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Original Article

Reliability Advancement and Availability Improvement of Industrial Systems using RPGT and Metaheuristic Algorithms

Savita Garg[1], Rachna Aggarwal[2], Shilpa Rani[3], Neetu Rani[4], Diksha Mangla[5]

[1,2]*Department of Mathematics, Mukand Lal National College, Yamuna Nagar, Haryana, India.*
[3,5]*Department of Mathematics and Humanities, MMEC, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India.*
[4]*Department of Mathematics, Shivaji College, University of Delhi, Delhi, India.*

[2]*Corresponding Author : raggarwal.math@mlncollegeynr.ac.in*

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Abstract *- This paper investigates the optimization of reliability and availability using the Regenerative Point Graphical Technique (RPGT). The study mainly focuses on system performance by systematically analyzing failure and repair rates through nature-inspired algorithms using Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Cuckoo Search Algorithm (CSA). A Markovian state model is developed to represent transitions between operational and failed states to compute reliability metrics such as Mean Time to System Failure (MTSF) and the system's availability. Mean sojourn times and transition probabilities are derived to solve state distributions. The methodology set a dataset comprising parameters such as workload, failure rate, and repair rate. Sensor index, which collectively determines maintenance priority, enables predictive maintenance strategies. The optimization framework employing GA, PSO, and CSA can effectively optimize failure and repair parameters, improving system reliability and operational continuity. Comparative analysis highlights the efficiency and behavior of each algorithm, as well as the trade-offs between exploration and exploitation in the optimization process.*

Keywords *- Availability Analysis, Cuckoo Search Algorithm, Genetic Algorithm, Particle Swarm Optimization, System performance optimization.*

1. Introduction

In modern engineering times, systems are designed to maintain high reliability and operational continuity to minimize downtime and reduce operating costs. This can be achieved via embedding redundancy and repair mechanisms to navigate component failures. Accurate modeling and optimization of a system enable proactive maintenance strategies and overall performance. Regenerative point Graphical Technique (RPGT) provides an approach to model system behavior, considering both failure and repair rates. By processing key parameters such as failure rates and repair rates, it is possible to maximize Mean Time to System Failure (MTSF) and enhance availability.

Ravi [1] has extended the great deluge algorithm into a modified great deluge algorithm for optimization of complex system reliability under cost. The algorithm is applied to redundancy allocation problems and reliability optimization, implemented in ANSIC, and compared with simulated annealing variants and other methods. Results show the Modified Great Deluge Algorithm (MGDA) achieves superior accuracy and speed, performing comparably to Ant Colony Optimization (ACO), making it an efficient alternative for reliability optimization. Zuo et al. [2] calculated the interpretation of a series-parallel system under different operating policy conditions. Haggag [3] discussed the 2-unit cold standby system with common cause failure and preventive maintenance using Kolmogorov's forward equations method. Kovalev et al. [4] presented an extensive review of different models and methods used for the optimization of electric power reliability. Yusuf et al. [5] studied the various measures of system performance using Kolmogorov's forward equations method. Khorshidi et al. [6] focused on optimizing the reliability of multistate weighted k-out-of-n systems using a dynamic modeling approach and compared the performance of genetic and imperialist competitive algorithms. Mirjalili [7] highlighted the effectiveness of Motth-Flame Optimization in handling complex constrained and unknown search spaces. The time-dependent behavior of a single-server queuing model was studied by Kumar et al. [8]. The Modified Gray Wolf Optimization algorithm was introduced by Sharma et al. [9] by integrating feature selection with classifiers such as Random Forest and decision trees. Gandhi et al. [10] solved a nonlinear noncomplex problem using the Shuffled Frog Leaping Algorithm. Zhang et al. [11] suggested a k-out-of-n model: G sub-system using active cold-standby or mixed redundancy strategy. Nath & Muhuri [13] introduced a novel formulation for the Reliability Redundancy Allocation Problem (RRAP). A hybrid evolutionary algorithm is recommended for various RRAP structures and achieves better performance over existing methods. With the help of a Machine Learning Algorithm using the Internet of Things, Muniandi et al. [14] developed a blueprint for condition-based monitoring and proactive maintenance of electronics systems. Gorji [12] examined the supply chain of green hydrogen for many stages. Key challenges were identified, and the use of metaheuristic optimization techniques to improve efficiency and sustainability was investigated. This paper provides a systematic approach to integrate maintenance with reliability-based optimization, using the application of Metaheuristic algorithms like GA, PSO, and CSA. The suggested strategy provides guidance and practical insights for the management to achieve enhanced operational reliability and high availability for the working model.

2. Notations and Nomenclature

Si: System's states, where i=0, 1, …, 6

β : Respective constant transition rate of the main unit going under preventive maintenance.

β_j: Respective constant transition rate causing the main unit to go into reduced state while the reserved unit is in standby/in operation (j = 1/2)

β_k: Failure constant transition rate of the reserved unit in standby mode / in operational mode (k=3/5)

β_4: Failure constant transition rate of the main unit

α/α_4: Constant transition rate of preventive maintenance/ repair rate of the main unit

α_j: Minor/ Major maintenance rate of the main unit (j = 1/2)

α_k: repair rate of the reserved unit in standby mode /in operational mode (k = 3/5)

3. System Transition Diagram and Model Description

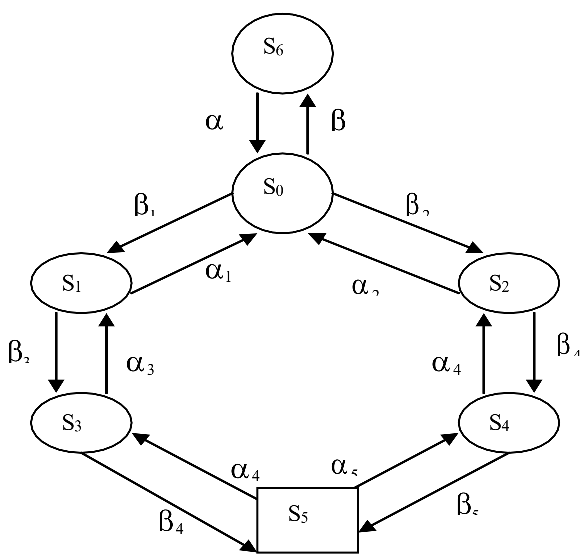

Fig. 1 Transition diagram

The model employed in this study (Figure 1) has been adopted from Yusuf et al. [5]. The transition diagram shows all possible states of the system, viz., transitions between the operational and non-operational states. The considered system contains two different units, namely, a main unit and a warm standby unit (reserve unit). Repairing of failed units is given a higher priority than major or minor maintenance by a repairman. The main unit is operational, and the reserve unit is on standby when the temperature is high. When the temperature changes from high to low, the main unit shifts to a reduced capacity, while the operation of the reserve unit depends upon the condition of the main unit. If both the main and the reserve system fail, the complete system goes into a failed state.

In Figure 1, each transition, labeled with a rate β for failure and α for repair, shows the corresponding transition probability. The system shows from S_0 to S_6, covering all operational and non-operational states. The Transition diagram is used for the calculation of mean sojourn times and probabilities for each state performance.

4. Mathematical Modelling

4.1. Transition Probabilities $q_{i,j}(t)$ and Mean Sojourn Time

Table 1. Transition Probabilities

From (State)	To (State)	Rate (label)	Transition probability ($q_{i,j}(t)$)
S_0	S_1	β_1	$\frac{\beta_1}{\beta+\beta_1+\beta_2}$
S_0	S_2	β_2	$\frac{\beta_2}{\beta+\beta_1+\beta_2}$
S_0	S_6	β	$\frac{\beta}{\beta+\beta_1+\beta_2}$
S_1	S_0	α_1	$\frac{\alpha_1}{\alpha_1+\beta_3}$
S_2	S_0	α_2	$\frac{\alpha_2}{\alpha_2+\beta_4}$
S_1	S_3	β_3	$\frac{\beta_3}{\alpha_1+\beta_3}$
S_3	S_1	α_3	$\frac{\alpha_3}{\alpha_3+\beta_4}$
S_3	S_5	β_4	$\frac{\beta_4}{\alpha_3+\beta_4}$
S_2	S_4	β_4	$\frac{\beta_4}{\alpha_2+\beta_4}$
S_6	S_0	α	1
S_4	S_5	β_5	$\frac{\beta_5}{\alpha_4+\beta_5}$

Table 2. Mean sojourn time

Outgoing rates	Mean sojourn time τ_i
$\beta_1+\beta_2$	$\tau_0=\frac{1}{\beta_1+\beta_2}$
α_1	$\tau_1=\frac{1}{\alpha_1}$
$\alpha_2+\beta_3$	$\frac{1}{\alpha_2+\beta_3}$
$\alpha_3+\beta_4$	$\frac{1}{\alpha_3+\beta_4}$
$\alpha_4+\alpha_5$	$\frac{1}{\alpha_4+\alpha_5}$
β_5	$\frac{1}{\beta_5}$

Table 1 shows the transition probability for moving from state 0 to state 6, whereas Table 2 represents the mean sojourn times after finding the reliability for various states.

4.2. Mean Time to System Failure (MTSF) (T_0)

The Mean Time to System Failure (MTSF, T_0) is the expected time the system operates before reaching complete failure, computed from state transition probabilities, repair rates, and visit probabilities of all system states.

$$\text{MTSF } (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \Big/ \left[1\text{-}\sum_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} \xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right] \tag{1}$$

$$T_0 = \frac{\frac{1}{\alpha_1}+\frac{1}{\alpha_3+\beta_4}+\frac{\alpha_3}{(\alpha_3+\beta_4)\,(\alpha_4+\alpha_5)}+\frac{1+\frac{\alpha_2}{\alpha_1}}{\beta_3}+\frac{1}{\beta_2}+\frac{1}{\alpha+\beta}(A+\frac{\beta_1}{\beta_2})}{1-A} \tag{2}$$

where $A=\frac{\beta_4}{\alpha_3+\beta_4}+\frac{\alpha_3\alpha_5}{(\alpha_3+\beta_4)(\alpha_4+\alpha_5)}$

4.3. Availability of System (A_0)

The system availability shows the time a system remains available and is calculated using repair rate, failure rate, and state transition probability.

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow}j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \Big/ \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right] \tag{3}$$

$$A_0 = 1-\overline{V}_6 \tag{4}$$

$$\overline{V}_6 = \frac{\beta_1}{\beta_1+\beta_2}+\frac{\beta_4}{\alpha_3+\beta_4}\frac{\alpha_5}{\alpha_4+\alpha_5}+1\cdot(probability\ of\ reaching\ from\ S5)$$

5. Methodology for Optimizing the Parameters

The dataset used in this study includes ten records. The four normalized features are 1). Workload (W) indicates usage (0-100), 2) Failure rate (α) reflects likelihood (0-100), 3) Sensor index (S) reflects unit health, a higher value shows more wear, and 4) Maintenance Priority (P), which ranges from 0-75 and indicates when maintenance is needed. The target is to train a supervised model to forecast P from W, α, and S for maintenance decisions.

Table 3. Range of parameters

Parameter	W	α	S	P
Range	0-100	0-100	0-100	0-75

The Optimization of RPGT-based reliability Metrics Using CSA, PSO, and GA is worked out below.

5.1. Mean Time to System Failure (T_0) with Optimized Parameter Values

The optimized value of MTSF and the related optimized failure and repair rates using the three algorithms: - Cuckoo search algorithm, Particle Swarn Optimization, and Genetic Algorithm, is computed as below

Table 4. Optimized Parameters and MTSF Value (T_0)

Algorithm	α_1	α_2	α_3	α_4	β_1	T_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.456714
PSO	1.1852	2.6891	0.9987	1.4723	3.0432	0.453109
GA	1.3794	2.8476	0.8742	1.6389	3.1954	0.443973

Table 4 shows the effectiveness of the algorithms to maximize MTSF to improve the reliability of the system.

5.2. Availability Optimization

The Optimized Failure and Repair rates to maximize the system availability using CSA, PSO, and GA are shown in Table 5 below.

Table 5. Optimized Parameters and Availability Value (T_0)

Algorithm	α_1	α_2	α_3	α_4	β_1	A_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.593937
PSO	1.3786	2.9874	0.9142	1.6349	3.0562	0.592282
GA	1.1842	2.6645	0.8963	1.5217	3.1325	0.595922

From above, we notice that GA computed the highest availability, followed by CSA, and the lowest value by PSO.

Using the three Metaheuristic algorithms, namely Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Cuckoo Search Algorithm (CSA) on a redundant system, the optimization of reliability indices was carried out using the RPGT technique. The relative results regarding MTSF optimisation show that CSA achieved the best performance (MTSF- .4567), showing its strong ability to maximize system lifetime, GA provided moderate results (MTSF- 0.4439), but PSO yielded the lowest MTSF (- 0.4531). In contrast, the results of the Steady state optimisation demonstrate that GA achieved the highest availability (-0.5959), CSA performed moderately (- 0.5939), and PSO resulted in the lowest availability (0.5922).

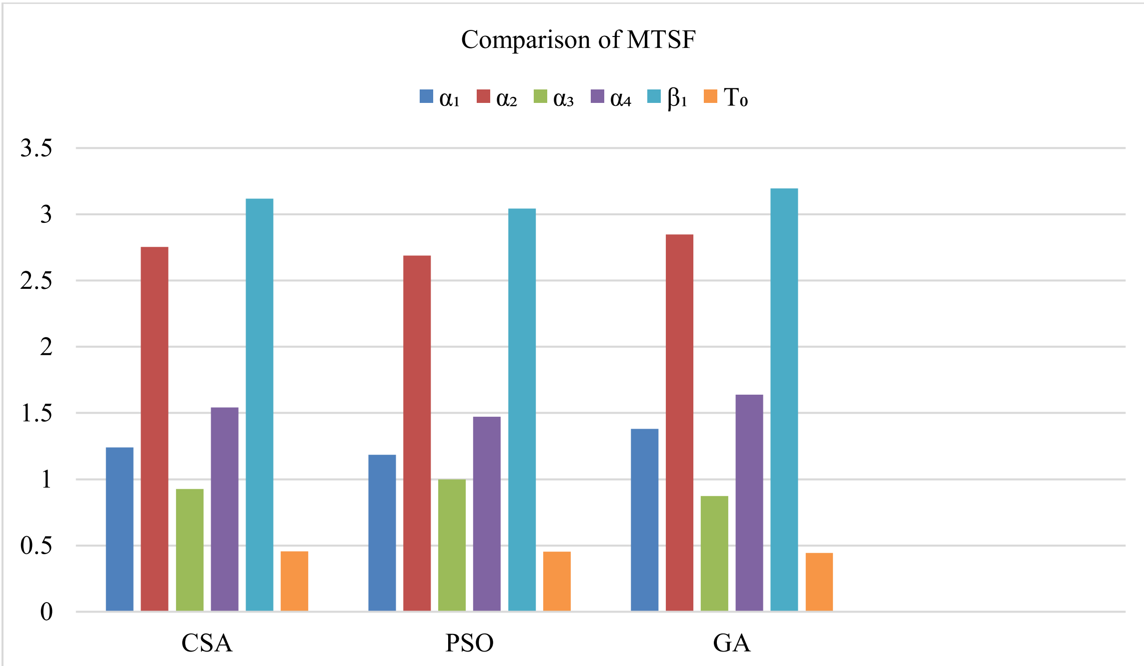

Fig. 2 Optimized Parameters and MTSF Value (To)

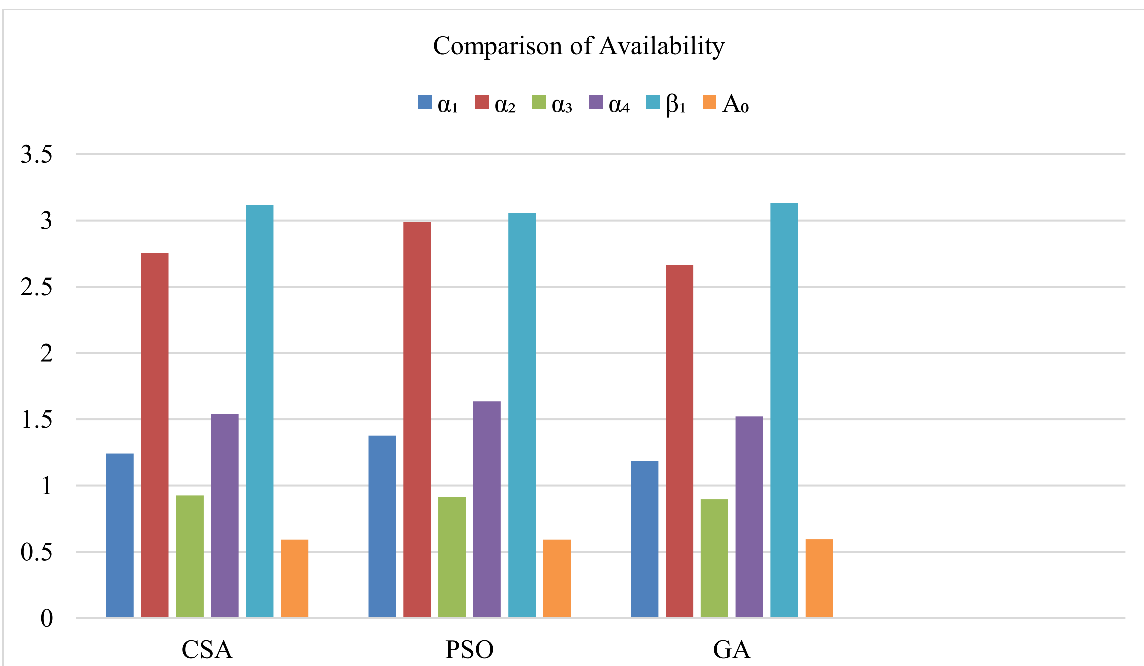

Fig. 3 Optimized Parameters and Availability Value (A_0)

6. Conclusion

The comparative analysis suggests that different algorithms demonstrate different results for varying conditions. In MTSF optimisation, the CSA is best suited, whereas the results of the steady state optimisation conclude GA as the effective technique. Furthermore, it also highlights the benefits of using the RTGP technique to optimize system availability, along with stochastic modeling and Metaheuristic optimization algorithms. Future work may focus on extending the approach to multi-component systems and real-time behavior. The methodology used in this work can be combined with another algorithm to enhance the overall reliability for the benefit of the industries.

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