 The switching modes at different times

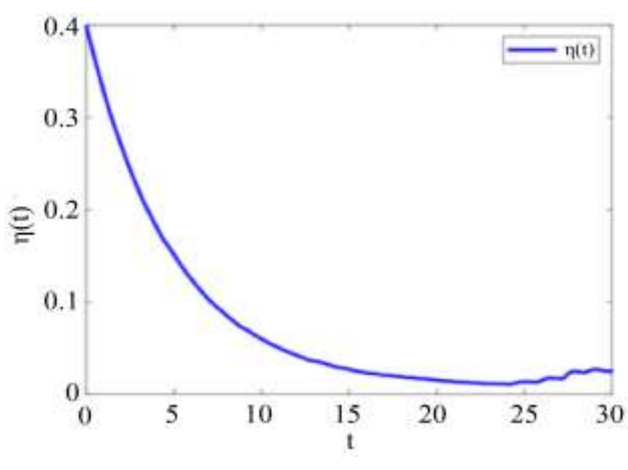

Fig. 4 The evolution of η (t)

Fig. 1 Show the state trajectories of the system (7) when perturbation input ω (t) = 0, which indicates that the system (12) is stochastic stability. Fig. 2 Presents the triggering intervals of the dynamic event-triggered mechanism. The switching modes at different times are displayed in Fig. 3. Fig. 4 shows the evolution of η (t), which indicates η (t) $\geq$ 0.

6. Conclusion

This study introduces a DETM with the aim of enhancing the flexibility and performance of control systems. Nonlinear are taken into account in the SMJSs. Strong conditions based on Lyapunov functions to ensure H_∞ performance and SS of the system. Finally, one numerical example is given to illustrate the effectiveness of the proposed methods.

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