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Original Article

Analysis of $SLIA_1A_2QR$ Model in Cyber Space

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Abstract - *$SLIA_1A_2QR$ (Susceptible – Latent - Infected – Antidotal -Antivirus – Quarantine – Antidotal - Recovered) is the suggested model in this article, which is an extension of the SEIR model. In this model, we discussed the basic Reproduction number for the MFE (Malicious object Free Equilibrium) point. We talked about the reproduction number R_0 in MFE and EE point using the Hurwitz criterion. If R_0 is less than 1, the MFE point is stable; if $R_0$is greater than 1, the VFE point is unstable. Numerous parameter graphs are discussed in two and three dimensions.*

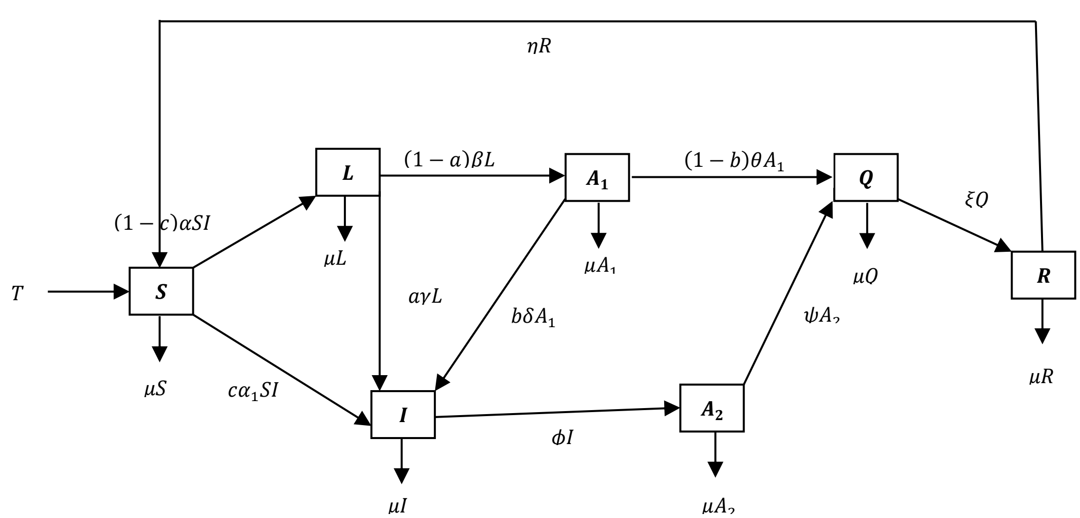
Keywords - *Antivirus, Endemic Equilibrium, Latent, Malicious Object Free Equilibrium, Threshold Number.*

1. Introduction

In computer networks, hazardous items such as viruses, worms, and other malware are represented, and their behavior is analyzed using a mathematical model. These models, which are frequently based on epidemiological models of transmission of disease, help in understanding the distribution of hazardous code, evaluating its effects, and creating mitigation and control plans. The two main ways by which the harmful items are spread within a computer network are through electronic mail and the use of secondary devices[1]. Similar to biological epidemic infections, the spread of malicious objects in the cyber network is epidemic in nature [2,3]. For predicting the spread of viruses, Richard and Mark suggested a better SEI[4] (Susceptible-Exposed- Infected) model. Switched from the SIR model to the SAIR (Susceptible- Antidotal- Infected- Recovered) model, which was investigated in [5-8]. Kaveri et.al. highlight different types of models [9-10]. In this article, we proposed $SLIA_1A_2QR$ (Susceptible – Latent - Infected – Antidotal -Antivirus – Quarantine – Antidotal - Recovered), which is an extension of the SAIR model.

2. Model Formulation

Here we use seven compartments, where S is the number of susceptible nodes, L is the number of latent nodes, I is the number of infected nodes, A_1 is the number of antidotal nodes, SA_2 is the number of antivirus nodes, Q is the number os quarantine nodes, and R is the number of recovered nodes. μ is the natural death rate.



$SLIA_1A_2QR$ Mathematicl Model

$$\left.\begin{aligned}
\dot{S} &= T-(1-c)\alpha SI - c\alpha_1 SI + \eta R - \mu S\\
\dot{L} &= (1-c)\alpha SI - \{(1-a)\beta + a\gamma + \mu\}E\\
\dot{I} &= a\gamma E + c\alpha_1 SI + b\delta A_1 - (\phi+\mu)I\\
\dot{A_1} &= (1-a)\beta E - \{b\delta + (1-b)\theta + \mu\}A_1\\
\dot{A_2} &= \phi I - (\psi+\mu)A_2\\
\dot{Q} &= (1-b)\theta A_1 + \psi A_2 - (\xi+\mu)Q\\
\dot{R} &= \xi Q - (\mu+\eta)R
\end{aligned}\right\} \tag{1}$$

With $S,\ L,I,A_1,A_2,Q,\ R \geq 0$. All the parameters are taken as positive.

Here $N = S + L + I + A_1 + A_2 + Q + R$

$$\dot{N} = \dot{S} + \dot{L} + \dot{I} + \dot{A_1} + \dot{A_2} + \dot{Q} + \dot{R}$$
$$\dot{N} = T - \mu N$$

As $t \to \infty,\quad N \to \frac{T}{\mu}$

3. Malicious Free Equilibrium Point

The region

$$\Lambda = \left\{(S, L, I, A_1, A_2, Q, R) \epsilon\ \mathbb{R}_+{}^7 : S \geq 0, L \geq 0, I \geq 0, A_1 \geq 0, A_2 \geq 0, Q \geq 0, R \geq 0\right\}$$

When the virus is absent, i.e., $I = 0$.

There exist an equilibrium point $P_0 = \left(S_0, L_0, I_0, A_{1_0}, A_{2_0}, Q_0, R_0\right)$.

$i.e.,\quad P_0 = \left(\frac{T}{\mu}, 0, 0, 0, 0, 0, 0\right)$.

The next generation matrix can be written as,

$$\frac{d\mho}{dt} = \mathcal{F} - \vartheta$$

Where , $\mathcal{F} = \begin{bmatrix} (1-c)\alpha SI \\ c\alpha_1 SI + b\delta A_1 \end{bmatrix}$ and $\vartheta = \begin{bmatrix} \{(1-a)\beta + a\gamma + \mu\}E \\ -a\gamma E + (\phi+\mu)I \end{bmatrix}$

$\therefore$ $\mathbb{f}$ is jacobian of $\mathcal{F} = \begin{bmatrix} 0 & (1-c)\alpha S \\ 0 & c\alpha_1 S \end{bmatrix}$

and $\mathbb{V}$ is jacobian of $\vartheta = \begin{bmatrix} (1-a)\beta + a\gamma + \mu & 0 \\ -a\gamma & (\phi+\mu) \end{bmatrix}$

$$\therefore\ \mathbb{V}^{-1} = \begin{bmatrix} \frac{1}{\{(1-a)\beta + a\gamma + \mu\}} & 0 \\ \frac{a\gamma}{\{(1-a)\beta + a\gamma + \mu\}(\phi+\mu)} & \frac{1}{(\phi+\mu)} \end{bmatrix}$$

$$\therefore\ \mathbb{fV}^{-1} = \begin{bmatrix} \frac{(1-c)\alpha^2\gamma S}{\{(1-a)\beta + a\gamma + \mu\}(\phi+\mu)} & \frac{(1-c)\alpha S}{(\phi+\mu)} \\ \frac{c\alpha_1\gamma S}{\{(1-a)\beta + a\gamma + \mu\}(\phi+\mu)} & \frac{c\alpha_1 S}{(\phi+\mu)} \end{bmatrix}$$

Then the threshold number $R_0 = \rho(\mathbb{fV}^{-1})$

$$R_0 = \frac{(1-c)\alpha^2\gamma S + c\alpha_1 S\{(1-a)\beta + a\gamma + \mu\}}{\{(1-a)\beta + a\gamma + \mu\}(\phi+\mu)} > 0$$

Theorem: The malicious object-free equilibrium point P_0 of the system (1) is locally asymptotically stable if $R_0 < 1$.

Proof: We construct a jacobian for P_0.

$$\mathcal{J}_{P_0} = \begin{bmatrix} -\mu & 0 & a_{13} & 0 & 0 & 0 & \eta \\ 0 & a_{22} & a_{23} & 0 & 0 & 0 & 0 \\ 0 & \alpha\gamma & a_{33} & \beta\delta & 0 & 0 & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 & a_{55} & 0 & 0 \\ 0 & 0 & 0 & a_{64} & \psi & a_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & a_{77} \end{bmatrix}$$

$a_{13} = \{(1-c)\alpha + c\alpha_1\}S_0$, $a_{22} = \{(1-a)\beta + a\gamma + \mu\}$

$a_{23} = (1-c)\alpha S_0$, $a_{33} = c\alpha_1 S_0 - (\phi+\mu)$

$a_{42} = (1-a)\beta$, $a_{44} = -\{b\delta + (1-b)\theta + \mu\}$

$a_{55} = -(\psi+\mu)$, $a_{64} = (1-b)\theta$

$a_{66} = -(\xi+\mu)$, $a_{77} = -(\eta+\mu)$.

The characteristic equation is $\left|\mathcal{J}_{P_0} - \lambda I\right| = 0$

$$\left|\mathcal{J}_{P_0} - \lambda I\right| = \begin{vmatrix} -\mu-\lambda & 0 & a_{13} & 0 & 0 & 0 & \eta \\ 0 & a_{22}-\lambda & a_{23} & 0 & 0 & 0 & 0 \\ 0 & \alpha\gamma & a_{33}-\lambda & \beta\delta & 0 & 0 & 0 \\ 0 & a_{42} & 0 & a_{44}-\lambda & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 & a_{55}-\lambda & 0 & 0 \\ 0 & 0 & 0 & a_{64} & \psi & a_{66}-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & a_{77}-\lambda \end{vmatrix} = 0$$

The characteristic equation is

$$(\mu+\lambda)(\mu+\eta+\lambda)(\mu+\xi+\lambda)(\mu+\psi+\lambda)(\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3) = 0$$

Where , $c_1 = -\{(1-a)\beta + a\gamma + c\alpha_1 S_0 - \phi - b\delta - (1-b)\theta - \mu\} > 0$

$$c_2 = \{(1-a)\beta + a\gamma + \mu\}(c\alpha_1 S_0 - \phi - \mu) - \alpha\gamma(1-c)\alpha S_0$$

$$-\{(1-a)\beta + a\gamma + c\alpha_1 S_0 - \phi\}\{b\delta + (1-b)\theta + \mu\} > 0$$

$c_3 = \{b\delta + (1-b)\theta + \mu\}[\{(1-a)\beta + a\gamma + \mu\}(c\alpha_1 S_0 - \phi - \mu) + \alpha\gamma(1-c)\alpha S_0] - (1-c)(1-a)b\alpha\beta S_0 > 0$

$\left|\mathcal{J}_{P_0} - \lambda I\right| = 0$, it has four roots that are negative, i.e., $\lambda_1 = -\mu,\ \lambda_2 = -(\mu+\eta)$,

$\lambda_3 = -(\mu + \xi)$ and $\lambda_4 = -(\mu + \psi)$. But for the remaining three roots, we can't so that they are all negative. So, we use the Hurwitz condition $c_1 > 0,\ c_2 > 0,\ c_3 > 0$ and $c_2 c_1 > c_3$. We observed that the system is stable by using the Hurwitz criterion.

4. Endemic Equilibrium Point

An endemic equilibrium point means where infection exists and they attacks the system.

E.E. point is calculated by $\dot{S} = \dot{L} = \dot{I} = \dot{A_1} = \dot{A_2} = \dot{Q} = \dot{R} = 0$

$T - (1-c)\alpha SI - c\alpha_1 SI + \eta R - \mu S = 0$

$(1-c)\alpha SI - \{(1-a)\beta + a\gamma + \mu\}E = 0$

$a\gamma E + c\alpha_1 SI + b\delta A_1 - (\phi + \mu)I = 0$

$(1-a)\beta E - \{b\delta + (1-b)\theta + \mu\}A_1 = 0$

$\phi I - (\psi + \mu)A_2 = 0$

$(1-b)\theta A_1 + \psi A_2 - (\xi + \mu)Q = 0$

$\xi Q - (\mu + \eta)R = 0$

We get e e point $P_1 = \left(S_1, L_1, I_1, A_{1_1}, A_{2_1}, Q_1, R_1\right)$

$$S_1 = \frac{T + \eta R_1}{\mu + \{(1-c)\alpha + c\alpha_1\}I_1}$$

$$L_1 = \frac{(1-c)\alpha S_1 I_1}{\{(1-a)\beta + a\gamma + \mu\}}$$

$$I_1 = \frac{a\gamma\{b\delta + (1-b)\theta + \mu\} + b(1-a)\delta\beta}{(\phi + \mu - c\alpha_1 S_1)} E_1$$

$$A_{1_1} = \frac{(1-a)\beta E_1}{b\delta + (1-b)\theta + \mu}$$

$$A_{2_1} = \frac{\phi I_1}{\psi + \mu}$$

$$Q_1 = \frac{(1-b)\theta(1-a)\beta(\psi + \mu)E_1 + \psi\phi I_1\{b\delta + (1-b)\theta + \mu\}}{\{b\delta + (1-b)\theta + \mu\}(\psi + \mu)(\xi + \mu)}$$

This shows that the E E point exist.

5. Conclusion

In this article, the model is an extension of the SAIR model. We discussed about threshold number. R_0. The malicious object free equilibrium point P_0 of the system (1) is locally asymptotically stable if $R_0 < 1$. at last, we calculate the Endemic equilibrium point.

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