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Original Article

Comparative Study of Defence Spending and Arms Race Models for Competitive Countries: A Combined Approach

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Abstract *- In this paper, three widely recognized mathematical models - the bureaucratic model, the strategic model, and the combined bureaucratic and strategic model are used to analyze how defense spending is implemented. Each model offers a different justification for why nations spend on defense. The predicted accuracy of the models is evaluated using data on Russia's and Ukraine's defense spending from 1994 to 2021. Additionally, the models are evaluated across six pairs of competitive countries to determine their general applicability. The results show that the models' accuracy in predicting defence spending trends varied. This comparative study contributes insights into the variables influencing defense spending and how they affect geopolitical dynamics. The study highlights the need for strong models to better understand and predict defence spending worldwide.*

Keywords *- Mathematical Modelling, Defence Spending, Richardson's Arms Race Model, Bureaucratic Model, Strategic Model.*

1. Introduction

Mathematical modelling is the process of describing a real-world problem in mathematical terms. The concept of modelling is used in all fields such as engineering, physics, chemistry, economics, computer science, biology, etc. [2]

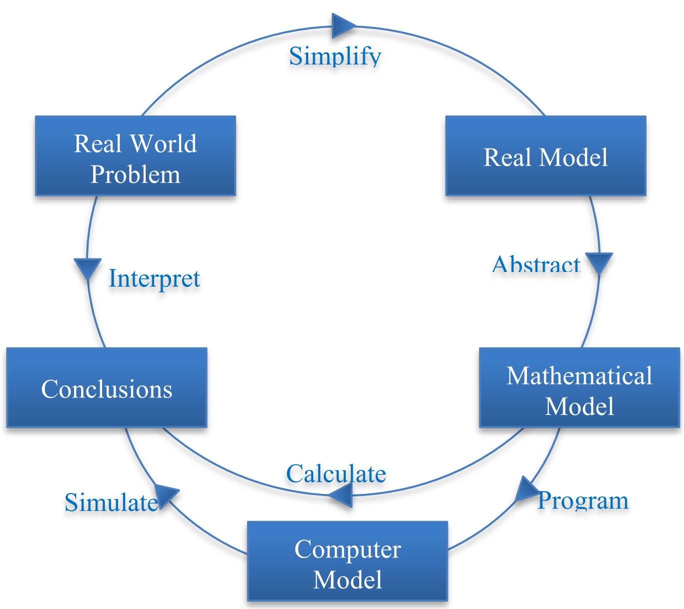


Fig. 1 Scientific Process to connect real world problem with Mathematics

The process of converting a real-world problem into a mathematical model involves several steps. First, the problem is identified and defined in the context of the real world. This problem is then translated into a mathematical model by formulating equations or formulas that represent the key aspects of the problem. The next step is to analyse the mathematical model to find some conclusions, often through solving these equations or using analytical methods to understand the system's behaviour. These mathematical conclusions are then implemented into a computer model, where computational techniques and software simulate the problem. The computer model generates predictions, which can be compared to actual observations or used to guide decision-making.

Mathematical models play a very significant and important role in solving problems in business, commercial, and military operations. The amount of money allotted by a state to building and sustaining armed forces or other strategies necessary for defense is referred to as the military budget, or defense budget. The term "defense expenditure" refers to all capital and ongoing investments made in the armed forces, including paramilitary forces deemed capable of conducting military operations and peacekeeping forces of defense ministries and other government organizations involved in defense projects. [3]

The United States, China, Russia, India, Saudi Arabia, the United Kingdom, Germany, Ukraine, France, and Japan are often regarded as great powers and have some of the highest military budgets in the world. The Stockholm International Peace Research Institute estimates that global military spending reached $2113 billion in 2021. Real defense spending has increased in both Russia and Ukraine over the last 27 years.

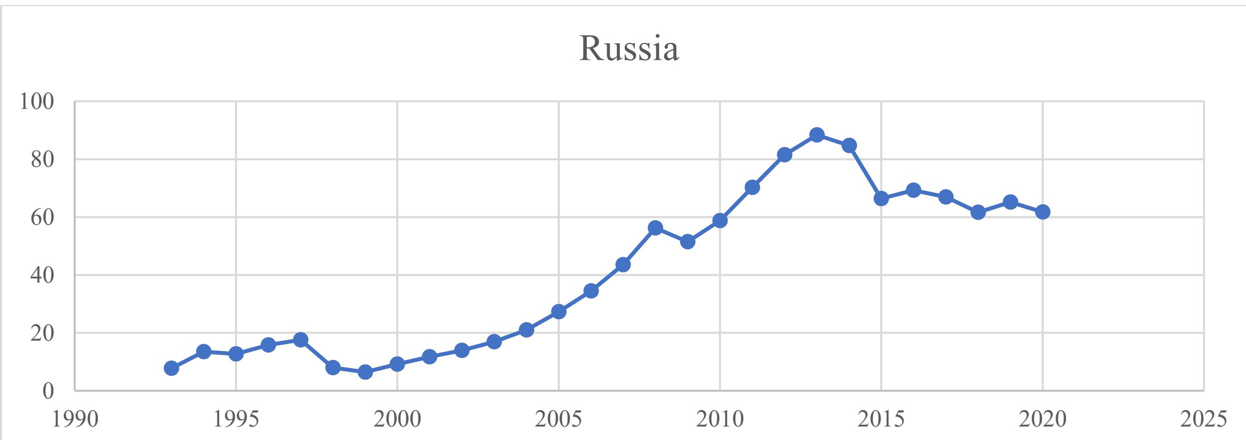


Fig. 2 Russia defence spending

Russia's military budget for the year 2021 was $65.91B, which is 6.8% higher than the year 2020.
Russia's military budget for the year 2020 was $61.71B, which is 5.35% lower than the year 2019.
Russia's military budget for the year 2019 was $65.20B, which is 5.83% higher than the year 2018.
Russia's military budget for the year 2018 was $61.61B, which is 7.93% lower than the year 2017.

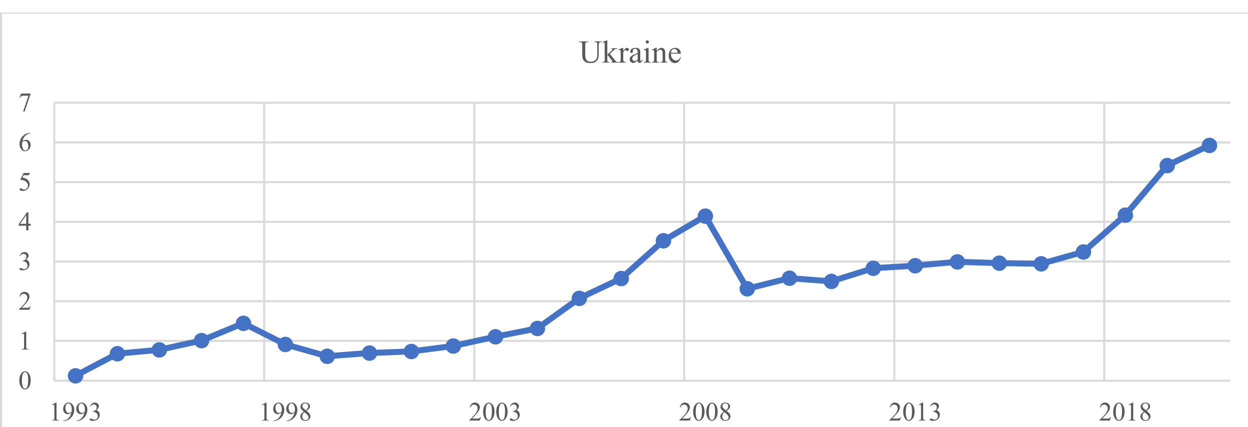


Fig. 3 Ukraine defence spending

Ukraine's military budget for the year 2021 was \$5.94B, which is 0.31% higher than the year 2020.
Ukraine's military budget for the year 2020 was \$5.92B, which is 9.32% higher than the year 2019.
Ukraine's military budget for the year 2019 was \$5.42B, which is 29.96% higher than the year 2018.
Ukraine's military budget for the year 2018 was \$4.17B, which is 28.43% higher than the year 2017.

When discussing defence spending, the arms race model is also pertinent, as it specifically examines defence dynamics.

Since it explicitly looks at defense dynamics, the arms race concept is equally relevant when talking about defense spending.

We take two nearby nations and let them represent their respective armament expenditures in a standardized monetary unit. Lewis F. Richardson (1881–1953) developed a simplified version of the Richardson Arms Race model, assuming that each nation's expenditure on weapons is directly proportionate to the other country's current expenditure. [2]

He also assumed that excessive spending on weapons compromises the nation's economy; therefore, the pace at which one nation's spending on weapons changes will also be exactly proportionate to its own spending. He believed that a nation's armaments buildup was influenced by both mutual stimulation and the long-standing, underlying grudges that each nation had against the other. [2]

$$\frac{dx}{dt} = \alpha y - \gamma x + r$$

$$\frac{dy}{dt} = \beta x - \delta y + s$$

Where $\alpha,\ \beta,\ \gamma,\ \delta$ are positive and r, s are constants which may have positive or negative signs.

The equilibrium points of the above equations are given by,

$$\alpha y_0 - \gamma x_0 + r \ = 0$$
$$\beta x_0 - \delta y_0 + s = 0$$

$$x_0 = \frac{r\delta + s\alpha}{\gamma\delta - \alpha\beta}$$
$$y_0 = \frac{r\beta + s\gamma}{\gamma\delta - \alpha\beta}$$

If r, s is positive, a steady state solution exists if $\gamma\delta - \alpha\beta > 0$

The characteristic equation is,

$$\lambda^2 + (\delta + \gamma)\lambda + \gamma\delta - \alpha\beta = 0$$

Let λ_1 and λ_2 be two roots of the equation.
$\lambda_1 + \lambda_2 = -(\delta + \gamma)$
$\lambda_1\lambda_2 = \gamma\delta - \alpha\beta$

Now the following cases arise.

Case I: $\gamma\delta - \alpha\beta > 0, r > 0, s > 0.$
in this case $x_0 > 0$ and $y_0 > 0$
$\lambda_1 < O \ and\ \lambda_2 < 0$

As such, there is a position of equilibrium, and the system is stable. This indicates that both countries spent on arms and military in a strategic manner so that the economy of the country is not compromised.

Case II: $\gamma\delta - \alpha\beta > 0, r < 0, s < 0.$
In this case $x_0 < 0,\ y_0 < 0$, so that there is no position of equilibrium.
However, since $\lambda_1 < 0$ & $\lambda_2 < 0$, $X(t) \rightarrow 0$, $Y(t) \rightarrow 0\ as\ t \rightarrow \infty$

$$x(t) \rightarrow x_0\,,\ y(t) \rightarrow y_0\ as\,t \rightarrow \infty$$

However, x_0 and y_0 are negative, and expenditure cannot be negative.
In any case, to become negative, they must pass through value zero.
As the value of $x(t)$ becomes zero then,

$$\frac{dy}{dt} = -\delta y + s$$

And since $s < 0$, the value of $y(t)$ decreases till it attains the value zero. Similarly, if the value of $y(t)$ becomes zero then $x(t)$ decreases till it attains a value of zero.

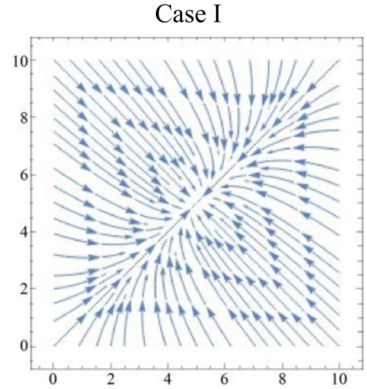
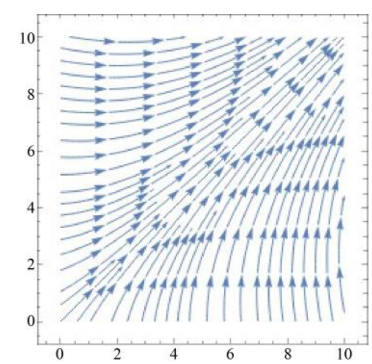
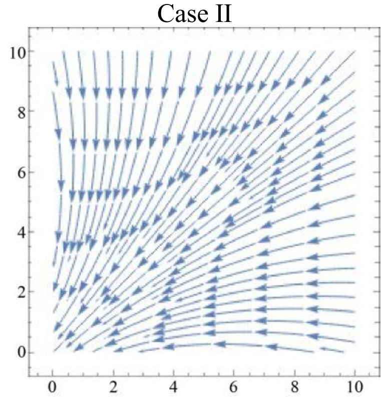
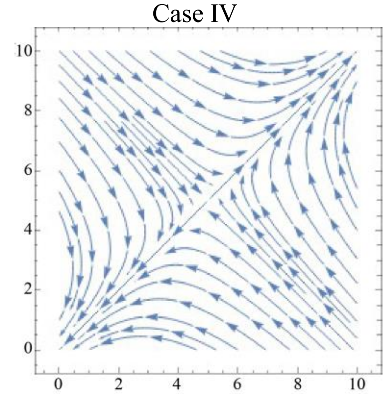
Thus, in this case, there will be complete disarmament in the long term.

Case III $\gamma\delta - \alpha\beta < 0, r > 0, s > 0$.
Which gives the values $x_0 < 0$ and $y_0 < 0$
One of $\lambda_{1,}$ is positive, and the other is negative.
In this case, there will be a runaway arms race.

Case IV $\gamma\delta - \alpha\beta < 0, r < 0, s < 0$.
Which gives the values $x_0 > 0$ and $y_0 > 0$
One of $\lambda_{1,}$ is positive, and the other is negative.
In this case, there will be a runaway arms race.

This describes four types of phase plane diagrams showing the dynamics of the model according to relations between the parameters $\alpha, \beta, \gamma, \delta, r, s$.

Table 1. Phase plane diagram of four cases of the arms race model

	$\gamma\delta > \alpha\beta$	$\gamma\delta < \alpha\beta$
$r > 0$, $s > 0$	Case I 	Case III 
$r < 0$, $s < 0$	Case II 	Case IV 

We use three mathematical models often employed in defense expenditure research to analyze data on Russia's and Ukraine's defense spending from 1994 to 2021.

While the arms race model has been widely applied in military and strategic studies, its use in analyzing modern business competition is limited. Most prior research does not consider broader economic factors such as GDP, which can significantly influence competitive dynamics. This paper explains three different types of arms race models incorporating GDP to capture the impact of macroeconomic conditions on firm competition.

Three widely used mathematical models based on the arms race model are applied to analyse Russia and Ukraine's defence spending data from 1994 to 2021. Each model assumes different fundamental causes or combinations of driving defence expenditure decisions. The models examine the amount of defence expenditure of countries from different perspectives:

1. Bureaucratic Model: The amount of money spent on defense at the internal level is primarily determined by the defense institution and the lengthy nature of defense procurement.
2. Strategic Model: The defence expenditure is primarily a response to the external threat and is restricted by a resource constraint.
3. Combined Bureaucratic and Strategic Model: This model attempts to measure the relative strengths of internal and external factors.

These models offer a thorough framework for comprehending and contrasting the variables affecting defense spending in Ukraine and Russia.

2. Literature Review

Moll, K. D., et al. (1980) [11] studied arms-building models, which describe how nations develop their military forces. They concluded the following points: (a) Social and psychological factors are less represented in existing arms race models. (b) Bureaucratic models often outperform Richardson-type models as predictors. (c) Arms-Using Models can assess military impact but currently do not give reliable policy instructions. (d) Future studies should explore neglected problems using recently developed empirical data showing promise for rapid progress.

Schneider, J. W. (1999) [13] applied five models widely used in defence spending studies. The goal was not only to find the single "best" model but also to inspect if a consistent pattern of behaviour emerged for every country using the combination of the models. They concluded that existing arms race models clarify the defence spending behaviour of the two nations, although they are by no means the final word and have only finite value for prediction.

Dunne, J. P., et al. (2003) [5] examined Richardson's action-reaction model of an arms race, which has prompted significant research attempting to empirically approximate such models. In general, these attempts were not successful. They used the latest developments in time-series econometrics to illustrate problems with estimates for Turkey and Greece, as well as India and Pakistan. They found little proof for a Richardson-type arms race between Greece and Turkey, whereas India and Pakistan exhibited a stable interaction with a clear equilibrium.

Dunne, J. P. et al. (2005) [7] examined Richardson's action-reaction model of arms races, which has inspired many empirical studies, though most have achieved limited success. His research revisited the estimation challenges associated with such models and, using recent advances in time-series econometrics, analyzed military expenditure data for Greece and Turkey. The findings revealed evidence of cointegration between the two countries' military spending, indicating a long-term relationship, but not one consistent with the classical Richardson-type arms race model.

Dunne, J. P. et al. (2007) [6] discussed the econometric modeling of arms races, defined as enduring rivalries between hostile powers that drive competitive military buildup. The study reviewed theoretical, data-related, and statistical issues in estimating such models, focusing on time-series Richardson-type models (e.g., India–Pakistan), Markov switching game-theory models (e.g., Greece–Turkey), as well as cross-section and panel approaches. The findings indicated that arms race parameters are not constant over time, though panel models can estimate average interaction effects and spillover costs of military spending. Rahul also emphasized that globalization requires considering broader strategic contexts beyond two-country models and highlighted the growing importance of qualitative-asymmetric arms races, especially between governments and non-state actors.

Chalikias, M. (2014) [3] applied Lewis Richardson's arms race model to the advertising expenditure of two competitive firms using secondary data from the mobile phone industry in Greece. They concluded that the theoretical models closely match reality, indicating that these models can be applied to firms under appropriate conditions.

Mondal, S. P. et al. (2018) [12] presented adaptive strategies to analyze first-order fuzzy differential equations (SFDE) in both fuzzy and crisp forms. Fuzzy solutions were obtained using Zadeh's extension principle and the generalized Hukuhara derivative, while crisp solutions were explored through various defuzzification techniques, including COA, BOA, LOM, SOM, MOM, RWPM, GMIV, and COAI. The study also applied these methods to the arms race model (ARM), a classical system of first-order differential equations with significance in military planning, which had not previously been studied in a fuzzy context.

Kevin Zhang et al. (2021) [8] demonstrated another potential application of Richardson's Arms Race model beyond its original focus on defence and international conflicts. They applied the model to illustrate the competitive behaviour of two oligopolistic companies using R&D as a parameter.

Shatyrko, D. K. A. (2023) [15] introduces a nuanced perspective by addressing these limitations. The study initially applied the ODE-based Richardson model, using statistical data spanning five years from various open sources, and conducted a numerical analysis with tools provided by the Maple software. This analysis enabled the observation of arms race behaviors through simulations and phase portraits, which provide visual insights into the stability and nature of equilibrium points within the model.

3. Methodology

3.1. Bureaucratic Model

The bureaucratic model was developed by Lucier in 1979. This model is the simplest model of defence spending. This model suggests inherent inertia in defence spending, suggesting that entrenched bureaucracies resist changes to their established positions. As a result, future budget decisions tend to be incremental adjustments based on past spending levels. This inertia is further compounded by the nature of defence procurement, which sometimes involves long-term, expensive programs that span several years or even decades.

The model can be expressed with the following equation:

$$M_t = \beta_0 + \beta_1 M_{t-1}$$

Where: M_t represents the amount of defence spending in year t, β_0 is a constant term, β_1 is the coefficient representing the relationship between the current year's spending and the previous year's spending, M_{t-1} is the amount of defence spending in year $t-1$. This equation implies that the current year's defence spending. M_t is determined by a constant factor β_0 plus a portion β_1 of the previous year's defence spending M_{t-1}.

The regression results for Russia's military spending from 1994 to 2021 using SPSS are given by the equation:

$$M_{r(t)} = 3.833 + 0.957 M_{r(t-1)}$$

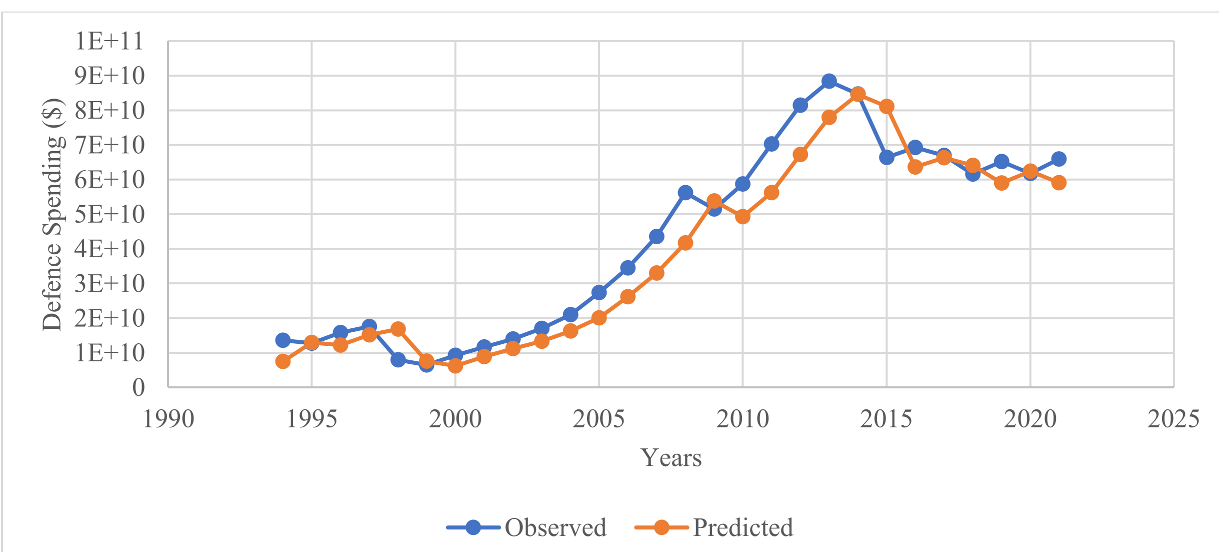


Fig. 4 Russia defence spending (Predicted Vs. Actual)

The coefficient for lagged spending is 0.957, which is very close to 1. This indicates that Russia's defence spending is very stable and predictable, with a strong tendency to continue spending at past levels. The model's high accuracy rate of 94.0% indicates that Russia's defence budget is consistent and largely influenced by past defence spending. This stability is likely due to Russia's established bureaucratic processes and long-term defence programs.

The regression results for Ukraine's military spending from 1994 to 2021 using SPSS are given by the equation:

$$M_{u(t)} = 0.218 + 0.996 M_{u(t-1)}$$

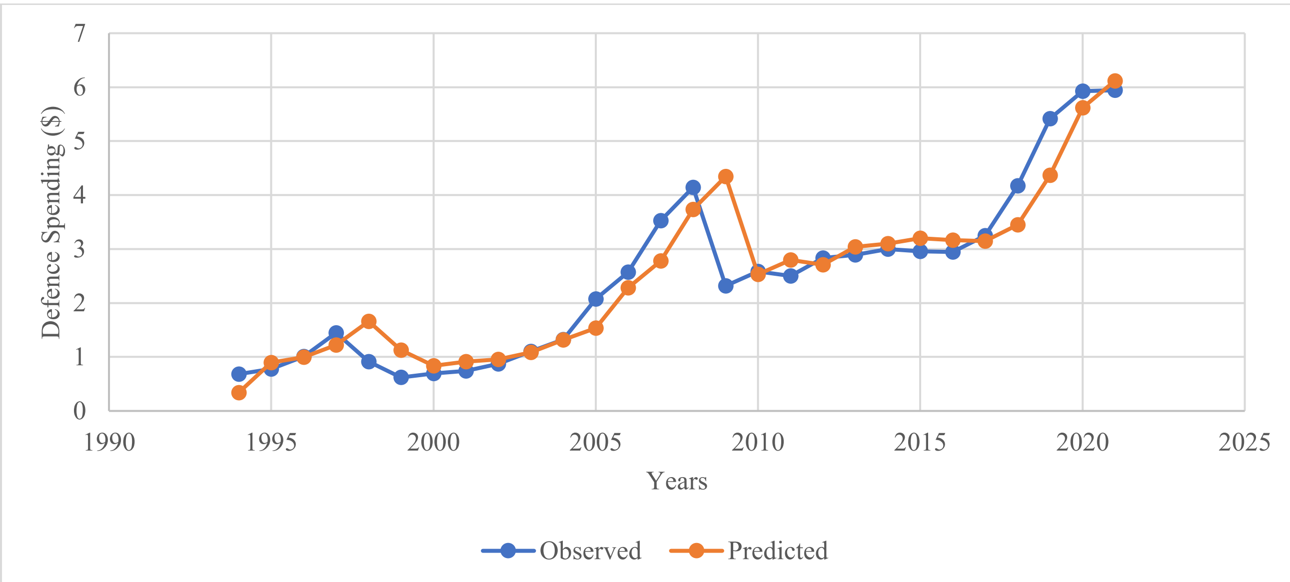


Fig. 5 Ukraine defence spending (Predicted Vs. Actual)

The coefficient for lagged spending is 0.996, which is very close to 1, showing strong inertia in Ukraine's defence spending. This means that past spending heavily influences current spending. However, the model's accuracy is 87.4%, which is slightly lower than Russia's. This lower accuracy suggests that Ukraine's defence spending might be more variable and less predictable. While the model is still effective, it indicates there may be other factors affecting Ukraine's spending that the model does not fully capture.

3.2. Strategic Model

Mancur Olson and Richard Zeckhauser's 1966 work is credited with helping to build strategic models for military spending. The concept of defense as an international public good was developed, and a few factors were identified as influencing defense expenditure, such as national income, the amount spent by allies, the relative costs of defense and non-defense goods, and the perceived level of threat.

The strategic model begins with the objective utility function:

$$U = U\,(y, q, Q, T)$$

Where y represents nondefence goods, q represents defence goods, Q represents the allies' contribution to the country's security, and T represents the known threat.

This model was expanded by Sandler and Hartley (1995), who showed that by setting the allied contribution to zero, the Olson-Zeckhauser model could be applied to nations without allies. In the cases of Russia and Ukraine, the model can be simplified by eliminating Q. The objective function is subject to the budget constraint.

$$I = p_y y + p_q q$$

Where I is income, generally measured in GDP or government revenue terms, and p_y and p_q are the prices of non-defense and of defense goods, respectively. This budget constraint forms the basis for the following econometric model:

$$M_t = \beta_0 + \beta_1 Y_t + \beta_2 PRICE + \beta_3 THREAT_{t-1}$$

However, in practice, PRICE can be difficult to obtain and can be omitted if it is assumed that the prices of defence and nondefence goods inflate at similar rates (Sandler and Hartley, 1995). Thus, the simplified econometric model becomes:

$$M_t = \beta_0 + \beta_1 Y_t + \beta_2 THREAT_{t-1}$$

Where M is military expenditure, Y is GDP, and THREAT is the military expenditure of the rival.

For Russia and Ukraine, we specify a linear function with the country's income Y and the lagged spending of the rival country, which is equivalent to the THREAT variable:

$$M_t = \beta_0 + \beta_1 Y_t + \beta_2 THREAT_{t-1}$$

Using SPSS on data from 1994 to 2021, the strategic model for Russia's military spending was derived as:

$$M_{r(t)} = -0.465 + 0.037 Y_{r(t)} + 1.294 M_{u(t-1)}$$

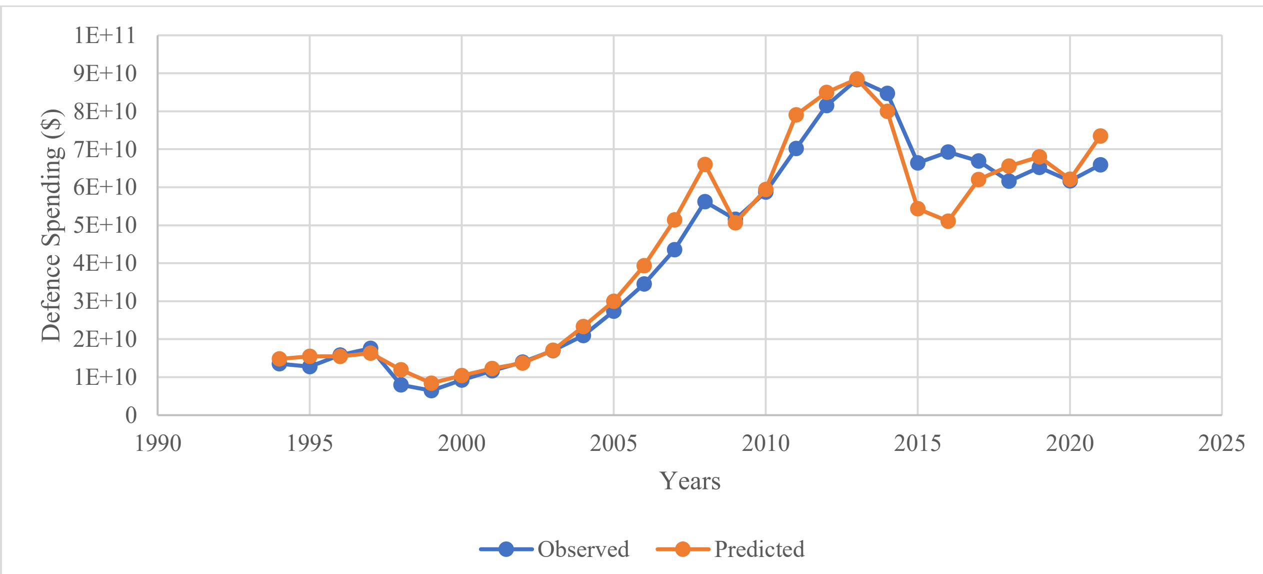


Fig. 6 Russia defence spending (Predicted Vs. Actual)

With a coefficient of 1.294, Russia's model shows a significant sensitivity to Ukraine's previous military spending. This indicates that Ukraine's military actions have a big impact on Russia, which shows a serious concern for regional domination. The GDP coefficient, which stands at 0.037, indicates that military spending is slightly influenced by the economy's capacity. The model's 95.2% accuracy rate shows that it successfully accounts for the key factors influencing Russia's military spending.

Using SPSS on data from 1994 to 2021, the strategic model for Ukraine's military spending was derived as:

$$M_{u(t)} = -0.013 + 0.019 Y_{u(t)} + 0.012 M_{r(t-1)}$$

With a coefficient of 0.019, Ukraine's model shows a moderate sensitivity to its own GDP, but its response to Russia's military spending is much weaker (coefficient of 0.012). It indicates that rather than being a direct response to Russian spending, Ukraine's defense expenditures are more influenced by its economic conditions. The model also shows significant inertia, with past spending heavily influencing current expenditure. The accuracy of 68.8% suggests that other factors may also influence Ukraine's military expenditure beyond those captured in the model.

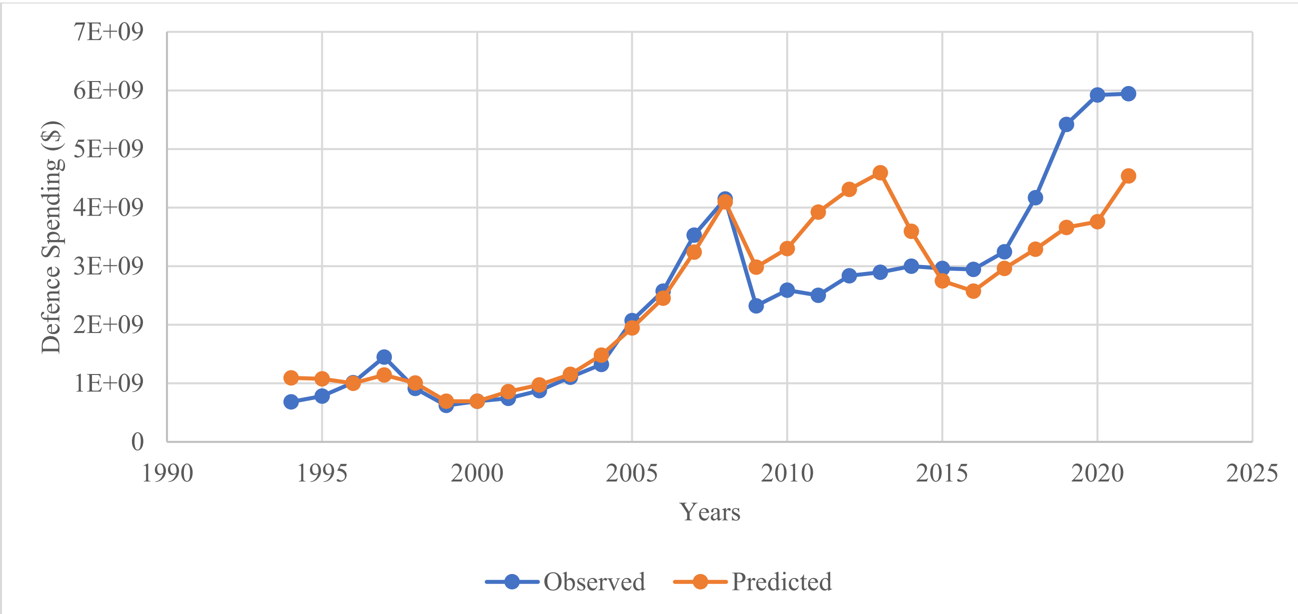


Fig. 8 Ukraine defence spending (Predicted Vs. Actual)

3.3. Bureaucratic and Strategic Model

A more comprehensive framework for understanding military expenditures can be achieved by combining the bureaucratic and strategic models with the nation's lagging spending, as discussed by Sandler and Hartley (1995) and Looney and Mehay (1990) for the US.

The combined model is represented as:

$$M_t = \beta_0 + \beta_1 Y_t + \beta_2 THREAT_{t-1} + \beta_3 M_{t-1}$$

Where M is military expenditure, Y is GDP, and THREAT is the military expenditure of the rival. The results using SPSS on data from 1994 to 2021 for Russia were:

$$M_{r(t)} = 0.309 + 0.481 M_{r(t-1)} - 0.499 M_{u(t-1)} + 0.022 Y_{r(t)}$$

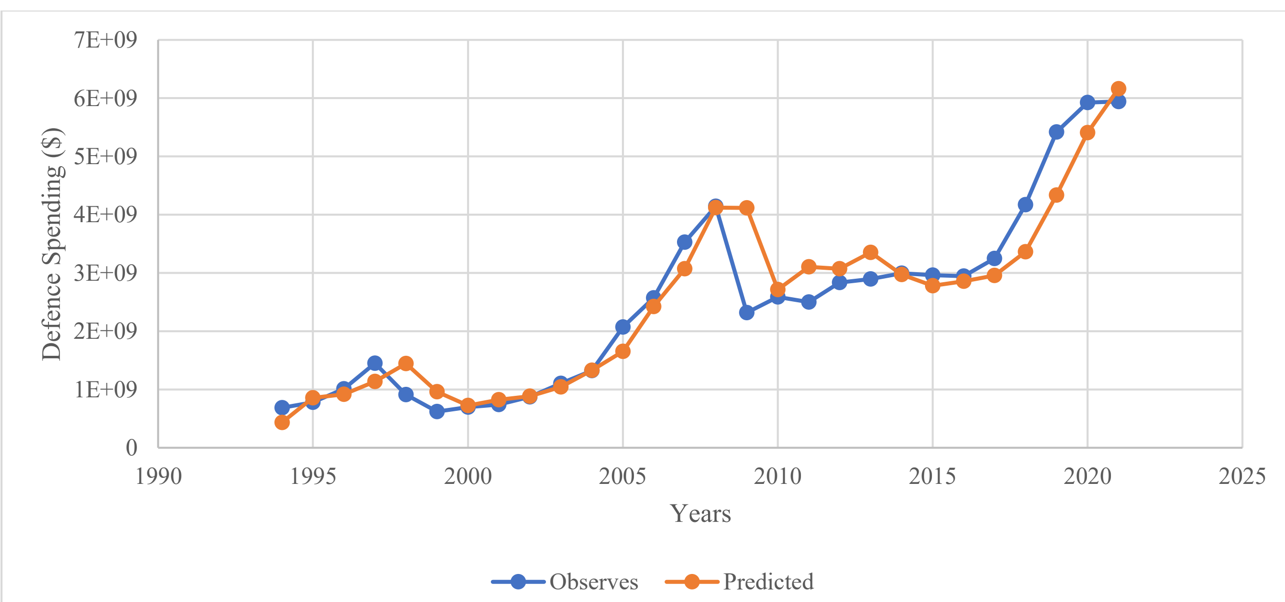


Fig. 8 Russia defence spending (Predicted Vs. Actual)

Russia's military budget has a significant amount of inertia, as shown by the coefficient of 0.481, which indicates that almost half of this year's expenditure is influenced by the previous year's expenditure. For Ukraine, the negative coefficient of -0.499 for past expenditure is unusual and may suggest that increased spending in the previous year could lead to a reduction in the current year's spending due to strategic adjustments or other geopolitical factors.

Economic capacity is a significant element, as indicated by the GDP's positive coefficient of 0.022, which suggests that Russia's military spending rises somewhat with economic expansion. The model predicts Russia's expenditure on defense with a 98.8% accuracy rate, showing a strong alignment with actual spending patterns.

The results using SPSS on data from 1994 to 2021 for Ukraine were:

$$M_{u(t)} = 0.012 - 0.005M_{r(t-1)} + 0.856M_{u(t-1)} + 0.007Y_{u(t)}$$

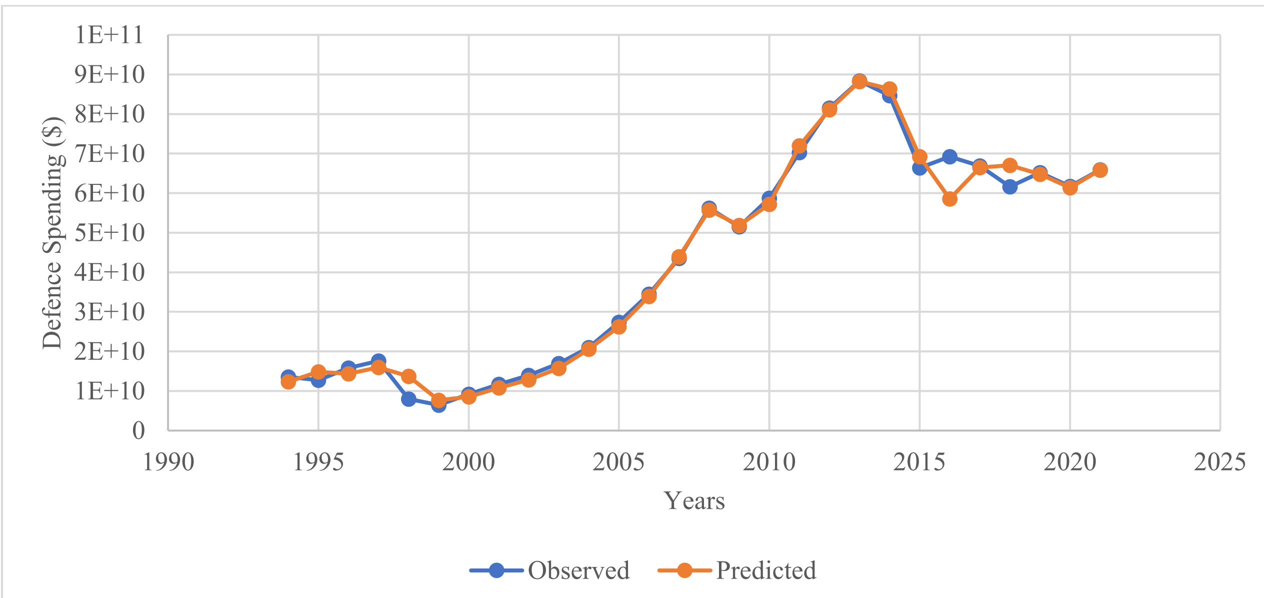


Fig. 9 Ukraine defence spending (Predicted Vs. Actual)

The large coefficient of 0.856 for Ukraine's military spending indicates that the current year's expenditure is strongly influenced by the previous year's spending, reflecting significant budgetary inertia in Ukraine's defence allocation. Russia's defense budget has had a minimal impact on Ukraine's current spending, according to the small negative coefficient of -0.005 for Russia's spending from the previous year. This means Ukraine's defence budget decisions are largely independent of Russia's spending levels. Though the effect is small, the positive coefficient of 0.007 indicates that Ukraine's military spending increases with its GDP. With an accuracy of 88.3%, this model predicts Ukraine's defence spending well, but not as accurately as the model for Russia.

4. Comparative Study

To find the model that determines the most accurate predictions, we examine three predictive models to analyze the defense spending data of selected country pairs. The data covered the period from 1994 to 2021, and the analysis was conducted using SPSS software.

We formed two groups. We used three combinations of Russia, Ukraine, and the United States in the first group.

1. Russia and Ukraine
2. Russia and the USA
3. Ukraine and the USA.

In the second group, we took three combinations of India, Pakistan, and China.

1. India and Pakistan

2. India and China
3. China and Pakistan

For each country pair, we applied three different predictive models and found the accuracy of each model's predictions. The accuracy values indicate the model's ability to predict actual defence spending accurately.

The analysis aimed to compare the performance of these models across different geopolitical combinations and to identify which model consistently provided the highest accuracy in predicting defence spending. This comparison helps in understanding the reliability of various modelling approaches in different geopolitical contexts.

Table 2

	Bureaucratic Model	Strategic Model	Bureaucratic Model and Strategic Model
Russia	0.940	0.952	0.988
Ukraine	0.874	0.688	0.883
Russia	0.940	0.956	0.988
U. S. A	0.973	0.831	0.986
U. S. A	0.973	0.822	0.974
Ukraine	0.874	0.694	0.883

Table 3

	Bureaucratic Model	Strategic Model	Bureaucratic Model and Strategic Model
India	0.989	0.991	0.995
Pakistan	0.969	0.978	0.985
India	0.989	0.991	0.995
China	0.995	0.998	0.999
China	0.995	0.998	0.999
Pakistan	0.969	0.983	0.986

5. Conclusion

The bureaucratic model explains why defence spending is stable and predictable, mostly for Russia. This model works well for Russia because its defence budget is influenced by long-standing bureaucratic processes and consistent planning. For Ukraine, the model is less accurate, suggesting its spending is more variable and affected by factors not fully captured by the model.

Strategic models further clarify military spending trends. Russia's spending is highly predictable and responds significantly to Ukraine's defence budget, indicating a focus on maintaining regional power. Ukraine's spending, however, seems more affected by its own economic situation rather than directly responding to Russia's spending, reflecting different strategic priorities.

Combining bureaucratic and strategic models offers a detailed view of defence spending patterns. While both countries have strong defence bureaucracies, Russia's model is more accurate, showing its spending is closely linked to economic and strategic factors. Ukraine's spending is more driven by its economic situation rather than reacting to Russia's budget.

After analyzing these three models on the first set of countries, we found that combining the bureaucratic and strategic models gave the most accurate predictions for defence spending. We saw the same result with the second set of countries. While this combined model worked well in our scenarios, it might not be as accurate in different scenarios. Therefore, no single model is best for every case. Instead, we need to decide the models based on the specific context of each country or scenario. Future research should explore additional factors and new methods to improve prediction accuracy.

References

[1] Charles H. Anderton, and Roxane A. Anderton, "The Economics of Conflict, Production, and Exchange," *The Economics of Conflict and Peace*, Brookfield: Avebury, pp. 54-82, 1997. [Google Scholar] [Publisher Link]

[2] Sandip Banerjee, *Mathematical Modeling: Models, Analysis, and Applications*, CRC Press, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[3] Miltiadis Chalikias, and Michalis Skordoulis, "Implementation of Richardson's Arms Race Model," *Applied Mathematical Sciences*, vol. 8, no. 81, pp. 4013-4023, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[4] Edward Hunter Christie, Caroline Buts, and Cind Du Bois, "Exploring the Speed of Change: European Defence Budgets in an Age of Danger," *Defense & Security Analysis*, vol. 40, no. 3, pp. 319-340, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[5] J. Paul Dunne, Eftychia Nikolaidou, and Ron P. Smith, "Arms Race Models and Econometric Applications," *Arms Trade, Security and Conflict*, 2000. [Google Scholar] [Publisher Link]

[6] J. Paul Dunne, and Ron P. Smith, "The Econometrics of Military Arms Races," *Handbook of Defense Economics*, vol. 2, pp. 913-940, 2007. [CrossRef] [Google Scholar] [Publisher Link]

[7] J. Paul Dunne, Eftychia Nikolaidou and Ron P. Smith, "Is there an Arms Race between Greece and Turkey?," *Peace Economics, Peace Science and Public Policy*, vol. 11, no. 2, pp. 140-174, 2005. [CrossRef] [Google Scholar] [Publisher Link]

[8] Kevin Zhang, and Zi Qian Chan, "Application of Richardson's Arms Race Model in Oligopolistic Competition," *International Journal of Science and Research*, vol. 10, no. 1, pp. 1205-1208, 2021. [CrossRef] [Publisher Link]

[9] Mary Kaldor, "The Role of Arms in Capitalist Economies: The Process of Overdevelopment and Underdevelopment," *Arms Control and Technological Innovation*, pp. 322-341, 2024. [Google Scholar] [Publisher Link]

[10] Charles E. Lucier, "Changes in the Values of Arms Race Parameters," *Journal of Conflict Resolution*, vol. 23, no. 1, pp. 17-40, 1979. [CrossRef] [Google Scholar] [Publisher Link]

[11] Kendall D. Moll, and Gregory M. Luebbert, "Arms Race and Military Expenditure Models: A Review," *Journal of Conflict Resolution*, vol. 24, no. 1, pp. 153-185, 1980. [CrossRef] [Google Scholar] [Publisher Link]

[12] Sankar Prasad Mondal et al., "Adaptive Strategies for System of Fuzzy Differential Equation: Application of Arms Race Model," *Journal of Mathematics and Computer Science*, vol. 18, no. 2, pp. 192-205, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[13] Jeffrey W. Schneider, "*What Drives Defense Spending in South Asia?: An Application of Defense Spending and Arms Race Models to India and Pakistan*," Thesis, Virginia Polytechnic Institute and State University, pp. 1-4, 1999. [Google Scholar]

[14] N. Suganya Baby, M. Dhavamani, and A. Kumaravel, "A New Perspective for Solving the System of Differential Equations Describing Arm Race Model in Neutrosophic Environment," *Mathematical Problems in Engineering*, vol. 2023, no. 1, pp. 1-11, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[15] Ron P. Smith, "The Influence of the Richardson Arms Race Model," *Lewis Fry Richardson: His Intellectual Legacy and Influence in the Social Sciences*, vol. 27, pp. 25-34, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[16] Denys Khusainov, and Andriy Shatyrko, "Simulation the Impact of Time-Delay in Richardson Arms Race Models," *Dynamical System Modeling and Stability Investigation*, pp. 89-97, 2023. [Google Scholar] [Publisher Link]

[17] Suklal Tudu, Sankar Prasad Mondal, and Shariful Alam, "Different Solution Strategy for Solving Type-2 Fuzzy System of Differential Equations with Application in Arms Race Model," *International Journal of Applied and Computational Mathematics*, vol. 7, no. 5, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[18] Michael D. Ward, "Back to the Future: Richardson's Multilateral Arms Race Model," *Lewis Fry Richardson: His Intellectual Legacy and Influence in the Social Sciences*, vol. 27, pp. 57-72, 2019. [CrossRef] [Google Scholar] [Publisher Link]