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Original Article

The Application of Proving Capability of Mathematics in Abstract Algebra

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Abstract *- A key skill for learning abstract algebra is mathematical proof. In this article, we describe students' mathematical proving abilities using descriptive and comparative methods in private universities with three different levels of accreditation. Data was gathered through testing and interviews. The Kruskal-Wallis test and Mann-Whitney U test were used to describe and compare the data statistically. The results suggest that students' mathematical proving aptitude varies across three categories: Extraordinary, Average, and Beginner levels. According to the evaluation results, significant differences were observed among universities with different accreditation levels. Based on these findings, it is suggested that students from Average and Beginner level accredited universities could improve their mathematical proving abilities by becoming more familiar with proof problems through regular practice, increasing their motivation to learn, and being provided with easily understandable learning materials.*

Keywords *- Accreditation of Private University, Comparison Analysis, The Kruskal-Wallis Test, Descriptive Analysis, Mathematical Proving Ability, Mann-Whitney (U) test.*

1. Introduction

In Abstract Algebra, the deductive approach is used to prove axiom. Understanding mathematics proof is obligatory [1-3]; a mathematical proof is a logical argument that demonstrates whether a given proposition is true or false [4]; each statement is obtained logically from the previous one and theorems whose proof has already been established[5], It implies that once a theorem is proven to be true, it will remain true forever. The proof in mathematics education is covered in a wide variety of literature, and learning and teaching how to prove it can be challenging. As per numerous earlier researches, many students still have difficulty forming mathematical proofs [6]. The capacity for developing mathematical proofs at the Advanced Leveler education level is more formal and specific compared to that of elementary and secondary school [7]; the process of writing proofs in abstract algebra differs significantly from calculus, geometry, or real analysis.

Abstract algebra is a mathematics course that helps students enhance their ability to understand and construct mathematical proofs [8]; in a mathematical proving task, we can observe how students provide logical arguments and use examples to support their reasoning, the kinds of misconceptions the students often experience, etc. In advanced-level education nowadays, students struggle with mathematical proofs, particularly in abstract algebra [9-10]. As formerly mentioned, the universities' students' knowledge wasn't satisfactory; we need to assess mathematical proving ability in abstract algebra and examine the differences in students' ability to solve problems.

2. The Kruskal-Wallis Test

The Kruskal-Wallis is a non-parametric statistical test used to compare the medians of three or more independent groups to determine whether they come from the same. Consider the independent random samples of sizes $n_1, n_2 \dots \dots . . n_c$ drawn from c univariate populations with unknown cumulative distribution functions $F_1, F_2, \dots . F_c$.
H_0= The populations have the same median
H_α= At least one population median differs from the others
$R_i$denote the sum of ranks for the i^{th} group, and n_i denote the sample size of the i^{th}group ($i = 1,2,3 \dots k$), where $k$is the

number of groups
The Kruskal-Wallis test statistic H is calculated as:

$$H = \frac{12}{\mathrm{N}(\mathrm{N}+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(\mathrm{N}+1)$$

where
$N = \sum_{i=1}^{k} n_i$ is the total number of observations across all groups
for the tied rank $T = 1 - \frac{\sum_j (t_j^3 - t_j)}{N^3 - N}$
where, t_j is the number of tied ranks in the $^{\text{jth}}$ group of ties.

3. Mann-Whitney(U) Test

This is a non-parametric test used to compare whether two independent groups have the same distribution. It is suitable for ordinal, interval, or ratio-level data and does not assume normality.

Let n_1 and n_2 be the number of observations in Group 1 and Group 2, respectively

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

The smaller U_1 and U_2 is used as the test statistic and $U = min(U_1, U_2)$
Under the null hypothesis, the expected value E(U) Var(U) are:

$$E(U) = \frac{n_1 n_2}{2}$$

$$Var(U) = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

For large sample sizes, the U-statistic can be approximated by a normal distribution

$$z = \frac{U - E(U)}{\sqrt{Var(U)}}$$

4. Motivation of Research

A descriptive quantitative methodology was used in this research, followed by a comparative analysis. The results of the test on mathematical proving abilities were described using descriptive analysis, while the comparative analysis was used to identify variations in students' mathematical proving abilities based on the accreditation ranking of their universities.

5. Domain of Research

Students from three different groups of private Indian universities participated in this study. The first group consisted of 36 students from an outstandingly accredited university, the second group included 32 students from a very good accredited university, and the third group comprised 32 students from a good university.

6. Data Study Method

We execute two distinct kinds of information analysis based on the study's objectives. Comparative analysis and descriptive analysis. The quantitative data were analyzed for 0 to 4 Grades [11]. Grade 0 was awarded for no proving ability, Grade 1 was awarded for one incorrect approach, Grade 2 was awarded for substantial progress, Grade 3 was awarded for minor mistakes, and Grade 4 for the students could make the completion of proving process. The overall Grade was 0 to 100 scales [12]

Table 1. Level of mathematical proving ability

Range	Deductive Reasoning Grade
Advanced Level	$75 \leq x \leq 100$
Average Level	$50 \leq x \leq 75$
Beginner Level	$0 \leq x \leq 50$

We used independent k-sample comparative analysis to determine mathematical proving skills based on their institution accreditation rating; further, the Mann-Whitney U test was for the non-parametric statistical.

7. Outcomes of Data Study

According to the given criteria, each level on the mathematical proving test was graded from 0 to 4 in order to assess the results.

Table 2. Outcomes for the mathematical proving ability

Data Description	Level of Accreditation		
	A	B	C
Mean	76.98	40.12	37.13
Median	84.13	34.08	42.12
Mode	84.13	34.08	42.12
Standard Deviation	13.18	19.09	12.05
Variance	161.74	360.18	125.98
Maximum Grade	92.13	92.13	59.16
Minimum Grade	51.15	17.08	17.08
Maximum. Theoretical Grade	100	100	100
Minimum Theoretical Grade	0	0	0

The A-accredited group had the highest mean grade in mathematical proving ability (76.98), followed by the B-accredited group (40.12), and the C-accredited group had the lowest mean grade (37.13). The grades for the B and C accredited groups were still significantly at the Beginner Level compared to the Advanced Level grade. The standard deviations for the grades in the A, B, and C groups were 13.18, 19.09, and 12.05, respectively. The Advanced Level grade achieved by students in Group C was 59.16, while Group B achieved a mean grade of 92.13.

The majority of students at A-accredited institutions Advanced Level (76.98%) compared to the majority of students at B- and C-accredited, who performed poorly (87.20% and 96.50%, respectively) displayed in table-3. Also, no student at a university with a C accreditation demonstrated outstanding ability; these phenomena showed that students in B- and C-rated universities still had Beginner Level mathematical reasoning skills. For more information about the mathematical proving abilities, we examined the Grades for each test level based on the accrediting group.

Table 3. Distribution of test grades depending on ability for mathematical proof

Range	Grade range	A-Rank Institute		B-Rank Institute		C-Rank Institute	
		Number	%	Number	%	Number	%
Advanced Level	$75 \leq x \leq 100$	27	76.98	5	12.8	2	0
Average Level	$50 \leq x \leq 75$	3	14.8	0	0	2	3.50
Beginner Level	$0 \leq x \leq 50$	6	8.07	27	87.20	28	96.50
Total		36	100	32	100	32	100

Table 4. Grade for each proof question

Accreditation	Rank level			Mean
	1	2	3	
A	72.66	88.60	70	76.98

B	51.15	28.93	40.06	40.12
C	55.93	29.99	25.14	37.13
Overall mean	58.89	50.35	45.45	50.28

Table 5. Distribution of mathematical proof ability grade

Rating Scale	Response Frequency															
	Grade 0			Grade 1			Grade 2			Grade 3			Grade 4			
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	Total
1	0	1	0	0	7	1	4	19	21	29	1	5	2	5	0	95
2	0	3	4	2	22	20	5	3	0	2	0	5	27	2	0	95
3	0	1	15	4	16	0	3	12	13	24	2	0	4	1	0	95
Total	0	5	19	6	45	21	12	34	34	55	3	10	33	8	0	285

In Table 4, the difficulty levels for each group varied. However, generally, Scale 3 was the most challenging, followed by Scale 2 and Scale 1. Table 5 displays the frequency distribution of the grades obtained by each group, allowing you to identify the issues that the students were facing. From the grade distribution in Table 5, 29 students in Group A correctly answered Question No. 2, achieving a grade of 4, while only one student correctly answered Questions No. 1 and No. 3, respectively. In Group B, 4 students received a grade of 4 on Question 1, 2 students on Question 2, and 1 student on Question 3.

Table 6. Normality test of mathematical proving ability data

Group	No of Students	Kolmogorov-Smirnov test	Sig.	Decision	Conclusion
A	36	0.402	0.000	Null hypothesis rejected	Not normally distributed
B	32	0.294	0.000	Null hypothesis rejected	Not normally distributed
C	32	0.289	0.000	Null hypothesis rejected	Not normally distributed

8. Co-Relation Study

The non-normal distribution of the research data and non-parametric statistical tests were used. Table 6 displays the outcomes of normality tests using the Kolmogorov-Smirnov test; the Kruskal-Wallis test with SPSS was performed to compare the two groups' mathematical proof capacities. It can be concluded that there were significant differences in mathematical proving ability between Groups A, B, and C. To determine which accreditation groups had significant differences in mathematical proving ability, post hoc testing was conducted using the Mann-Whitney U test. A summary of the test results is presented in Table 8.

Table 7. The Kruskal-Wallis test result

Chi-Square (χ^2)	Hypothesis
55.89	Null Hypothesis Rejected

Table 8. The Mann-Whitney (U) test

Groups to Compare	Mann-Whitney(U)	Result
A-B	85.90	Statistically relevant
A-C	10.96	Statistically relevant
B-C	429.98	Not- Statistically relevant

This indicates that Group A and Group B, as well as A and Group C, have very different mathematical proving abilities. The probability value between groups B and C exceeded the significance level of 0.05. It indicates that there was little variation in the capacity to prove mathematical concepts.

9. Description of Mathematical Proving Ability

According to the study's findings, students from the A-accredited University demonstrated an advanced level of mathematical proof ability, with an average grade of 76.98, while students from the B- and C-accredited universities had average grades of 40.12 and 37.13, respectively.

Table 9. Grade Distribution of A-accredited University

Rating Scale	Response Frequency					Total
	Grade 0	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	
1	0	0.5	3.5	29	2	35
2	0	1.5	4.5	1	28	35
3	0	3	3.5	25	2.5	34
Total	0	5	11.5	55	32.5	104

Table 9 showed a majority of Grades 3 and 4 are more. Four students have 3 issues, while one has only one in question 1; most students received a Grade of 4 for problem number 2. The most challenging issue for the A-accredited group, at problem number 1, the students were asked if Q with the * operation is a commutative group defined by and G is a set of rational numbers. $a * b = a + b + 2ab, \forall\, a, b \in G$

We then looked at the subgroup proving problem in relation to problem number 3.

Table 10. Grade distribution for B- accredited University

Rating Scale	Response Frequency					
	Grade 0	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Total**
1	1	7	21	1	3	33
2	3	24	4	1	3	35
3	1	16	12	2	1	32
Total	5	47	35	2	7	96

Furthermore, just one student was able to react appropriately to question 3 Problem-3 given the relation is set $\mathfrak{R}$ is the real number we define

$$k = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \forall\, a, b, c, d \in \mathfrak{R}.\, \text{ad} - \text{bc} \neq 0 \right\}$$

With the operator of multiplication

$$L = \left\{ \begin{pmatrix} x & -y \\ y & x \end{pmatrix}, \forall\, x, y \in \mathfrak{R}.\, x^2 + y^2 \neq 0 \right\} \text{ Subgroup of K}$$

Students need to learn how to apply theorems in subgroup proofs for Problem Number 3. This phenomenon has also been observed in various research studies., which indicated that the most common error happened with Grade 2. Problem number 2 said, "lf G is a Group$a \in G$, a is said to be idempotent if $a^2 = a$. The approach used by the students was not suitable for proving Problem Number 2, indicating that they were unable to prove the theorems for a new mathematical statement.

Table 11. Grade distribution for C- accredited University

Rating Scale	Response Frequency					Total
	Grade 0	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	
1	1	1.5	20	5	0	27.5
2	5	19.5	1.5	6	1	32
3	14	0	14.5	1	1	30.5
Total	20	20	36	12	2	90

Only one student from the C-accredited university group provided the proper response to the problem1, and no other student did so.

10. Result Discussion

The results of this research revealed that students from the A, B, and C-accredited universities had average mathematical proving ability grades of 76.98, 40.12, and 37.13, respectively; the students' mathematical proofing skills between A-B-accredited and A-C-accredited universities proved that there were substantial differences in the students' capabilities. There was little variation in ability. From the findings, we can also conclude that students in B- and C-rated universities have weaker mathematical analytical abilities, which need to be improved. Here is the corrected version of the sentence; it was also possible to identify the challenges faced by students while attempting to solve tasks involving challenging mathematical proofs, particularly in completing subgroup proofs. Students' carelessness often resulted in incomplete conditions being written.

We recommend improving students' mathematical proving skills in the abstract algebra course by encouraging them to work on proof problems, reinforcing their foundational knowledge of the concepts, inspiring their motivation to learn, and providing them with easily understandable learning materials. We also advise private universities with B and C accreditation to upgrade their instructional facilities.

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