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Original Article

Fuzzy Transportation Problem Using New Ranking Techniques to Order Pentagonal Fuzzy Numbers with Error Using Lagrange's Interpolation Formula

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Abstract - *In this research article, we transform the conventional transportation problem into a fuzzy transportation problem employing symmetric pentagonal fuzzy numbers (a1-2d, a1-d, a1, a1+d, a2+2d). The ordering of fuzzy pentagonal numbers is accomplished through the alpha-cut method. To quantify the discrepancy between the crisp and fuzzy transportation problems, we examine the error for varying values of d (1, 2, and 3). The data obtained is subjected to Lagrange's polynomial fit to model the error term. Subsequently, we conduct a comparative analysis of the errors derived from the fuzzy transportation method and those obtained through Lagrange's polynomial for different d values, specifically d=4.*

Keywords - *Fuzzy, Ranking, α-cut, Pentagonal, Transportation, Python, Lagrange's, Error.*

1. Introduction

In the contemporary landscape of optimization and decision-making, addressing uncertainty and imprecision has become a crucial aspect of problem-solving. This article delves into the intricate realm of transportation problems, presenting a transformative approach that converts conventional crisp scenarios into fuzzy counterparts. The cornerstone of this transformation lies in the utilization of pentagonal fuzzy numbers, introducing a nuanced layer of ambiguity to the traditional problem formulations.

Central to our investigation is the exploration of a novel method for ranking pentagonal fuzzy numbers intricately tied to the concept of alpha cuts within this fuzzy numerical framework. By establishing a robust foundation for comparing two pentagonal fuzzy numbers, this article pioneers a distinctive ranking methodology that lays the groundwork for solving fuzzy optimization problems. The integration of pentagonal fuzzy numbers and the proposed ranking technique not only contributes to theoretical advancements but also holds practical significance in addressing real-world challenges characterized by uncertain and imprecise parameters.

This research seeks to broaden the understanding of fuzzy optimization methodologies, offering insights into their application in diverse problem-solving contexts. The implications of this work extend beyond the theoretical realm, promising practical solutions that resonate with the complexities and uncertainties inherent in contemporary optimization scenarios.

2. Basic Concepts

2.1. Pentagonal Fuzzy Number

A pentagonal fuzzy number is a type of fuzzy number that has a pentagonal membership function. The membership function of a pentagonal fuzzy number is defined by five parameters: a, b, c, d, and e, where $a \leq b \leq c \leq d \leq e$.

Pentagonal fuzzy number denoted by $A = (a_1, a_2, a_3, a_4, a_5)$ and its membership function is defined as follows The shape of the membership function is pentagonal, with a plateau between b and c, and two sloping sides between a and b, and between c and d. The height of the plateau is 1, and the height of the sloping sides varies linearly from 0 to 1. Pentagonal fuzzy numbers are commonly used in decision-making and optimization problems where uncertain or imprecise information is present. They can represent a range of possible values for a decision variable or an objective function coefficient and can be used to model uncertainty in the problem parameters. Pentagonal fuzzy numbers can be added, subtracted, multiplied, and divided using appropriate operations on their membership functions. These operations can be used to perform fuzzy arithmetic and to derive fuzzy solutions for decision-making and optimization problems.

$$\mu_A(x)=\begin{cases} 0 & x < x_1 \\ \dfrac{x-x_1}{x_2-x_1} & x_1 \le x < x_2 \\ \dfrac{x-x_2}{x_3-x_2} & x_2 \le x < x_3 \\ 1 & x = x_3 \\ \dfrac{x_4-x}{x_4-x_3} & x_3 < x \le x_4 \\ \dfrac{x_5-x}{x_5-x_4} & x > x_5 \end{cases}$$

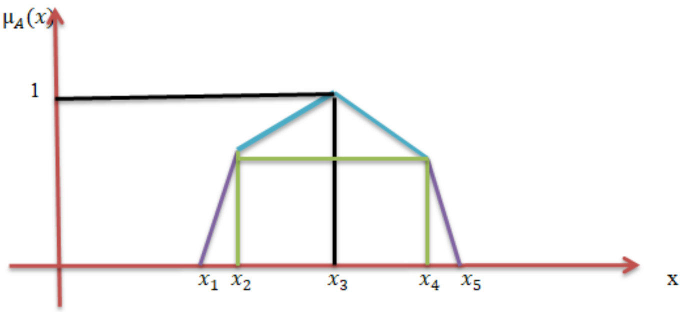

Fig. 1 Pentagonal fuzzy number [x_1, x_2, x_3, x_4 , x_5]

2.2. Operations on Pentagonal Fuzzy Numbers

$\bar{A}$ = (a_1, a_2, a_3, a_4, a_5), $\bar{B}$ = (b_1, b_2, b_3, b_4, b_5)

i) Addition: $\bar{A}$ (+) $\bar{B}$ = (a_1, a_2, a_3, a_4, a_5) + (b_1, b_2, b_3, b_4, b_5)
= ($a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$, $a_4 + b_4$, $a_5 + b_5$)

ii) Subtraction: $\bar{A}$ (-) $\bar{B}$ = (a_1, a_2, a_3, a_4, a_5) - (b_1, b_2, b_3, b_4, b_5)
= ($a_1 - b_5$, $a_2 - b_4$, $a_3 - b_3$, $a_4 - b_2$, $a_5 - b_1$)

iii) Multiplication: $\bar{A}$ (×) $\bar{B}$= (a_1, a_2, a_3, a_4, a_5) × (b_1, b_2, b_3, b_4, b_5)
= ($a_1 \times b_1$, $a_2 \times b_2$, $a_3 \times b_3$, $a_4 \times b_4$, $a_5 \times b_5$)

2.3. Python code to find Addition, Subtraction, Multiplication and Division of Two Pentagonal Fuzzy Numbers

```
print('Enter Your first Pentagonal number=') # display the message for user
a1=float(input(''))  #first component of first fuzzy number
b1=float(input('')) #second  component of first fuzzy number
c1=float(input('')) #third  component of first fuzzy number
d1=float(input('')) #fourth component of first fuzzy number
e1=float(input('')) #fifth component of first fuzzy number
a=[a1,b1,c1,d1,e1]  # list of first pentagonal fuzzy number
print('Enter Your second Pentagonal number')
a2=float(input('')) #first component of second  fuzzy number
b2=float(input('')) #second component of second  fuzzy number
c2=float(input('')) #third  component of second  fuzzy number
d2=float(input('')) #fourth component of second  fuzzy number
e2=float(input('')) #fifth component of second  fuzzy number
b=[a2,b2,c2,d2,e2]  # list of second  pentagonal fuzzy number
print('Enter Your Choice 1= Addition 2= Subtraction  3 = Multiplication ,4 = order')
ch=int(input(''));  # switch statement for operation on fuzzy number choice
if(ch==1):
```

```
    #case 1 for the addition of the fuzzy number
    'Addition of two fuzzy Pentagonal numbers is'
    c=[a1+a2,b1+b2,c1+c2,d1+d2,e1+e2]
    print(c)
elif(ch==2):
    #case 2  % case 2 for the subtraction  of the fuzzy number
    'subtraction of two fuzzy Pentagonal is(a-b)'
    c=[a1-e2,b1-d2,c1-c2,d1-b2,e1-a2]    # difference of fuzzy numbers
    print(c)
elif(ch==3):
    #case 3 % case 3 for multiplication  of fuzzy number#case 3 % case 3 for multiplication  of fuzzy number
    'Multiplication of two fuzzy Pentagonal is(a-b)'
    c=[a1*a2,b1*b2,c1*c2,d1*d2,e1*e2]
    print(c)
```

2.4. α- Cut for Pentagonal Fuzzy Number

For any $\alpha \in [0, 1]$

$$\frac{a_1^\alpha - a_1}{a_2 - a_1} = \alpha, \quad \frac{a_2^\alpha - a_2}{a_3 - a_2} = \alpha \quad \frac{a_4 - a_4^\alpha}{a_4 - a_3} = \alpha, \quad \frac{a_5 - a_4^\alpha}{a_5 - a_4} = \alpha$$

$a_1^\alpha = (a_2 - a_1)\,\alpha + a_1 \quad a_2^\alpha = (a_3 - a_2)\,\alpha + a_2$

$a_4^\alpha = -(a_4 - a_3)\,\alpha + a_4 \quad a_5^\alpha = -(a_5 - a_4)\,\alpha + a_5$

Thus $\bar{A}_\alpha = [a_1^\alpha \quad a_3^\alpha \quad a_4^\alpha \quad a_5^\alpha]$

2.5. Ordering Fuzzy Number

To order any two pentagonal fuzzy numbers $\bar{X} = [a_1, a_2, a_3, a_4, a_5]$ and $\bar{Y} = [b_1, b_2, b_3, b_4, b_5]$, we find here α cut say $\bar{X}_\alpha$ and $\bar{Y}_\alpha$. Thus $\bar{X}_\alpha = [a_1^\alpha, a_2^\alpha]$ and $\bar{X}_\alpha = [b_1^\alpha, b_2^\alpha]$ Then $\bar{X} \le \bar{Y}$ if $a_1^\alpha \le b_1^\alpha$ and $a_2^\alpha \le b_2^\alpha$ otherwise $\bar{Y} \le \bar{X}$

3. Numerical Example

Consider the following crisp transportation problem: Suppose there are three factories (F1, F2, and F3) and four warehouses (W1, W2, W3, and W4). The transportation costs per unit from each factory to each warehouse are given in the following table:

Table 1. Crisp transportation problem

Source \ Target	$\bar{W}_1$	$\bar{W}_2$	$\bar{W}_3$	$\bar{W}_4$	Supply
F_1	6.00	3.00	5.00	4.00	23.00
F_2	5.00	9.00	2.00	7.00	17.00
$\bar{F}_3$	5.00	**7.00**	8.00	6.00	9.00
Demand	8.00	13.00	18.00	10.00	

The objective is to determine the optimal transportation plan that minimizes the total transportation cost while satisfying the supply and demand constraints.

To solve this crisp transportation problem, we can use any of the traditional methods, such as the Least Cost Method. Minimum transportation cost = 161.0

3.1 Fuzzy Transportation Problem by Using Pentagonal Fuzzy Number

Table 2. FLCM for central pentagonal fuzzy number(d=1)

	Destination				Supply
Originl	[4 5 6 7 8]	[1 2 3 4 5] **(10 11 12 13 14)**	[3 4 5 6 7] **(-5 -2 1 4 7)**	[2 3 4 5 6] **(8 9 10 11 12)**	(21 22 23 24 25)
Demand	[3 4 5 6 7]	[7 8 9 10 11]	[0 1 2 3 4] **(14 15 16 18 19)**	[5 6 7 8 9]	(15 16 17 18 19)
	[3 4 5 6 7] **(5 6 7 8 9)**	[5 6 7 8 9]	[6 7 8 9 10] **(-4 -2 1 4 6)**	[4 5 6 7 8]	(7 8 9 10 11)
	(6 7 8 9 10)	(11 12 13 14 15)	(16 17 18 19 20 21)	(8 9 10 11 12)	
	Minimum Fuzzy Transportation Cost = [9,20,39,58, 62] + [-4,5,20,44,50] + [14,23,41,57,68] + [-2,13,32,56,57] + [18,28,40,54,59] + [12,7,32,54,79] = [-19,24,105,208,319] = **176.64**				

Table 3. FLCM for central pentagonal fuzzy number(d=2)

	Destination				Supply
Origin	[2 4 6 8 10]	[-1 1 3 5 7] **(12 10 12 14 15)**	[1 3 5 7 9] **(-7 -4 1 6 8)**	[0 2 4 6 8] **(6 8 10 12 14)**	(19 21 23 25 27)
	[1 3 5 7 9]	[5 7 9 11 13]	[-2 0 2 4 6] **(12 14 16 18 20)**	[3 5 7 9 11]	(13 15 17 19 21)
	[1 3 5 7 9] **(3 5 7 9 11)**	[3 5 7 9 11]	[4 6 8 10 11] **(-5 -1 1 5 8)**	[2 4 6 8 10]	(5 7 9 11 13)
Demand	(4 6 8 10 12)	(9 11 13 15 17)	(14 16 18 20 22)	(6 8 10 12 14)	
Minimum Fuzzy Transportation Cost = [10,23,39,57, 59] + [-3,7,20,46,49] + [14,29,41,57,67] + [-4,13,32,56,58] + [14,25,40,58,50] + [10,6,32,56,89] = [-20,25,105,205,317] = **328.68**					

Table 4. FLCM for central pentagonal fuzzy number (d=3)

	Destination				Supply
Origin	[0 3 6 9 12]	[-3 0 3 6 9] **(10 11 12 13 14)**	[-1 2 5 8 11] **(-5 -2 1 6 9)**	[-2 1 4 7 10] **(4 7 10 13 16)**	(17 20 23 26 29)
	[-1 2 5 8 11]	[3 6 9 12 15]	[-4 -1 2 5 8] **(10 13 16 19 22)**	[1 4 7 10 13]	(11 14 17 20 23)
	[-1 2 5 8 11] **(1 4 7 10 13)**	[1 4 7 10 13]	[2 5 8 11 14] **(-6 -3 1 4 7)**	[0 3 6 9 12]	(3 6 9 12 15)
Demand	(2 5 8 11 14)	(7 10 13 16 19)	(12 15 18 21 24)	(4 7 10 13 16)	
Minimum Fuzzy Transportation Cost = [10,26,39,58, 59] + [-4,2,20,44,49] + [14,28,41,57,67] + [-2,13,32,56,57] + [17,26,40,56,57] + [11,5,30,52,78] = [-22,26,105,204,316] = **558.52**					

3.2. Interpolation

Table 5. Data for Lagrange's interpolation formula to find polynomial

Error = f(x) = Minimum crisp transportation cost - minimum fuzzy transportation cost				
x		1	2	3
f(*x*)		28.61	167.68	397.52

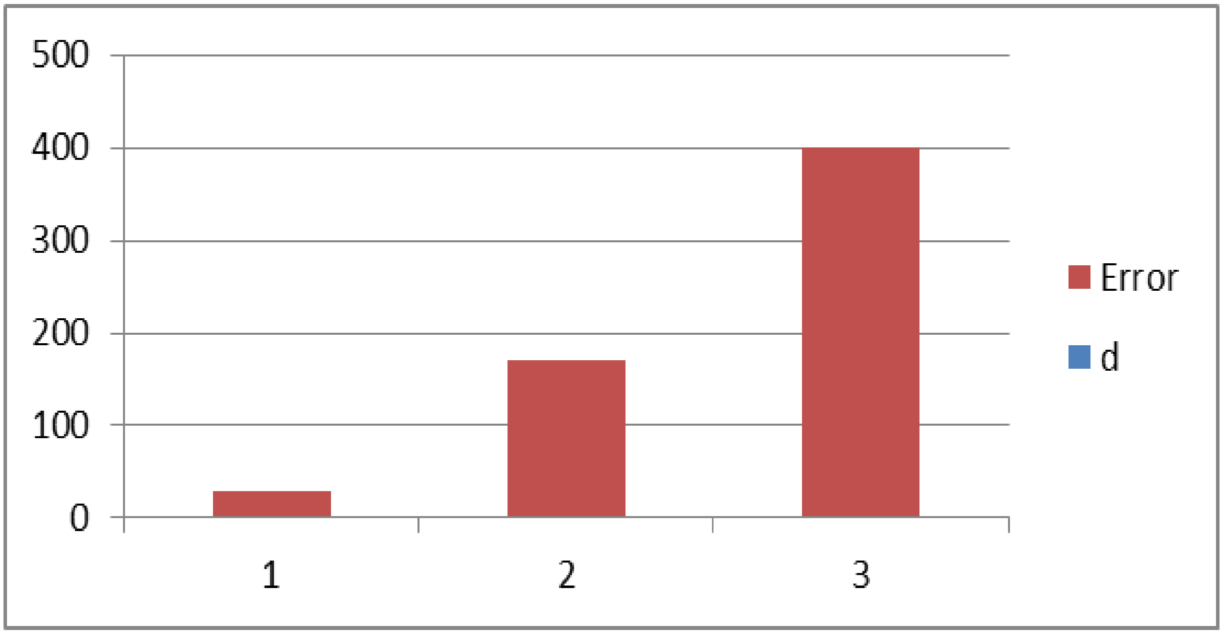

Fig. 2 Graphical presentation of distance d and error

3.3. Lagrange's Interpolation Polynomial

Lagrange's interpolation formula is a mathematical method used to approximate a polynomial that passes through a given set of data points. This interpolation technique provides a way to construct a polynomial of degree (n-1), where n is the number of data points.

$$f(x)=45.385x^2+2.915x-19.69$$

3.4. Verification

To verify the result, we take the value of d=5 in the central pentagonal fuzzy number. We calculate minimum fuzzy transportation cost and Error by using Lagrange's interpolation polynomial.

Then, we check whether the error obtained by both methods is equal or not.

Algorithm

Step 1: Obtain the solution to the transportation problem using the matrix Minima method.

Step 2: Solve the fuzzy transportation problem by using a central trapezoidal fuzzy number by taking $d = 1$

Step 3: Calculate the error by using the difference between crisp solution and fuzzy solution.

Step 4: Repeat steps 2 and 3 for $d = 2$ and $d = 3$.

Step 5: Find the divide difference interpolation polynomial, which passes through the above data.

Step 6: Verify the result for $d = 5$.

Table 6. FLCM for pentagonal fuzzy number (d=5)

Destination				Supply
[-4 1 6 11 16]	[-7 -2 3 8 13] **(2 7 12 17 22)**	[-5 0 5 10 15] **(-9 -4 1 6 11)**	[-6 -1 4 9 14] **(8 9 10 11 12)**	(13 18 23 28 33)
[-5 0 5 10 15]	[-1 4 9 14 19]	[-8 -3 2 7 13] **(6 11 16 21 26)**	[-3 2 7 12 17]	(7 12 17 22 27)
[-5 0 5 10 15] **(-3 2 7 12 17)**	[-3 2 7 12 17]	[-2 3 8 13 18] **(-9 -4 1 6 11)**	[-4 1 6 11 16]	(-1 4 9 14 19)
(-2 3 8 13 18)	(3 8 13 18 23)	(8 13 18 23 28)	(0 5 10 15 20)	
Minimum Fuzzy Transportation Cost = [10,22,39,59, 58] + [-3,1,20,46,49] + [10,29,41,59,68] + [-5,11,32,58,58] + [16,38,40,57,69] + [11,4,32,58,89] = [-21,29,105,209,318] = **1290.51**				

Error using Lagrange’s Interpolation polynomial: f(5) =1129.51
Minimum Fuzzy transportation cost = 1129.51

4. Conclusion

In summary, our study successfully converted the crisp transportation problem into a fuzzy counterpart using symmetric pentagonal fuzzy numbers. By employing the alpha-cut method for ordering, we addressed uncertainty systematically.

Analysing the error between crisp and fuzzy transportation problems at different values of d revealed insights into the impact of uncertainty. The application of Lagrange's polynomial fit provided a robust means of modelling the error term. Comparing errors from the fuzzy transportation method and Lagrange's polynomial, particularly at d=4, highlighted the effectiveness of our approach. Our research contributes a systematic method for fuzzy transportation problems, emphasizing the implications of uncertainty and providing a reliable framework for error analysis and comparison. This work opens avenues for further exploration of fuzzy methodologies for transportation optimization.

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