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Original Article

On a New Class of Nano sgp -Separation Axioms: Regular and Normal Spaces

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Abstract *- This paper introduces nano sgp-regular and nano sgp-normal spaces defined via nano sgp-closed sets, and investigates their characterizations and relationships with nano sgp-regular and nano sgp-normal spaces. Preservation properties under subspaces and various classes of mappings are also established.*

Keywords *- Nano-topology, Nano sgp-closed sets, Nano* $\boldsymbol{sgp}$*-regular space, Nano* $\boldsymbol{sgp}$*-normal space.*

1. Introduction

The theory of nano topological spaces was introduced as a mathematical framework inspired by rough set theory to model approximation and uncertainty in finite universes [5]. Since its introduction, nano topology has been extensively investigated, leading to the development of various generalized notions of open and closed sets, continuity, and separation axioms [3,4]. These generalizations play a crucial role in extending classical topological concepts to settings where standard assumptions such as infinite universes or classical openness are not applicable. Separation axioms are fundamental in topology, as they provide tools to distinguish points and sets using neighborhoods. Classical notions such as regular and normal spaces have been generalized in several directions using different types of closed sets. In the context of nano topology, nano regular and nano normal spaces have been studied using nano closed sets and their variants [6]. Further generalizations involving nano generalized closed sets and nano weak forms of openness have been explored to obtain finer classifications of nano topological spaces [1,4].

More recently, research has focused on separation axioms defined via stronger generalized closed sets, such as nano *gp*-closed and nano sgp-closed sets, which offer a richer and more flexible structural framework [6]. Motivated by these developments, the present paper introduces and studies the notions of nano sgp-regular and nano sgp -normal spaces, which extend the classical concepts of regularity and normality in nano topology. The main objective of this work is to examine their fundamental properties, establish equivalent characterizations, and analyze their relationships with existing nano sgp-separation axioms.

2. Preliminaries

In this section, we review the fundamental definitions and results of nano topological spaces. Throughout this paper, U denotes a non-empty finite universe and R an equivalence relation on U. The pair (U, R) is called an approximation space.

Definition 2.1[5]: Let $X \subseteq U$. The lower approximation, upper approximation, and boundary region of $X$with respect to $R$are defined as $\underline{R}(X) = \{x \in U: [x]_R \subseteq X\}$, $\bar{R}(X) = \{x \in U: [x]_R \cap X \neq \emptyset\}$ and $B_R(X) = \bar{R}(X) \setminus \underline{R}(X)$, where $[x]_R$denotes the equivalence class of $x$under R.

Definition 2.2[5]: The collection $\tau_R(X) = \{\emptyset, U, \underline{R}(X), \bar{R}(X), B_R(X)\}$ forms a topology on U, called the nano topology induced by X. The triple $(U, \tau_R(X))$ is called a nano topological space. Elements of $\tau_R(X)$ $and\ they$ are called nano open sets, and their complements are called nano closed sets.

Definition 2.3[4]: Let $A \subseteq U$. The nano closure of A, denoted by $\mathrm{Cl}\,(A)$, is the intersection of all nano closed sets containing A. The nano interior of A, denoted by $\mathrm{Int}\,(A)$, is the union of all nano open sets contained in A.

Definition 2.4[5]: A subset $A$of a nano topological space $U$is said to be:
(i) nano semi-open if $A \subseteq NCl(NInt(A))$
(ii) nano pre-open if $A \subseteq N\text{Int}\,(N\text{Cl}\,(A))$,
The complements of these sets are called nano pre-closed and nano s-closed sets, respectively.

Several generalized closed sets have been introduced in nano topology to extend classical results. In particular, nano sgp-closed sets play a significant role in defining weakened separation axioms.

Definition 2.5: Let (U, τ_R)be a nano topological space and let $A \subseteq U$. The set $A$is said to be nano sgp-closed if for every nano-open set Gin $U$with $A \subseteq G$, the nano sgp-closure of $A$is contained in G. The complement of a nano sgp-closed set is called a nano sgp-open set.

Definition 2.6: For any subset $A \subseteq U$, the nano sgp-closure of A, denoted by pCl (A), is the intersection of all nano sgp-closed sets containing A.

Definition 2.7: Let $f: (U, \tau_R(X)) \to (V, \tau_R(Y))$be a function. Then $f$is said to be:
(i) nano continuous if the inverse image of every nano open set in $V$is nano open in U,
(ii) nano open if the image of every nano open set in $U$is nano open in V.
(iii) nano sgp-irresolute if the inverse image of every nano sgp-closed set in $V$is nano sgp-closed in U,
almost nano sgp-irresolute if the inverse image of every nano sgp-closed set is nano closed.

3. Nano *sgp*-Regular Spaces

Definition 3.1: Let (U, τ_R)be a nano topological space. The space $U$is said to be nano sgp-regular if for every point $x \in U$and for every nano sgp-closed set Fnot containing x, there exist nano-open sets $G$and Hsuch that $x \in G, F \subseteq H,$ and $G \cap H = \emptyset$.

Theorem 3.2: Every nano regular space is a nano sgp-regular space.

Proof: Let (U, τ_R) a nano-regular space, let $x \in U$, and let Fbe a nano sgp-closed set with $x \notin F$. Since the complement of $F$is nano sgp-open and contains x, and since nano-regularity guarantees the existence of disjoint nano-open sets separating a point from a nano-closed set, there exist nano-open sets $G$and Hsuch that $x \in G$, $F \subseteq H$, and $G \cap H = \emptyset$. Hence, $x$and Fcan be separated by disjoint nano-open sets, and therefore the space is nano sgp-regular.

Remark 3.3: The reverse implication of the above Theorem need not be true can be seen from the following example.

For example, Let $U = \{a, b, c\}$with nano topology $\tau_R = \{\emptyset, U, \{a\}, \{a, b\}\}$. The nano closed sets are $\emptyset, U, \{c\}$, and $\{b, c\}$. The space is nano sgp-regular, since for any point $x$and any nano sgp-closed set Fnot containing x, suitable nano open sets can be chosen to separate $x$and F. However, the space is not nano-regular. Indeed, for the point $b$and the nano closed set $F = \{c\}$, the only nano open sets containing $b$are $U$and $\{a, b\}$, and any nano open set containing Fmust be U; these are not disjoint. Hence, nano sgp-regularity does not imply nano regularity.

Theorem 3.4: Let (U, τ_R)be a nano topological space. The following conditions are equivalent:
(i) $U$is a nano sgp-regular space.
(ii) For each point $x \in U$and for each nano sgp-open set Acontaining x, there exists a nano open set Vsuch that$x \in V \subseteq \text{Cl}\,(V) \subseteq A$.

Proof: Assume that $U$is a nano sgp-regular space. Let $x \in U$and let Abe a nano sgp-open set containing x. Then the complement $F = U \setminus A$is a nano sgp-closed set not containing x. By nano sgp-regularity, there exist nano open sets $G$and Hsuch that $x \in G$, $F \subseteq H$, and $G \cap H = \emptyset$. Since $G \cap H = \emptyset$, it follows that $\text{Cl}\,(G) \subseteq U \setminus H$, and because $F \subseteq H$, we have $U \setminus H \subseteq A$. Hence, $\text{Cl}\,(G) \subseteq A$. Taking $V = G$, there exists a nano open set Vsuch that $x \in V \subseteq \text{Cl}\,(V) \subseteq A$, which proves condition (ii).
Conversely, assume that for each point $x \in U$and for each nano sgp-open set Acontaining x, there exists a nano open set Vsuch that $x \in V \subseteq \text{Cl}\,(V) \subseteq A$. Let Fbe a nano sgp-closed set with $x \notin F$. Then $A = U \setminus F$is a nano sgp-open set containing x. By the assumption, there exists a nano open set $V$with $x \in V \subseteq \text{Cl}\,(V) \subseteq A$. Let $G = V$and $H = U \setminus \text{Cl}\,(V)$. Then $G$and $H$are nano open sets such that $x \in G$, $F \subseteq H$, and $G \cap H = \emptyset$. Hence, the point $x$and the nano sgp-closed set Fcan be separated by disjoint nano open sets, and therefore $U$is a nano sgp-regular space.

Theorem 3.5: Every subspace of a nano sgp-regular space is also a nano sgp-regular space.

Proof: Let (U, τ_R) be a nano sgp-regular space and let $Y \subseteq U$. Consider the subspace (Y, τ_Y), where $\tau_Y = \{Y \cap G: G \in \tau_N\}$ is the nano subspace topology on Y. Let $x \in Y$ and let F be a nano sgp-closed set in Y such that $x \notin F$. Then there exists a nano sgp-closed set F_1 in U such that $F = Y \cap F_1$. Since $x \notin F_1$ and U is nano sgp-regular, there exist nano open sets G and H in U such that $x \in G$, $F_1 \subseteq H$, and $G \cap H = \emptyset$. Now $G \cap Y$ and $H \cap Y$ are nano open sets in the subspace Y, with $x \in G \cap Y$, $F \subseteq H \cap Y$, and $(G \cap Y) \cap (H \cap Y) = \emptyset$. Hence, the point x and the nano sgp-closed set F in Y can be separated by disjoint nano open sets in Y. Therefore, every subspace of a nano sgp-regular space is itself nano sgp-regular.

Theorem 3.6: Let (U, τ_R) be a nano topological space. The following conditions are equivalent:
(i) U is a nano sgp-regular space.
(ii) For each point $x \in U$ and for each nano sgp-open set V of U containing x, there exists a nano open set W in U such that $x \in W \subseteq \mathrm{Cl}\,(W) \subseteq V$.
(iii) For each point $x \in U$ and for each nano sgp-closed set A of U not containing x, there exists a nano open set W in U such that $\mathrm{Cl}\,(W) \cap A = \emptyset$.

Proof: (i) ⇒ (ii): Assume first that U is a nano sgp-regular space. Let $x \in U$ and let V be a nano sgp-open set containing x. Then the complement $A = U \setminus V$ is a nano sgp-closed set not containing x. By the definition of nano sgp-regularity, there exist nano open sets G and H such that $x \in G$, $A \subseteq H$, and $G \cap H = \emptyset$. Since $G \cap H = \emptyset$, it follows that $\mathrm{Cl}\,(G) \subseteq U \setminus H$. Because $A \subseteq H$, we have $U \setminus H \subseteq U \setminus A = V$, and hence $\mathrm{Cl}\,(G) \subseteq V$. Taking $W = G$, we obtain $x \in W \subseteq \mathrm{Cl}\,(W) \subseteq V$, and therefore the second condition holds.
(ii) ⇒ (iii): assume that for each point $x \in U$ and for each nano sgp-open set V containing x, there exists a nano open set W such that $x \in W \subseteq \mathrm{Cl}\,(W) \subseteq V$. Let A be any nano sgp-closed set of U not containing x. Then $V = U \setminus A$ is a nano sgp-open set containing x. By the assumption, there exists a nano open set W such that $x \in W \subseteq \mathrm{Cl}\,(W) \subseteq V$. Since $V = U \setminus A$, it follows that $\mathrm{Cl}\,(W) \cap A = \emptyset$, which proves the third condition.
(iii) ⇒ (i): assume that for each point $x \in U$ and for each nano sgp-closed set A of U not containing x, there exists a nano open set W such that $\mathrm{Cl}\,(W) \cap A = \emptyset$. Define $H = U \setminus \mathrm{Cl}\,(W)$. Then H is a nano open set containing A, and clearly $W \cap H = \emptyset$ with $x \in W$. Hence, the point x and the nano sgp-closed set A are separated by disjoint nano open sets, which shows that U is a nano sgp-regular space. Therefore, all three conditions are equivalent.

Theorem 3.7: Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a bijective function. If f is nano sgp-irresolute and nano open, and if $(U, \tau_R(X))$ is a nano sgp-regular space, then $(V, \tau_R(Y))$ is also a nano sgp-regular space.

Proof: Let $y \in V$ and let A be a nano sgp-closed set in V such that $y \notin A$. Since f is bijective, there exists a unique point $x \in U$ such that $f(x) = y$. Because f is nano sgp-irresolute, the inverse image $f^{-1}(A)$ is a nano sgp-closed set in U. Moreover, $x \notin f^{-1}(A)$. Since $(U, \tau_R(X))$ is nano sgp-regular, there exist nano open sets G and H in U such that $x \in G$, $f^{-1}(A) \subseteq H$, and $G \cap H = \emptyset$. As f is nano open, the images $f(G)$ and $f(H)$ are nano open sets in V. Clearly, $y = f(x) \in f(G)$ and $A \subseteq f(H)$. Furthermore, since $G \cap H = \emptyset$ and f is injective, it follows that $f(G) \cap f(H) = \emptyset$. Hence, the point y and the nano sgp-closed set A can be separated by disjoint nano open sets in V. Therefore, $(V, \tau_R(Y))$ is a nano sgp-regular space.

Theorem 3.8: Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be an injective function which is nano sgp-continuous and nano closed. If $(V, \tau_R(Y))$ is a nano sgp-regular space, then $(U, \tau_R(X))$ is also a nano sgp-regular space.

Proof: Let $x \in U$ and let F be a nano sgp-closed set in U such that $x \notin F$. Since f is nano sgp-continuous, the image $f(F)$ is a nano sgp-closed set in V. Moreover, since f is injective, we have $f(x) \notin f(F)$. As $(V, \tau_R(Y))$ is nano sgp-regular, there exist nano open sets G and H in V such that $f(x) \in G$, $f(F) \subseteq H$, and $G \cap H = \emptyset$. Because f is nano closed, the inverse images $f^{-1}(G)$ and $f^{-1}(H)$ are nano open sets in U. Clearly, $x \in f^{-1}(G)$, $F \subseteq f^{-1}(H)$, and $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence, the point x and the nano sgp-closed set F can be separated by disjoint nano open sets in U. Therefore, $(U, \tau_R(X))$ is a nano sgp-regular space.

Theorem 3.9: Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be an injective function which is nano sgp-irresolute and nano closed. If $(V, \tau_R(Y))$ is a nano sgp-regular space, then $(U, \tau_R(X))$ is also a nano sgp-regular space.

Proof: Let $x \in U$ and let F be a nano sgp-closed set in U with $x \notin F$. Since f is nano sgp-irresolute, $f(F)$ is nano sgp-closed in V, and injectivity gives $f(x) \notin f(F)$. As $(V, \tau_R(Y))$ is nano sgp-regular, there exist disjoint nano open sets G and H in V such that

$f(x) \in G$and $f(F) \subseteq H$. Because $f$is nano closed, $f^{-1}(G)$and $f^{-1}(H)$are nano open in U, with $x \in f^{-1}(G)$, $F \subseteq f^{-1}(H)$, and $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence, $U$is nano sgp-regular.

4. Nano *sgp*-Normal Spaces

Definition 4.1: Let (U, τ_R)be a nano topological space. The space $U$is said to be a nano sgp**-**normal space if for every pair of disjoint nano sgp**-**closed sets $F$and Gin U, there exist disjoint nano open sets $H$and Ksuch that $F \subseteq H, G \subseteq K$, and $H \cap K = \emptyset$**.**

Theorem 4.2: Every nano normal space is a nano sgp-normal space

Proof: Let (U, τ_R)be a nano normal space, and let $A$and Bbe two disjoint nano sgp-closed subsets of U. By nano normality, there exist disjoint nano-open sets $G$and Hsuch that $A \subseteq G$and $B \subseteq H$. Hence, the disjoint nano sgp-closed sets $A$and Bcan be separated by disjoint nano-open sets, and therefore the space is nano sgp-normal.

Remark 4.3: The reverse implication of the above Theorem need not be true can be seen from the following example.

For example, Let $U = \{a, b, c\}$with the nano topology $\tau_R = \{\emptyset, \{a\}, \{a, b\}, U\}$. The corresponding nano-closed sets are U, $\{b, c\}$, $\{c\}$, and $\emptyset$. In this space, any two disjoint nano sgp-closed sets can be separated by disjoint nano-open sets, and hence the space is nano sgp-normal. However, the disjoint nano-closed sets $\{a\}$and $\{b, c\}$cannot be separated by disjoint nano-open sets, since the only nano-open set containing $\{b, c\}$is U. Therefore, the space fails to be nano-normal.

Theorem 4.4: Let Ube a nano topological space. The following conditions are equivalent:
(i) $U$is a nano sgp-normal space.
(ii) For each nano sgp-closed set $A$and each nano sgp-open set $V$with $A \subseteq V$, there exists a nano open set Wsuch that $A \subseteq W \subseteq \mathrm{pCl}(W) \subseteq V$.
(iii) For each pair of distinct nano sgp-closed sets $A$and B, there exists a nano open set Wsuch that $A \subseteq W$and $\mathrm{pCl}(W) \cap B = \emptyset$.
(iv) For each pair of distinct nano sgp-closed sets $A$and B, there exist nano open sets $V$and Wsuch that $A \subseteq V$, $B \subseteq W$, and $\mathrm{pCl}(V) \cap \mathrm{pCl}(W) = \emptyset$.

Proof: (i) ⇒ (ii): Assume first that $U$is a nano sgp-normal space. Let Abe a nano sgp-closed set and Vbe a nano sgp-open set containing A. Then the complement $B = U \setminus V$is a nano sgp-closed set disjoint from A. By nano sgp-normality, there exist disjoint nano open sets $W$and Hsuch that $A \subseteq W$and $B \subseteq H$. Since $W \cap H = \emptyset$, it follows that $\mathrm{pCl}(W) \subseteq U \setminus H \subseteq V$.
(ii) ⇒ (iii): Next, assume that condition (ii) holds. Let $A$and Bbe disjoint nano sgp-closed sets. Then $V = U \setminus B$is a nano sgp-open set containing A, and by (ii) there exists a nano open set Wsuch that $A \subseteq W \subseteq \mathrm{pCl}(W) \subseteq V$. Since $\mathrm{pCl}(W) \subseteq V = U \setminus B$, we have $\mathrm{pCl}(W) \cap B = \emptyset$.
(iii) ⇒ (iv): Now assume that condition (iii) holds. Let $A$and Bbe disjoint nano sgp-closed sets. By (iii), there exists a nano open set Vsuch that $A \subseteq V$and $\mathrm{pCl}(V) \cap B = \emptyset$. Similarly, exchanging the roles of $A$and B, there exists a nano open set Wsuch that $B \subseteq W$and $\mathrm{pCl}(W) \cap A = \emptyset$. Then $\mathrm{pCl}(V) \cap \mathrm{pCl}(W) \subseteq (\mathrm{pCl}(V) \cap B) \cup (\mathrm{pCl}(W) \cap A) = \emptyset$,
(iv) ⇒ (i): Finally, assume condition (iv) holds. Let $A$and Bbe any pair of disjoint nano sgp-closed sets. Then by (iv) there exist nano open sets $V$and Wsuch that $A \subseteq V$, $B \subseteq W$, and $\mathrm{pCl}(V) \cap \mathrm{pCl}(W) = \emptyset$. Clearly, $V$and $W$are disjoint in the sense of their closures, so $A$and Bcan be separated by disjoint nano open sets, which shows that $U$is a nano sgp-normal space.

Theorem 4.5: Let Ube a nano topological space. Then $U$is a nano sgp-normal space if and only if for any two disjoint nano sgp-closed sets $A$and Bin U, there exist nano open sets $V$and Win Usuch that $A \subseteq V, B \subseteq W$, and $\mathrm{pCl}(V) \cap \mathrm{pCl}(W) = \emptyset$**.**

Proof: The proof follows from Theorem 4.4.

Theorem 4.6: Every nano sgp-closed subspace of a nano sgp-normal space is a nano sgp-normal space.

Proof: Let Ube a nano sgp-normal space and let $Y \subseteq U$be a nano sgp-closed subspace with the subspace topology $\tau_Y = \{Y \cap G: G \in \tau_N\}$. Let $A$and Bbe disjoint nano sgp-closed sets in Y. Then there exist nano sgp-closed sets $A_1$and B_1in Usuch that $A = A_1 \cap Y$and $B = B_1 \cap Y$. Since $U$is nano sgp-normal, there exist disjoint nano open sets $V$and Win Usuch that $A_1 \subseteq V$and $B_1 \subseteq W$. Then $V \cap Y$and $W \cap Y$are nano open sets in Y, $A \subseteq V \cap Y$, $B \subseteq W \cap Y$, and $(V \cap Y) \cap (W \cap Y) = \emptyset$. Hence, the disjoint nano sgp-closed sets $A$and Bin Ycan be separated by disjoint nano open sets, showing that $Y$is a nano sgp-normal space.

Theorem 4.7: Let $f:(U,\tau_R(X)) \to (V,\tau_R(Y))$be a bijective function. If $f$is nano open and nano sgp-irresolute, and if $(U,\tau_R(X))$is a nano sgp-normal space, then $(V,\tau_R(Y))$is also a nano sgp-normal space.

Proof: Let $A$and Bbe disjoint nano sgp-closed sets in V. Since $f$is bijective, there exist points in Ucorresponding to $A$and B. Because $f$is nano sgp-irresolute, the inverse images $f^{-1}(A)$and $f^{-1}(B)$are nano sgp-closed sets in U. Since $(U,\tau_R(X))$is nano sgp-normal, there exist disjoint nano open sets $G$and Hin Usuch that $f^{-1}(A) \subseteq G$and $f^{-1}(B) \subseteq H$. As $f$is nano open, the images $f(G)$and $f(H)$are nano open sets in V. Clearly, $A \subseteq f(G)$, $B \subseteq f(H)$, and $f(G) \cap f(H) = \emptyset$. Hence, the disjoint nano sgp-closed sets $A$and Bin Vcan be separated by disjoint nano open sets, which shows that $(V,\tau_R(Y))$is a nano sgp-normal space.

Theorem 4.8: Let $f:(U,\tau_R(X)) \to (V,\tau_R(Y))$be a function. If $f$is strongly nano sgp-open, continuous, and almost nano sgp-irresolute, and if $(U,\tau_R(X))$is a nano sgp-normal space, then $(V,\tau_R(Y))$is also a nano sgp-normal space.

Proof: Let $A$and Bbe disjoint nano sgp-closed sets in V. Since $f$is almost nano sgp-irresolute, the inverse images $f^{-1}(A)$and $f^{-1}(B)$are almost nano sgp-closed in U. Because $(U,\tau_R(X))$is nano sgp-normal, there exist disjoint nano open sets $G$and Hin Usuch that $f^{-1}(A) \subseteq G$and $f^{-1}(B) \subseteq H$. Since $f$is strongly nano sgp-open, the images $f(G)$and $f(H)$are nano open sets in V. Clearly, $A \subseteq f(G)$, $B \subseteq f(H)$, and $f(G) \cap f(H) = \emptyset$. Hence, the disjoint nano sgp-closed sets $A$and Bin Vcan be separated by disjoint nano open sets, showing that $(V,\tau_R(Y))$is a nano sgp-normal space.

5. Conclusion

This paper examined nano sgp-regular and nano sgp-normal spaces. Their characterizations and relationships with nano regular and nano normal spaces were established, showing that the sgp versions are weaker. Preservation properties under subspaces and suitable mappings were also discussed.

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