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Original Article

Orthogonality of Reverse $(\alpha, 1)$ Derivation and Symmetric Reverse $(\alpha, 1)$ Biderivation in Semiprime Rings

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Abstract - *This paper examines the orthogonality between reverse $(\alpha, 1)$-derivation and symmetric reverse $(\alpha, 1)$-biderivation in a 2 torsion-free semiprime ring. Several equivalences are established through lemmas and theorems that describe necessary and sufficient conditions for such mappings to be orthogonal. In particular, it is shown that orthogonality enforces bilinear identities in which special cases ensure that the associated mapping becomes a biderivation. These results extend previous studies on orthogonal derivations and biderivations while offering new perspectives on the structural properties of semiprime rings. This framework presented an open gap by characterizing the interaction of orthogonality between reverse $(\alpha, 1)$-derivation and symmetric reverse $(\alpha, 1)$-biderivation.*

Keywords - *Derivation, $(\alpha, 1)$-derivation, reverse $(\alpha, 1)$-derivation, reverse (α, 1)-biderivation, Semiprime Ring.*

1. Introduction

Earlier studies have addressed orthogonality in several contexts, including reverse derivations in semiprime rings [1], generalized derivations [2], and classical derivations linked to Posner-type results [3]. Extensions to reverse, Jordan, left biderivations were explored in [4], while orthogonality of (σ, τ)-derivations and bi-(σ, τ)-derivations was analyzed in [5]. Orthogonality between classical derivations and biderivations was considered in [6]. Recent studies have examined structural properties of semiprime rings under derivations [7], extensions of Jordan derivations in non-associative algebraic structures [8], functional identities involving symmetric derivations [9], and orthogonal derivations in generalized near rings [10]. Although these works provided significant insights, they either focused on generalized derivations, classical derivations, or restricted cases such as (σ, τ)-derivations, leaving the symmetric reverse setting largely unexamined. The novelty of the work lies in addressing this gap by examining the orthogonality between reverse $(\alpha, 1)$-derivations and symmetric reverse $(\alpha, 1)$-biderivations in 2-torsion-free semiprime rings.

2. Preliminaries

Throughout this work, $\mathcal{R}$ stands for an associative ring. A ring $\mathcal{R}$is 2-torsion-free provided that, for every. $p_{u1} \in \mathcal{R}$, the condition $2p_{u1} = 0$ entails $p_{u1} = 0$. A ring $\mathcal{R}$ is termed prime if $p_{u1}\mathcal{R}q_{v1} = 0$ implies $p_{u1} = 0$ or $q_{v1} = 0$; and $\mathcal{R}$ is semiprime if $p_{u1}\mathcal{R}p_{u1} = 0$implies $p_{u1} = 0$.An example of a 2-torsion-free semiprime ring is $(3\mathbb{Z}, +, \cdot)$.An additive map $\delta: \mathcal{R} \to \mathcal{R}$is termed a derivation on $\mathcal{R}$ if $\delta(p_{u1}q_{v1}) = \delta(p_{u1})\, q_{v1} + p_{u1}\, \delta(q_{v1})$. An additive map $\delta: \mathcal{R} \to \mathcal{R}$ is termed a reverse derivation on $\mathcal{R}$ if $\delta(p_{u1}q_{v1}) = \delta(q_{v1})\, p_{u1} + q_{v1}\, \delta(p_{u1})$. A mapping $\mathbb{B}(.,.): \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is symmetric if $\mathbb{B}(p_{u1}, q_{v1}) = \mathbb{B}(q_{v1}, p_{u1})$.

A symmetric biadditive mapping $\mathbb{B}: \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is referred to as a biderivation if
$\mathbb{B}(p_{u1}q_{v1}, r_{w1}) = \mathbb{B}(p_{u1}, r_{w1})\, q_{v1} + p_{u1}\, \mathbb{B}(q_{v1}, r_{w1})$, and $\mathbb{B}(p_{u1}, q_{v1}r_{w1}) = \mathbb{B}(p_{u1}, q_{v1})\, r_{w1} + q_{v1}\, \mathbb{B}(p_{u1}, r_{w1})$.
A function $\alpha: \mathcal{R} \to \mathcal{R}$is called an endomorphism if it satisfies:
Additivity $\alpha(a + b) = \alpha(a) + \alpha(b)$;
Multiplicativity $\alpha(ab) = \alpha(a)\alpha(b)$;

Identity preservation $\alpha(1) = 1$.
An additive map $\delta: \mathcal{R} \to \mathcal{R}$ is called an $(\alpha, 1)$-derivation if $\delta(p_{u1}q_{v1}) = \delta(p_{u1})\,\alpha(q_{v1}) + p_{u1}\,\delta(q_{v1})$.
An additive map $\delta: \mathcal{R} \to \mathcal{R}$ is called a reverse $(\alpha, 1)$-derivation if $\delta(p_{u1}q_{v1}) = \delta(q_{v1})\,\alpha(p_{u1}) + q_{v1}\,\delta(p_{u1})$.
An additive map $\mathbb{B}: \mathcal{R} \times \mathcal{R} \to \mathcal{R}$is termed a symmetric $(\alpha, 1)$-biderivation if
$\mathbb{B}(p_{u1}q_{v1}, r_{w1}) = \mathbb{B}(p_{u1}, r_{w1})\,\alpha(q_{v1}) + p_{u1}\,\mathbb{B}(q_{v1}, r_{w1})$, and
$\mathbb{B}(p_{u1}, q_{v1}r_{w1}) = \mathbb{B}(p_{u1}, q_{v1})\,\alpha(r_{w1}) + q_{v1}\,\mathbb{B}(p_{u1}, r_{w1})$.
An additive map $\mathbb{B}: \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is termed a symmetric reverse$(\alpha, 1)$-biderivation if $\mathbb{B}(p_{u1}q_{v1}, r_{w1}) =$
$\mathbb{B}(q_{v1}, r_{w1})\,\alpha(p_{u1}) + q_{v1}\,\mathbb{B}(p_{u1}, r_{w1})$, and
$\mathbb{B}(p_{u1}, q_{v1}r_{w1}) = \mathbb{B}(p_{u1}, r_{w1})\,\alpha(q_{v1}) + r_{w1}\,\mathbb{B}(p_{u1}, q_{v1})$, Where α is an endomorphism on $\mathcal{R}$.
Let $\mathcal{R}$be a semiprime ring. Then δ and g are orthogonal if $\delta(p_{u1})\,\mathcal{R}\,g(q_{v1}) = 0 = g(q_{v1})\,\mathcal{R}\,\delta(p_{u1}), \forall p_{u1}, q_{v1} \in \mathcal{R}$. Also, if δ is a reverse derivation and $\mathbb{B}$a a reverse biderivation on $\mathcal{R}$, they are called orthogonal when $\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1}, r_{w1}) = 0 =$ $\mathbb{B}(q_{v1}, r_{w1})\,\mathcal{R}\,\delta(p_{u1}), \forall\, p_{u1}, q_{v1}, r_{w1} \in \mathcal{R}$.

Lemma 1: ([3], Lemma 1)
Let $\mathcal{R}$ be a 2-torsion-free semiprime ring for every. $p_{u1}, q_{v1} \in \mathcal{R}$. Then the following are equivalent:
1)$p_{u1} r q_{v1} = 0$
2)$q_{v1} r p_{u1} = 0$
3)$p_{u1} r q_{v1} + q_{v1} r p_{u1} = 0, \forall p_{u1}, q_{v1} \in \mathcal{R}$.
If one of the above conditions is fulfilled, then $p_{u1}q_{v1} = q_{v1}p_{u1} = 0$.

Lemma 2: ([10], Lemma 2)
If $\mathcal{R}$ is a 2-torsion-free semiprime ring, δ and β are derivations of $\mathcal{R}$ such that. $(\delta(p_{u1})r)\,\beta(q_{v1}) = 0 =$ $\beta(q_{v1})\big(r\,\delta(p_{u1})\big)$, then $\beta(p_{u1})\big(r\,\delta(q_{v1})\big) = 0 = (\delta(q_{v1})\,r)\,\beta(p_{u1}), \forall p_{u1}, q_{v1}, r \in \mathcal{R}$.

Lemma 3: ([10], Lemma 3)
Let $\mathcal{R}$ be a 2-torsion-free semiprime ring. Suppose that a reverse derivation $\delta: \mathcal{R} \to \mathcal{R}$ and a symmetric reverse. Biderivation $\mathbb{B}: \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ satisfy $\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(p_{u1}, q_{v1}) = 0 = \mathbb{B}(p_{u1}, q_{v1})\,\mathcal{R}\,\delta(p_{u1}), \forall p_{u1}, q_{v1} \in$ $\mathcal{R}$, then $\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1}, r_{w1}) = 0 = \mathbb{B}(q_{v1}, r_{w1})\,\mathcal{R}\,\delta(p_{u1}), \forall p_{u1}, q_{v1}, r_{w1} \in \mathcal{R}$.

Lemma 4:
Let δ be a reverse $(\alpha, 1)$-derivation and $\mathbb{B}$ be a symmetric reverse $(\alpha, 1)$-biderivation of a semiprime ring $\mathcal{R}$. Then the following identities hold.
$\delta\mathbb{B}(p_{u1}q_{v1}, r_{w1}) = \delta(\mathbb{B}(p_{u1}q_{v1}, r_{w1}))$
$\delta\mathbb{B}(p_{u1}q_{v1}, r_{w1}) = \delta(\mathbb{B}(q_{v1}, r_{w1})\,\alpha(p_{u1}) + q_{v1}\,\mathbb{B}(p_{u1}, r_{w1}))$
$\delta\mathbb{B}(p_{u1}q_{v1}, r_{w1}) = \delta(\alpha(p_{u1}))\,\alpha(\mathbb{B}(q_{v1}, r_{w1})) + \alpha(p_{u1})\,\delta\mathbb{B}(q_{v1}, r_{w1}) + \delta\mathbb{B}(p_{u1}, r_{w1})\,\alpha(q_{v1}) + \mathbb{B}(p_{u1}, r_{w1})\,\delta(q_{v1})$.
If $\alpha(p_{u1}) = p_{u1}$ and $\alpha(\mathbb{B}(q_{v1}, r_{w1})) = \mathbb{B}(q_{v1}, r_{w1})$, then we get
$\delta\mathbb{B}(p_{u1}q_{v1}, r_{w1}) = \delta(p_{u1})\,\mathbb{B}(q_{v1}, r_{w1}) + p_{u1}\,\delta\mathbb{B}(q_{v1}, r_{w1}) + \delta\mathbb{B}(p_{u1}, r_{w1})\,\alpha(q_{v1}) + \mathbb{B}(p_{u1}, r_{w1})\,\delta(q_{v1})$.

3. Results and Discussions

Theorem 1: Let $\mathcal{R}$ be a 2-torsion-free semiprime ring, $\alpha: \mathcal{R} \to \mathcal{R}$ be an endomorphism on $\mathcal{R}$.
If $\delta$is a reverse $(\alpha, 1)$-derivation and $\mathbb{B}$is a symmetric reverse $(\alpha, 1)$-biderivation, they are orthogonal if and only if $\mathbb{B}(p_{u1}, q_{v1})\,\delta(r_{w1}) + \delta(p_{u1})\,\mathbb{B}(r_{w1}, q_{v1}) = 0, \forall p_{u1}, q_{v1}, r_{w1} \in \mathcal{R}$.

Proof:
First, we assume δ be a reverse $(\alpha, 1)$-derivation and $\mathbb{B}$ be a symmetric reverse $(\alpha, 1)$-biderivation, then such that.

$$\mathbb{B}(p_{u1}, q_{v1})\,\delta(r_{w1}) + \delta(p_{u1})\,\mathbb{B}(p_{u1}, q_{v1}) = 0 \tag{1}$$

$$\mathbb{B}(p_{u1}, q_{v1})\,\delta(r_{w1}) + \delta(p_{u1})\,\mathbb{B}(q_{v1}, r_{w1}) = 0 \tag{2}$$

Replace $r_{w1}$by $p_{u1}r_{w1}$in (2), we get
$\mathbb{B}(p_{u1}, q_{v1})\,\delta(p_{u1}r_{w1}) + \delta(p_{u1})\,\mathbb{B}(q_{v1}, p_{u1}r_{w1}) = 0$ $\mathbb{B}(p_{u1}, q_{v1})\big(\delta(r_{w1})\,\alpha(p_{u1}) + r_{w1}\,\delta(p_{u1})\big) +$
$\delta(p_{u1})\big(\mathbb{B}(q_{v1}, r_{w1})\,\alpha(p_{u1}) + r_{w1}\,\mathbb{B}(q_{v1}, p_{u1})\big) = 0$

$\mathbb{B}(p_{u1},q_{v1})\,\delta(r_{w1})\,\alpha(p_{u1})+\mathbb{B}(p_{u1},q_{v1})\,r_{w1}\,\delta(p_{u1})+\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})\,\alpha(p_{u1})+\delta(p_{u1})\,r_{w1}\,\mathbb{B}(q_{v1},p_{u1})=0$

$$\big(\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})+\mathbb{B}(p_{u1},q_{v1})\,\delta(r_{w1})\big)\,\alpha(p_{u1})+\delta(p_{u1})\,r_{w1}\,\mathbb{B}(q_{v1},p_{u1})+\mathbb{B}(p_{u1},q_{v1})\,r_{w1}\,\delta(p_{u1})=0 \quad (3)$$

Substituting (2) in (3), we get

$\delta(p_{u1})\,r_{w1}\,\mathbb{B}(q_{v1},p_{u1})+\mathbb{B}(p_{u1},q_{v1})\,r_{w1}\,\delta(p_{u1})=0,\forall p_{u1},q_{v1},r_{w1}\in\mathcal{R}.\ \delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(p_{u1},q_{v1})+\mathbb{B}(p_{u1},q_{v1})\,\mathcal{R}\,\delta(p_{u1})=0$

By Lemma (1), we have
$\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(p_{u1},q_{v1})=0$

By Lemma (2), we have
$\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1},r_{w1})=0$
$\therefore\ \delta$ and $\mathbb{B}$ are orthogonal.

Conversely, if δ and $\mathbb{B}$ are orthogonal, then $\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1},r_{w1})=0$ also
$\mathbb{B}(p_{u1},q_{v1})\,\mathcal{R}\,\delta(r_{w1})=0$.
Since $1\in\mathcal{R}$ implies $\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})=0$ and $\mathbb{B}(p_{u1},q_{v1})\,\delta(r_{w1})=0$.
$\therefore\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})+\mathbb{B}(p_{u1},q_{v1})\,\delta(r_{w1})=0$.

***Theorem 2*:** Let $\mathcal{R}$ be a 2-torsion-free semi-prime ring, $\alpha:\mathcal{R}\to\mathcal{R}$ be an endomorphism on $\mathcal{R}$.
If $\delta$is a reverse $(\alpha,1)$-derivation and $\mathbb{B}$ is a symmetric reverse $(\alpha,1)$-biderivation, they are orthogonal if and only if $\delta(p_{u1})\,\mathbb{B}(p_{u1},q_{v1})=0,\forall p_{u1},q_{v1}\in\mathcal{R}$.

Proof:

Assume that $\delta$and $\mathbb{B}$are reverse $(\alpha,1)$-derivation and symmetric reverse $(\alpha,1)$-biderivation

Such that $$\delta(p_{u1})\,\mathbb{B}(p_{u1},q_{v1})=0 \quad (4)$$

Replace $q_{v1}$by $r_{w1}q_{v1}$in (4)
$\delta(p_{u1})\,\mathbb{B}(p_{u1},r_{w1}q_{v1})=0$
$\delta(p_{u1})(\mathbb{B}(p_{u1},q_{v1})\,\alpha(r_{w1})+q_{v1}\,\mathbb{B}(p_{u1},r_{w1}))=0$

$$\delta(p_{u1})\,\mathbb{B}(p_{u1},q_{v1})\,\alpha(r_{w1})+\delta(p_{u1})\,q_{v1}\,\mathbb{B}(p_{u1},r_{w1})=0 \quad (5)$$

Substitute (4) in (5), we get

$$\delta(p_{u1})\,q_{v1}\,\mathbb{B}(p_{u1},r_{w1})=0,\forall p_{u1},q_{v1},r_{w1}\in\mathcal{R}. \quad (6)$$

Replace $r_{w1}$by q_{v1}in (6), we get
$\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(p_{u1},q_{v1})=0$.

By Lemma (2),

$$\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1},r_{w1})=0,\forall p_{u1},q_{v1},r_{w1}\in\mathcal{R}. \quad (7)$$

$\therefore\delta$ and $\mathbb{B}$ are orthogonal.

Conversely, $\delta$and $\mathbb{B}$are orthogonal, i.e., $$\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1},r_{w1})=0. \quad (8)$$

Substitute $r_{w1}=p_{u1}$in (8), we get
$\delta(p_{u1})\,\mathcal{R}\,\mathbb{B}(q_{v1},p_{u1})=0$.
Since1 $\in\mathcal{R}$, i.e.,$\delta(p_{u1})\,\mathbb{B}(p_{u1},q_{v1})=0$.
Hence,$\delta(p_{u1})\,\mathbb{B}(p_{u1},q_{v1})=0$.

***Theorem 3*:** Let $\mathcal{R}$ be a 2-torsion-free semiprime ring, $\alpha:\mathcal{R}\to\mathcal{R}$ be an endomorphism on $\mathcal{R}$.
If $\delta$is a reverse $(\alpha,1)$-derivation and $\mathbb{B}$ is a symmetric reverse $(\alpha,1)$-biderivation, they are orthogonal if and only if $\delta\mathbb{B}=0$.

Proof:
Let δbe a reverse $(\alpha,1)$-derivation and $\mathbb{B}$be a reverse $(\alpha,1)$-biderivation such that $\delta\mathbb{B}=0$.
By using Lemma (4), we have
$\delta\mathbb{B}(p_{u1}q_{v1},r_{w1})=\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})+p_{u1}\,\delta\mathbb{B}(q_{v1},r_{w1})+\delta\mathbb{B}(p_{u1},r_{w1})\,\alpha(q_{v1})+\mathbb{B}(p_{u1},r_{w1})\,\delta(q_{v1})$
$\delta\mathbb{B}(p_{u1},r_{w1})\,\alpha(q_{v1})+p_{u1}\,\delta\mathbb{B}(q_{v1},r_{w1})=\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})+p_{u1}\,\mathbb{B}(q_{v1},r_{w1})+\delta\mathbb{B}(p_{u1},r_{w1})\,\alpha(q_{v1})+$
$\mathbb{B}(p_{u1},r_{w1})\,\delta(q_{v1})\qquad\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1})+\mathbb{B}(p_{u1},r_{w1})\,\delta(q_{v1})=0 \quad (9)$
Interchanging $r_{w1}$and q_{v1}in (9), we get

$\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1}) + \mathbb{B}(q_{v1},p_{u1})\,\delta(r_{w1}) = 0.$
By Theorem (1),
$\therefore\ \delta$ and $\mathbb{B}$ are orthogonal.
Conversely, suppose that $\delta$and $\mathbb{B}$are orthogonal, then.
$\delta(p_{u1})\,r\,\mathbb{B}(q_{v1},r_{w1}) = 0.$
$\delta(\delta(p_{u1})\,r\,\mathbb{B}(q_{v1},r_{w1})) = 0.$
$\delta(r\,\mathbb{B}(q_{v1},r_{w1}))\,\alpha(\delta(p_{u1})) + r\,\mathbb{B}(q_{v1},r_{w1})\,\delta(\delta(p_{u1})) = 0.$

$$(\delta\mathbb{B}(q_{v1},r_{w1})\,\alpha(r) + \mathbb{B}(q_{v1},r_{w1})\,\delta(r))\,\alpha(\delta(p_{u1})) + r\,\mathbb{B}(q_{v1},p_{u1})\,\delta\delta(p_{u1}) = 0. \quad (10)$$

Replace$\alpha(r) = r,\ \delta(p_{u1}) = \delta(p_{u1})$, Then
$\delta\mathbb{B}(q_{v1},r_{w1})\,r\,\delta(p_{u1}) + \mathbb{B}(q_{v1},r_{w1})\,\delta(r)\,\delta(p_{u1}) + r\,\mathbb{B}(q_{v1},r_{w1})\,\delta(\delta(p_{u1})) = 0.$
$\delta\mathbb{B}(q_{v1},r_{w1})\,r\,\delta(p_{u1}) = 0.$
Replace $p_{u1} = \mathbb{B}(q_{v1},r_{w1})$in the above equation, we get
$\delta\mathbb{B}(q_{v1},r_{w1})\,r\,\delta(\mathbb{B}(q_{v1},r_{w1})) = 0.\ \delta\mathbb{B}(q_{v1},r_{w1})\,r\,\delta\mathbb{B}(q_{v1},r_{w1}) = 0.$
Since $\mathcal{R}$ is a semiprime ring,
$\delta\mathbb{B}(q_{v1},r_{w1}) = 0, \forall q_{v1},r_{w1} \in \mathcal{R}.$
$\therefore\ \delta\mathbb{B} = 0.$

Theorem 4**:** Let $\mathcal{R}$ be a 2-torsion-free semiprime ring, $\alpha:\mathcal{R} \to \mathcal{R}$be an endomorphism on $\mathcal{R}$.If δ is a reverse $(\alpha,1)$-derivation and $\mathbb{B}$ is a symmetric reverse $(\alpha,1)$-biderivation such that $\delta$and $\mathbb{B}$ are orthogonal if and only if $\delta\mathbb{B}$is an $(\alpha,1)$-biderivation.

Proof:
Let $\delta$and $\mathbb{B}$be reverse $(\alpha,1)$-derivation and symmetric reverse $(\alpha,1)$-biderivation, then $\delta\mathbb{B}$is an $(\alpha,1)$-biderivation.
i.e $\quad \delta\mathbb{B}(p_{u1}q_{v1},r_{w1}) = \delta\mathbb{B}(p_{u1},r_{w1})\,\alpha(q_{v1}) + p_{u1}\,\delta\mathbb{B}(q_{v1},r_{w1}).$ (11)
To prove that $\delta$and $\mathbb{B}$are orthogonal.

By using Lemma (4), we have

$$\delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1}) + \mathbb{B}(p_{u1},r_{w1})\,\delta(q_{v1}) = 0. \quad (12)$$

Interchanging $r_{w1}$and q_{v1}we get
$\delta(p_{u1})\,\mathbb{B}(r_{w1},q_{v1}) + \mathbb{B}(p_{u1},q_{v1})\,\delta(r_{w1}) = 0.$
By Theorem (1),
$\therefore\ \delta$ and $\mathbb{B}$ are orthogonal.

Conversely, if $\delta$and $\mathbb{B}$are orthogonal,
By Lemma (4), we get

$$\delta\mathbb{B}(p_{u1}q_{v1},r_{w1}) = \delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1}) + p_{u1}\,\delta\mathbb{B}(q_{v1},r_{w1}) + \delta\mathbb{B}(p_{u1},r_{w1})\,\alpha(q_{v1}) + \mathbb{B}(p_{u1},r_{w1})\,\delta(q_{v1}). \quad (13)$$

Substituting equation (12) in (13), we get
$\delta\mathbb{B}(p_{u1}q_{v1},r_{w1}) = \delta\mathbb{B}(p_{u1},r_{w1})\,\alpha(q_{v1}) + p_{u1}\,\delta\mathbb{B}(q_{v1},r_{w1}).$
$\therefore\ \delta\mathbb{B}$ is an $(\alpha,1)$-biderivation.

4. Conclusion

This study analyzes the orthogonality between reverse $(\alpha,1)$-derivation and symmetric reverse $(\alpha,1)$-biderivation in a 2-torsion-free semiprime ring. If $\delta$is a reverse $(\alpha,1)$-derivation and $\mathbb{B}$ is a reverse $(\alpha,1)$-biderivation of $\mathcal{R}$, then $\delta$and $\mathbb{B}$are orthogonal if and only if any one of the following conditions holds for all $p_{u1},q_{v1},r_{w1} \in \mathcal{R}$:

1. $\delta\mathbb{B} = 0$
2. $\mathbb{B}(p_{u1},q_{v1})\,\delta(r_{w1}) + \delta(p_{u1})\,\mathbb{B}(q_{v1},r_{w1}) = 0$
3. $\delta(p_{u1})\,\mathbb{B}(p_{u1},q_{v1}) = 0$
4. $\delta\mathbb{B}$ is an $(\alpha,1)$-biderivation.

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