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Original Article

Flow and Heat Transport of Kiwifruit Juice past a Divergent Channel with Suction/Injection by Power-Law Fluid Model

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Abstract *- The pseudoplastic nature of kiwifruit juice is considered in this study under certain prescribed conditions. The flow and heat transport modeling of kiwifruit juice along the wall of a divergent channel with applied suction or injection is investigated with a suitably fitted non-Newtonian Power-Law fluid model. To simplify the equations governing fluid flow, heat transfer, and the necessary boundary conditions, similarity variables are utilized to obtain self-similar equations. The reduced form of underlying equations is evaluated by the finite difference method-based MATLAB solver 'bvp4c'. The velocity and temperature profiles are graphically depicted by examining different values of the pertinent parameters that characterize the flow. The influence of rheological flow parameters on velocity and temperature are analyzed from graphs, and conclusions are drawn from the processing application's points of view.*

Keywords *- Divergent channel, Heat transport, Kiwifruit juice, Power-Law fluid, Pseudoplastic, Similarity variables.*

1. Introduction

Kiwi fruit is the edible berry of a woody vine. Actinidia deliciosa is the scientific name for the kiwifruit. Its natural habitat is the southern Chinese highlands. Kiwi is also known by many other names, such as Mihoutau, Macaque peach and the very popular Chinese gooseberry. It has green flesh and black seeds that are also edible. The fruit is petite and has a tawny skin. It is grown in large quantities, countries like New Zealand, Italy, California, France, Greece, Chile, Japan and South Korea. This fruit is successfully cultivated in Jammu and Kashmir, Himachal Pradesh, Assam, Nagaland, Mizoram, Meghalaya, and the highlands of Tripura. Vitamin C, fiber, calcium, iron, and phosphorus are among its many beneficial nutrients. It has a variety of health benefits, improves heart health, speeds up digestion, is beneficial for weight loss, lowers blood pressure, helps to clear out toxins, helps to fight cancer, etc.

For designing products, evaluating manufacturing processes, and planning packaging and storage, the knowledge of rheological flow characteristics of fluid fruit and vegetable products is essential. Existing literature primarily focuses on experimental studies of these properties by food scientists and theoretical analyses by mathematicians using appropriate fluid models for Newtonian and non-Newtonian fluids. However, there is a notable gap in mathematical modelling that specifically addresses the rheological flow parameters of juices from fruits and vegetables, whether they exhibit Newtonian or non-Newtonian behaviour. Addressing this gap is crucial for optimizing the design and layout of flow systems in food processing industries, thereby enhancing efficiency and effectiveness.

A comprehensive analysis of the most current research on the rheological characteristics of fluid food products has been presented by Diamante and Umemoto [1]. Krokida *et al.* [2] compiled information from the literature on the rheological features of fluid products. Kaya *et al.* [3] investigated the heat and mass transport processes of Hayward kiwi fruits during the drying process using both experimental and numerical methods. Goula *et al.* [4] conducted research on the rheological properties of kiwifruit juice at various levels of solid concentration and temperatures. Gerschenson *et al.* [5] looked at how processing affected the kiwi fruit's tissue structure and dynamic rheological behaviour. Traditional fruit from the Amazon area that has high bioactivity was studied for its rheological behaviour at different temperatures and soluble solid concentrations by Santos *et al.* [6].

A diverging channel is one whose cross-sectional area steadily rises in the direction of the stream's flow. Divergent channels are frequently employed by engineers to control the flow of fluids. Increasing the channel's cross-sectional area increases its manageability and control by reducing the flow rate. This is of the utmost importance in systems where constant or regulated flow is required. Duryodhan *et al.* [7] utilized both numerical and experimental methods to conduct a study where they investigated the flow of a single-phase liquid through microchannels that were either diverging or converging. Gepner and Floryan [8] examined flows in converging-diverging channels to determine which channel shapes result in the most effective mixing. Heat and mass transport in stretchable convergent/divergent channels was studied by Khan *et al*. [9]. Akinshilo *et al.* [10] investigated how a nanofluid moves and transmits heat across porous converging or diverging channels. Banerjee *et al.* [11] examined the behaviour of Casson fluids through permeable diverging channels, taking into account factors such as heat transfer, suction or blowing at the channel walls, and viscous dissipation.

The power-law fluid model is commonly employed to study the flow behaviour of fruit juices, as it can accurately describe the non-Newtonian behaviour that these fluids exhibit. It offers an easy and effective approach for describing the flow behaviour of liquid fruit products, taking into account their shear-thinning or shear-thickening properties. Juices from many different fruits have been shown to have a shear-thinning nature. The Power-law fluid model represents this behaviour by a Power-law law relationship, where the flow behaviour index defines the degree of shear-thinning and the consistency index quantifies the fluid's resistance to flow at a particular shear rate. The power-law fluid flow in the MHD boundary layer was investigated by Hirschhom et al. [12]. Zhang et al. [13] examined how power-law fluids transferred heat in pipes with varying cross sections. Saritha et al. [14] investigated the effects of Soret and Dufour phenomena on magnetohydrodynamic boundary layer flow over a flat plate using a power-law fluid model. Li et al. [15] conducted a numerical study to analyze fluid flow and heat transfer characteristics of a power-law fluid in an irregular channel with non-uniform suction and blowing conditions, focusing on the heating process. Evangelista et al. [16] explored the rheological characteristics and thermophysical properties of Malbec grape juice, examining how these properties vary with temperature and soluble solids content. Their aim was to optimize the unit operations involved in beverage industries. Quek et al. [17] used a rheometer to analyze how temperature and concentration affect the rheological behaviour of freeze-dried soursop juice concentrates across a wide range of conditions. Costa et al. [18] investigated both time-dependent and steady-state shear rheological properties of Jambolan pulp at various temperatures.

This study aims to investigate the flow behaviour of kiwi fruit juice past a divergent channel by Power-law fluid model for processing applications. The study examines how velocity and temperature profiles are influenced by changes in suction or injection parameters for distinct values of flow index and consistency coefficients. The numerically obtained results are plotted for discussion and to get physical insight into the results.

2. Mathematical Formulation

Kiwi fruit juice flowing steadily through a diverging channel is taken into consideration. The assumption of porous, immobile channel walls allows for the imposition of mass suction or blowing. The diverging channel wall serves as the x-axis, while the y-axis is orthogonal to it. The point where two channel walls converge is the origin.

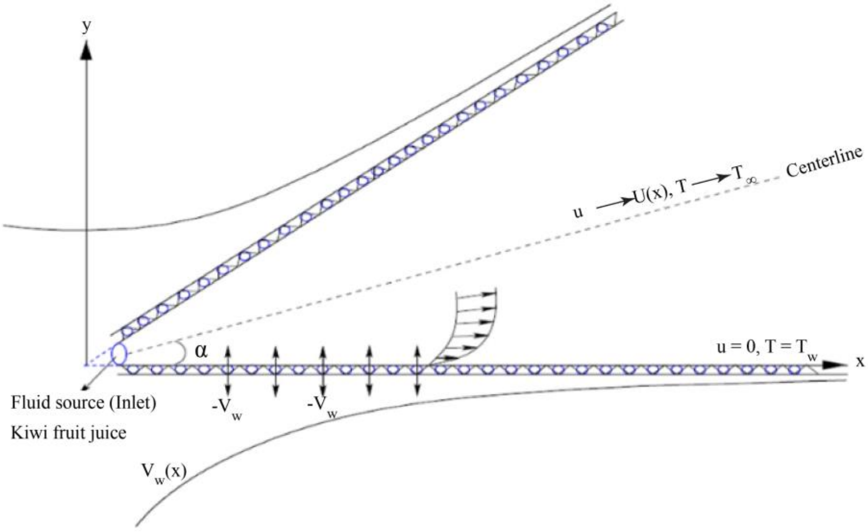

Fig. 1 Sketch of fluid domain geometry

The non-Newtonian shear-thinning nature of kiwifruit juice flow under certain prescribed conditions is guided by the following equations:

$$\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \frac{\partial \mathrm{v}}{\partial \mathrm{y}} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} - \nu\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^{n} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left|\frac{\partial u}{\partial y}\right|^{n+1} \tag{3}$$

and boundary conditions are:

$$At\ \mathrm{y} = 0\text{: } \mathrm{u} = 0\ , v = v_w\ (x),\ T = T_W \tag{4}$$

$$\text{As }\ y \to \infty\text{: }\ u = \mathrm{U(x)}, \mathrm{v} = 0\ ,\ \ T \to T_\infty \tag{5}$$

where,

u- velocity along x-axis

v- velocity along y-axis

U(x)-free stream velocity

n- flow index

T_W - temperature of the plate

T_∞- temperature far away from plate

v_w - constant normal wall velocity

c_p- specific heat at constant pressure

ρ-fluid density,

Potential flow velocity $U(x)$ at the wall of divergent channel:

$$U(x) = \frac{u_1}{x},\ \ u_1 > 0 \tag{6}$$

The dimensionally reduced version of the self-similar governing equations is obtained by introducing similarity variables:

$$\Psi = \sqrt[n+1]{\nu x U^{2n-1}}\, f(\eta), \qquad \eta = \sqrt[n+1]{\frac{U^{2-n}}{\nu \mathrm{x}}}\, y, \qquad T = T_\infty + (T_w - T_\infty)\theta(\eta) \tag{7}$$

Stream function $\Psi(x, y)$ is connected with velocity components as

$$u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x} \tag{8}$$

Using these transformations, the governing equations (2) and (3) are reduced as follows:

$$n(n+1)f'''(f'')^{n-1} + ff'' - (\mathrm{n}+1) = 0 \tag{9}$$

$$\theta'' + \frac{Pr}{n+1}f\theta' + PrEc|f''|^{n+1} = 0 \tag{10}$$

where,

Ψ – dimensionless stream function

Ec - Eckert number

θ – dimensionless temperature

Pr- Prandtl number

$\eta -$ similarity variables

The relevant boundary conditions (4) and (5) are transformed as

$$At\ \eta = 0\text{: } \mathrm{f} = f_w, f' = 0, \qquad \theta = 1 \tag{11}$$

$$As\ \eta \to \infty\text{: } f' = 1, \theta \to 0 \tag{12}$$

Where,

f_w - suction/injection parameter

3. Method of Solution

The Matlab solver ‘bvp4c’ is employed to evaluate a system of linear or non-linear BVP. This approach is distinct from the shooting approach and is algorithm-based. It can efficiently calculate the approximation of y(x) for every x in [a, b] while taking boundary conditions into account at each stage. With this approach, the boundary conditions at infinity are swapped out for those at a position that reasonably solves the presented issue. To apply the finite difference method-based solver ‘bvp4c’, the resultant governing equations (9) and (10) are transformed as follows:

$$f = y_1, f' = y_2,\ f'' = y_3, \theta = y_4, \theta' = y_5 \tag{13}$$

$$y_1' = y_2,\ y_2' = y_3, \qquad y_4' = y_5 \tag{14}$$

Making use of (13) and (14) in equations (9) and (10):

$$y_3' = -\frac{1}{n(n+1)} y_3^{1-n}[(y_1 y_3 - (n+1)] \tag{15}$$

$$y_5' = -\frac{Pr}{n+1}\,(y_1 y_5) - PrEc|y_3|^{n-1} \tag{16}$$

Boundary conditions (11) and (12) transformed as follows:

$$y_1(0) = f_w,\ y_2(0) = 0, y_4(0) = 1 \tag{17}$$

$$y_2(\infty) = 0, \qquad y_4(\infty) = 0 \tag{18}$$

4. Results and Discussion

The fluid flow modeling of kiwi fruit juice along the wall of a divergent channel with applied suction $(f_w < 0)$ and injection $(f_w > 0)$ is investigated with a suitably fitted non-Newtonian Power-Law fluid model. The velocity $(f'(\eta))$ and temperature $(\theta(\eta))$ distributions for three distinct values of flow index $n\ (= 0.357, 0.394, 0.402)$ exhibiting pseudo-plastic nature of fluid, in combination with other involved flow parameters, are presented graphically in Figure 2-8.

Table 1. Rheological parameters of kiwifruit juice [1]

Product	T (^{0}C)	k (Pa s^n)	n
Kiwifruit juice	35	0.298	0.357
	45	0.233	0.394
	55	0.189	0.402

The velocity profiles for variation of flow index (n) with applied injection $(f_w > 0)$ and suction $(f_w < 0)$ are presented in Figure 2(a-b). The fluid velocity enhances with the rise of flow index values in both cases but a significant increase is observed for suction. For fluids that adhere to the Power-law fluid model, the velocity of a fluid increases in direct proportion to the shear rate raised to the exponent 1/n. Therefore, as the value of the flow index ‘n’ increases, the velocity of the fluid also increases for a given shear rate.

Figure 3(a-c) demonstrates that fluid velocity accelerates as the injection parameter increases, across three different flow index (n) values, whereas it is observed from Figure 4(a-c) that the fluid velocity diminishes with the growth of the suction parameter. Injection enhances fluid motion because it gives additional energy with growing fluid. In the case of suction, the fluid particles are absorbed through the wall of the divergent channel, as such the fluid motion diminishes and causes a reduction in velocity boundary layer thickness.

The temperature profiles for variation of flow index (n) with applied injection and suction parameters are plotted in Figure 5(a-b). As the flow index increases during injection, the fluid temperature rises, whereas it decreases during suction. This phenomenon occurs because, with higher flow index values, the fluid exhibits higher resistance to deformation (shear thickening behaviour) under injection conditions, leading to increased kinetic energy conversion into thermal energy. Conversely, under suction conditions, lower flow index values result in easier deformation (shear thinning behaviour), reducing the conversion of kinetic energy into thermal energy and hence lowering the fluid temperature.

Figure 6(a-c) illustrates that the fluid temperature decreases with injection parameter growth for three distinct values of flow index (n) whereas it is noticed from Figure 7(a-c) that the fluid temperature rises with the growth of the suction parameter. In the case of injection, the fluid is rapidly expanded as it is injected into the receiving fluid. This rapid expansion causes a decrease in pressure, which in turn causes a decrease in temperature. The friction that suction creates between the fluid and the suction device's surface may raise the fluid's temperature even further.

The profiles of temperature for variation of the Prandtl number with applied injection $(f_w > 0)$ and suction $(f_w < 0)$ are shown in Fig. 8 (a-b). The fluid temperature reduces with the rise of Prandtl number with applied injection, but it enhances in the case of applied suction. When the Prandtl number increases with injection, it means that the fluid is more efficient at transferring momentum than heat. This can result in a reduction in the temperature of the fluid, but the suctioning process compresses the fluid, which increases its internal energy and temperature.

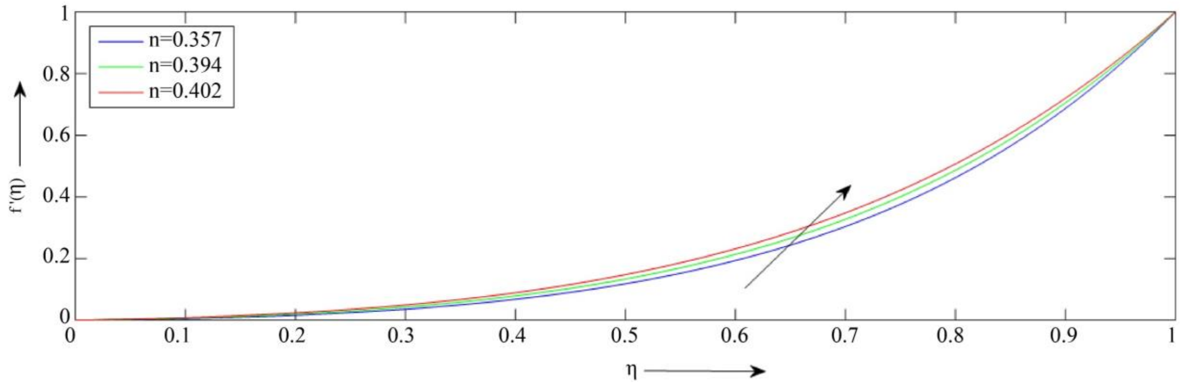

Fig. 2(a). $\boldsymbol{f'(\eta)}$ **against** $\boldsymbol{\eta}$ **for variation of** $\boldsymbol{n}$ **for** $\boldsymbol{f_w = 0.5}$

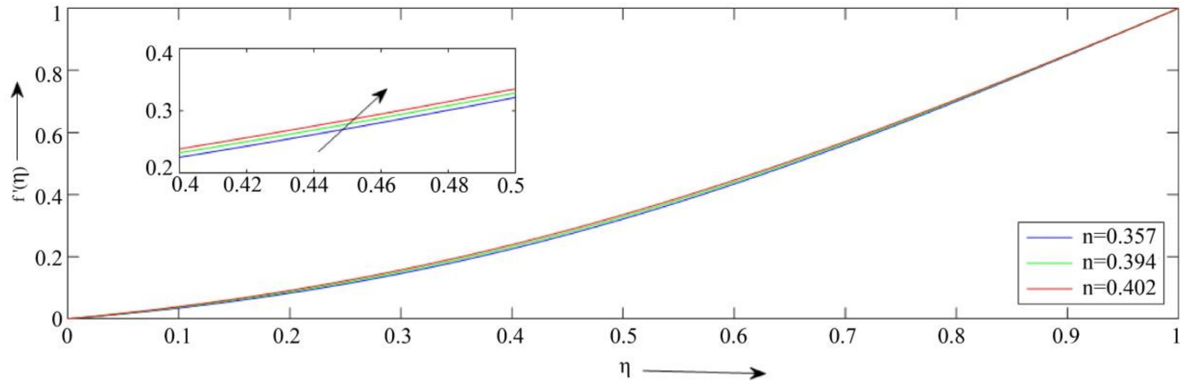

Fig. 2(b). $\boldsymbol{f'(\eta)}$ **against** $\boldsymbol{\eta}$ **for variation** $\boldsymbol{n}$ **for** $\boldsymbol{f_w = -0.5}$

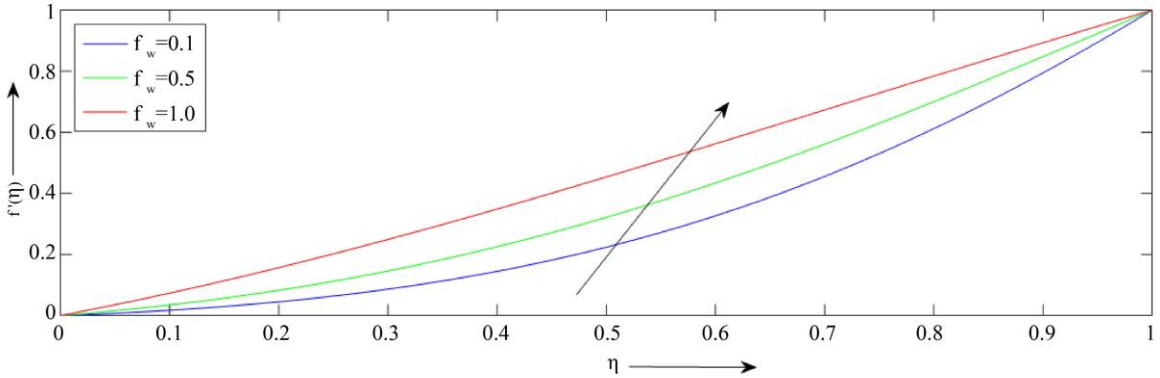

Fig. 3(a). $f'(\eta)$ **against** η **for variation of** $f_w > 0$ **for** n **= 0.357**

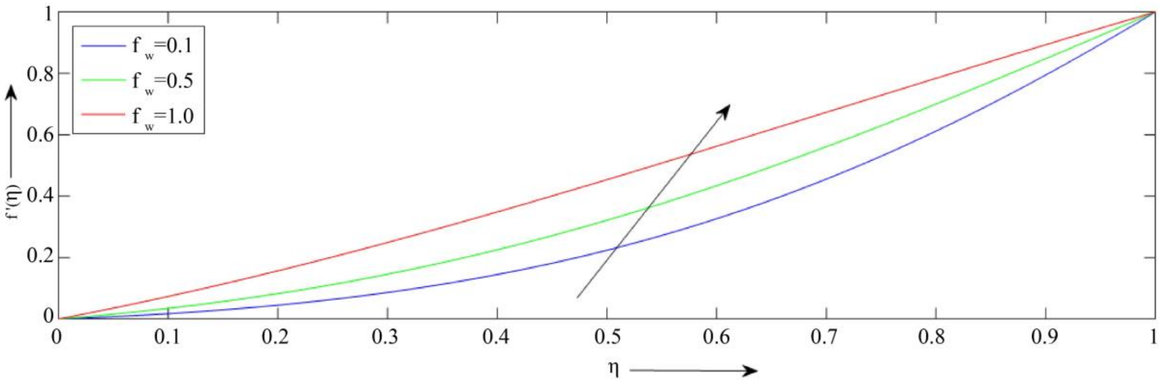

Fig. 3(b). $f'(\eta)$ **against** η **for variation of** $f_w > 0$ **for** n **= 0.394**

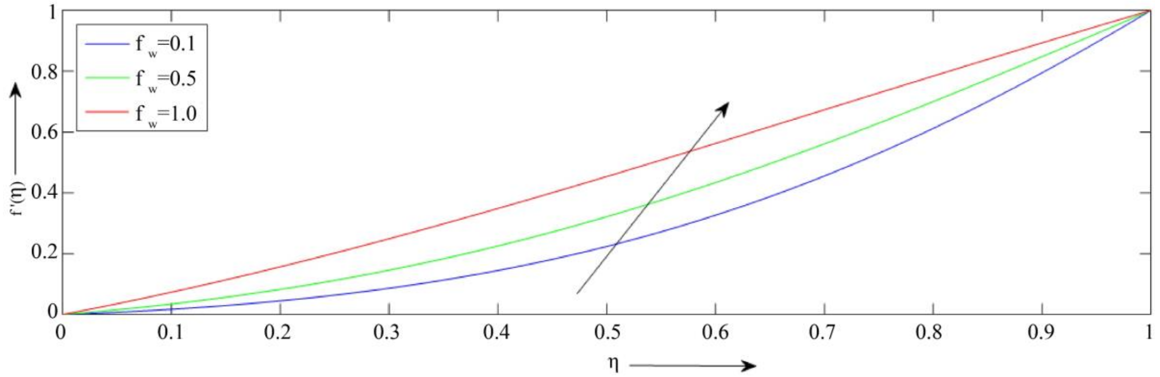

Fig. 3(c). $f'(\eta)$ **against** η **for variation of** $f_w > 0$ **for** n **= 0.402**

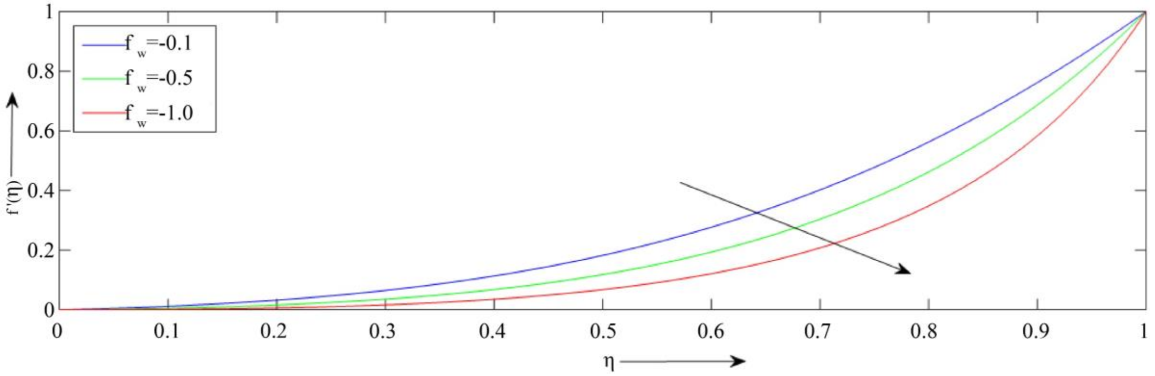

Fig. 4 (a). $f'(\eta)$ **against** η **for variation of** $f_w < 0$ **for** n **= 0.357**

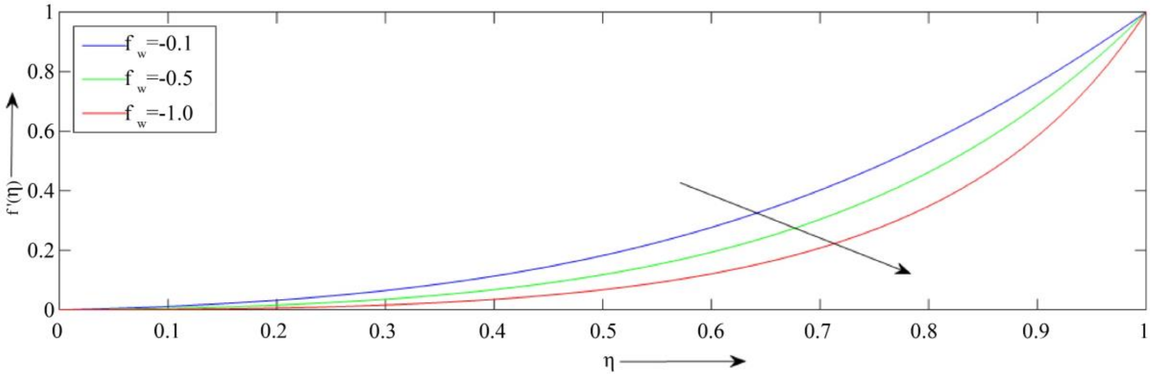

Fig. 4(b). $f'(\eta)$ **against** η **for variation of** $f_w < 0$ **for** n **= 0.394**

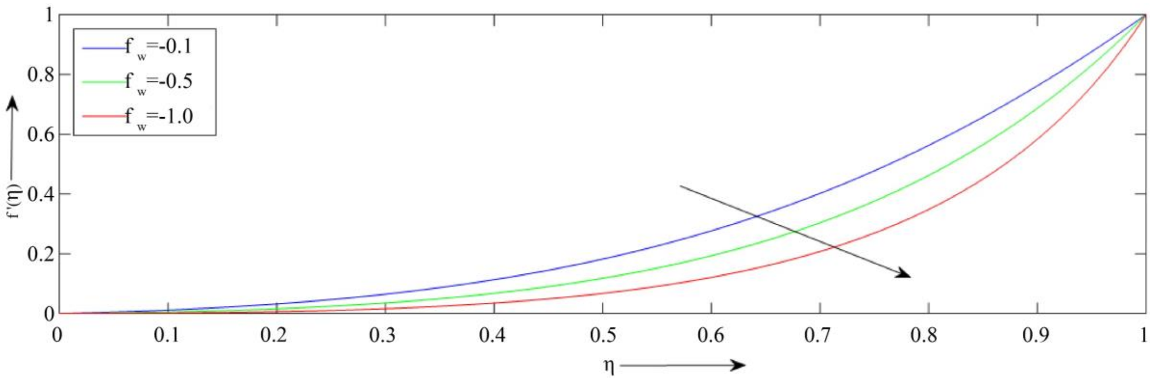

Fig. 4(c). $f'(\eta)$ **against** η **for variation of** $f_w < 0$ **for** n **= 0.402**

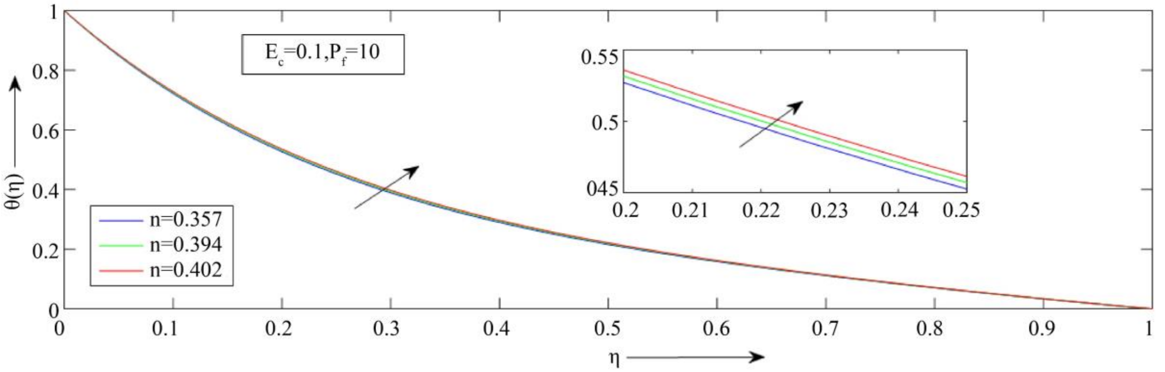

Fig. 5(a). $\theta(\eta)$ **against** η **for variation of** n **for (a)** $f_w = 0.5$

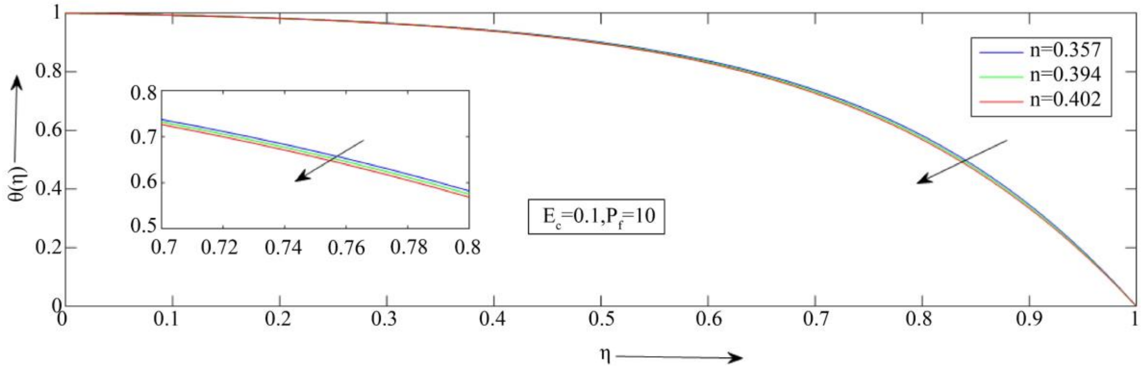

Fig. 5(b). (η) **against** η **for variation of** n **for (a)** $f_w = -0.5$

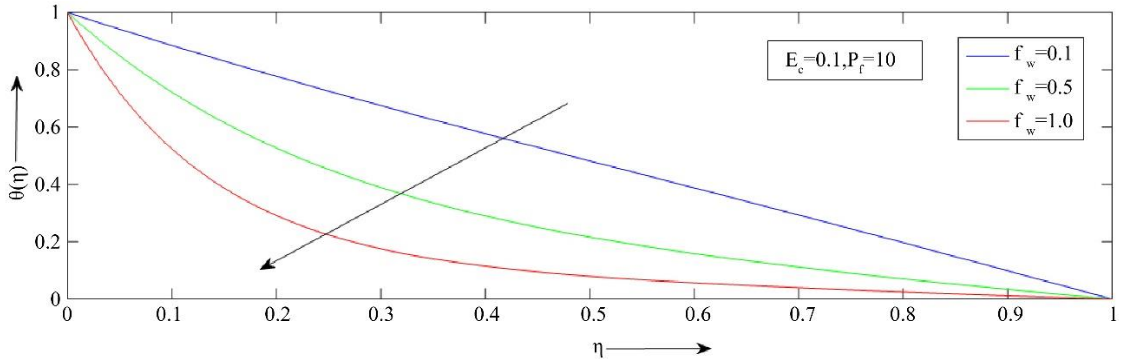

Fig. 6(a). $\theta(\eta)$ **against** η **for variation of** $f_w > 0$ **for** n **= 0.357**

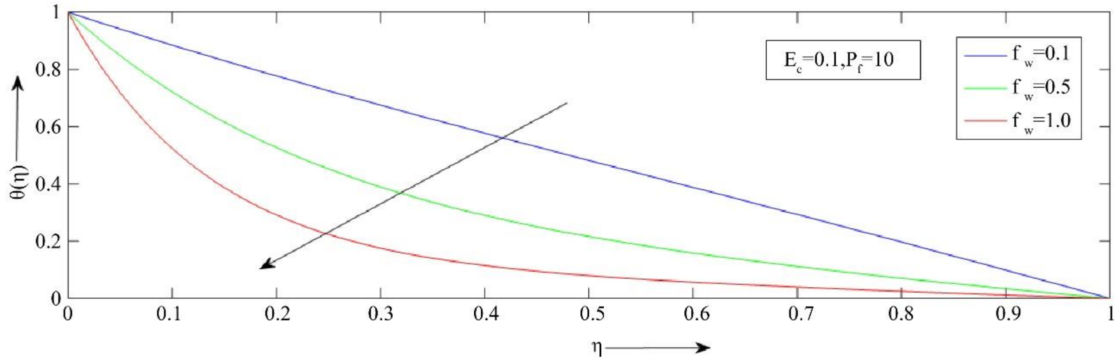

Fig. 6(b). $\theta(\eta)$ **against** η **for variation of** $f_w > 0$ **for** n **= 0.394**

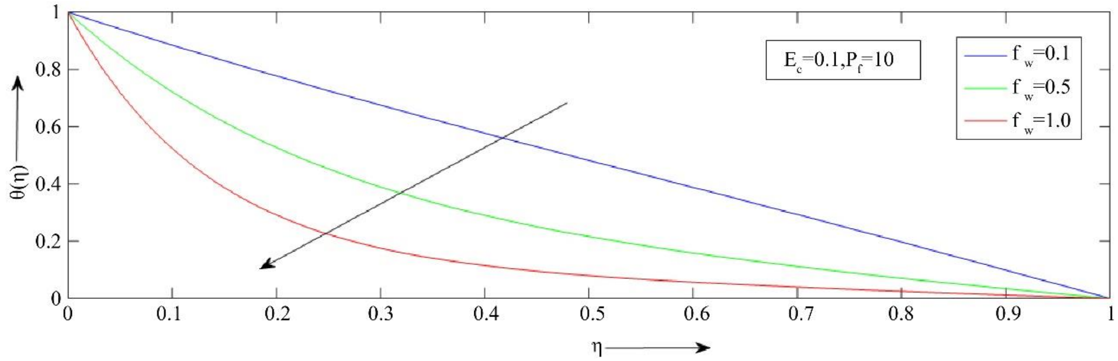

Fig. 6(c). $\theta(\eta)$ **against** η **for variation of** $f_w > 0$ **for** n **= 0.402**

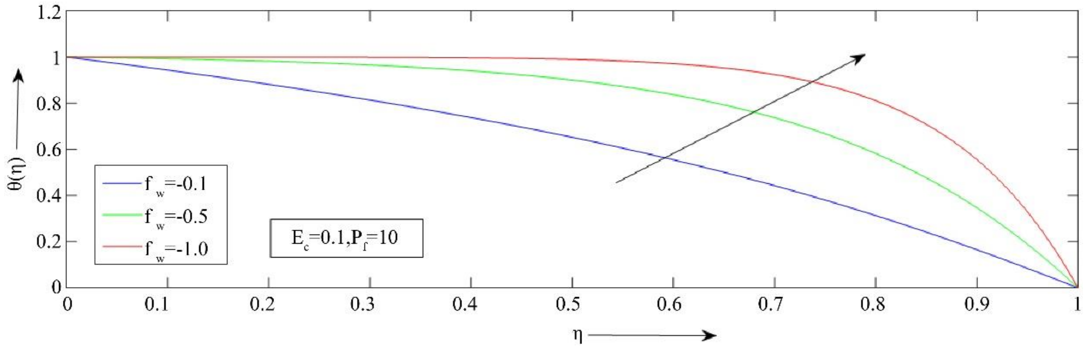

Fig. 7(a). $\theta(\eta)$ **against** η **for variation of** $f_w < 0$ **for** n **= 0.357**

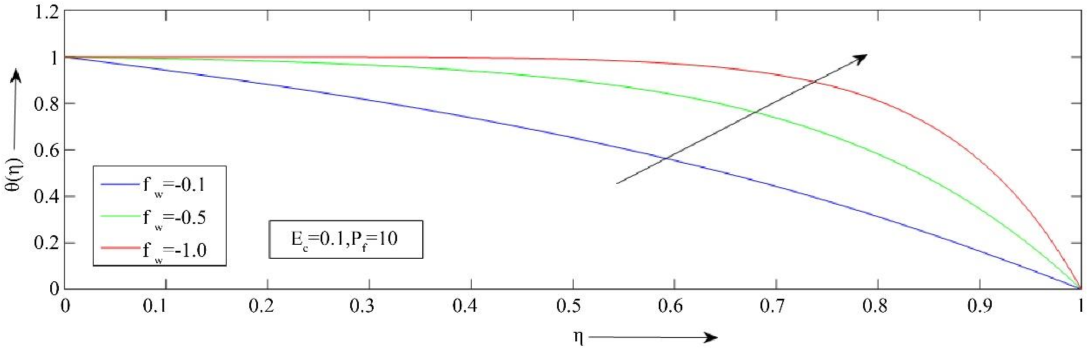

Fig. 7(b). $\theta(\eta)$ **against** η **for variation of** $f_w < 0$ **for** n **= 0.394**

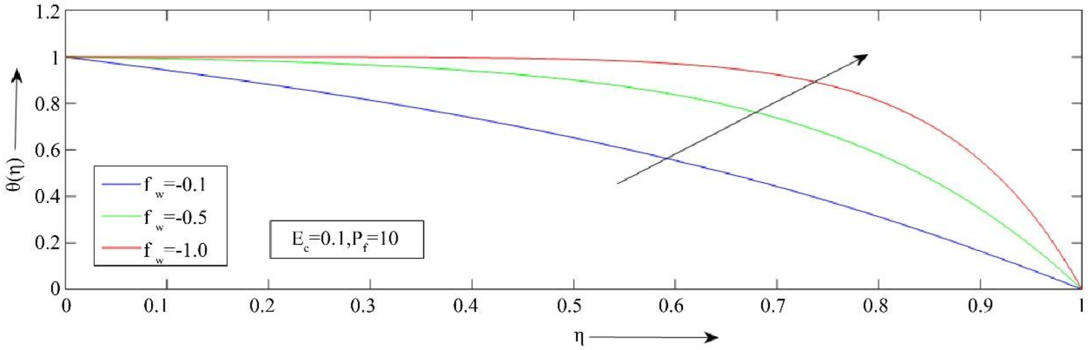

Fig. 7(c). $\boldsymbol{\theta(\eta)}$ **against** $\boldsymbol{\eta}$ **for variation of** $\boldsymbol{f_w < 0}$ **for** $\boldsymbol{n}$ **= 0.402**

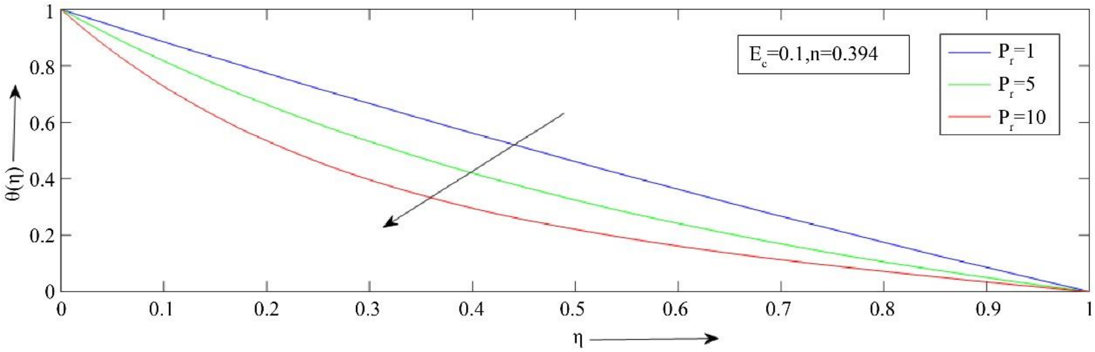

Fig. 8(a). $\boldsymbol{\theta(\eta)}$ **against** $\boldsymbol{\eta}$ **for variation of** $\boldsymbol{P_r}$ **for** $\boldsymbol{f_w = 0.5}$

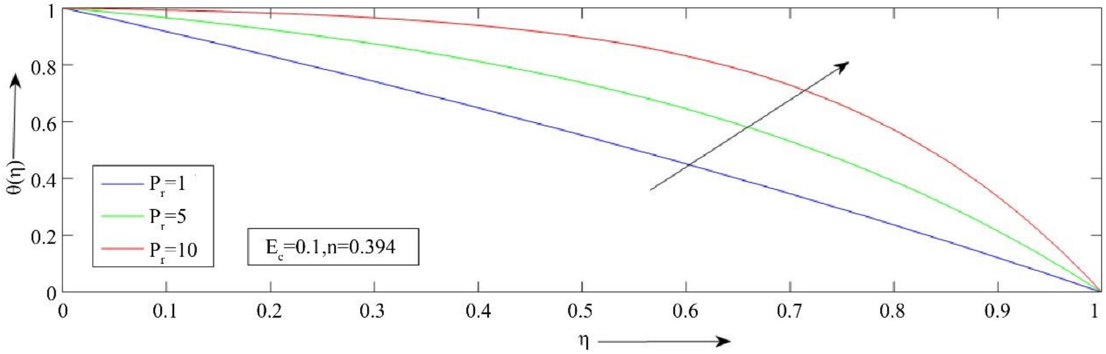

Fig. 8(b). $\boldsymbol{\theta(\eta)}$ **against** $\boldsymbol{\eta}$ **for variation of** $\boldsymbol{P_r}$ **for** $\boldsymbol{f_w = -0.5}$

5. Conclusion

The fluid velocity enhances with the rise of flow index, but significant increase is observed for suction. Injection enhances fluid motion because it gives additional energy with growing fluid while suction helps fluid particles for absorption and thus fluid motion diminishes and causing in a decrease in the thickness of the velocity boundary layer. The fluid is rapidly expanded as it is injected into the receiving fluid, which causes a decrease in pressure and thus diminishes the temperature. Friction that results from suction between the surface of the suction device and the fluid may raise the fluid's temperature further. A growing Prandtl number makes the fluid more efficient at transferring momentum than heat and thus reduces the temperature of the fluid, but the suctioning process compresses the fluid, which increases its internal energy and temperature.

6. Scope for future work

There is ample scope to extend this work for further research due to the diversified applications in engineering and food sciences. Non-Newtonian fluid flows of different fruit and vegetable products can also be studied with the help of the same fluid model. Fluid flow problems of different geometry can also be considered as per the requirements of the food industry. Some fluid properties have been discussed in this study, but there are many rheological properties of fluids of engineering interest that may be incorporated for further research. A flow simulation of the problem may give a clear picture of the obtained results.

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