International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V71I4P103

Volume 71 Issue 4, 21-26, April 2025

Original Article

Forecasting Stock Prices of the Vietnamese Stock Market under the Assumption of Log-Normally Distributed Stock Prices

Doan Thanh Son

Faculty of Management Information, University of Finance and Business Administration, Hung Yen, Viet Nam.

Corresponding Author : ngay060779@gmail.com

Received: 26 February 2025 Revised: 29 March 2025 Accepted: 15 April 2025 Published: 29 April 2025

Abstract *- The aims of this paper present a stock price model and a method for forecasting stock prices under the assumption that stock prices follow the log-normally distribution law, simulating stock prices using geometric Brownian motion. The empirical analysis is conducted on stock codes in the VN30 index on the Vietnamese stock market. The experimental results show that the forecasting method presented in the report can be used to forecast stock prices in short-term investment periods of less than one month.*

Keywords *- Stock price model, Brownian motion, Forecasting, Log-normal.*

1. Introduction

The stock market is constantly fluctuating, directly affecting investment strategies. Stock investors always want to accurately forecast stock prices to build appropriate investment strategies to bring the most profit possible. In recent decades, there have been many studies in the world on models for forecasting stock prices in general and stock prices in particular [6, 8];... Each model has its own advantages and disadvantages, and no model can predict with absolute accuracy. Stock prices fluctuate every day and are affected by many factors, such as financial market information, economics, society, and information related to the issuing company... Therefore, forecasting stock prices is not easy. Researchers always strive to come up with forecasting models to improve accuracy and minimize costs. Different forecasting strategies have been studied by many scientists with very famous works such as optimization models [3, 10, 11, 15-17, 23, 24], machine learning models [4, 9, 12, 13, 19, 21, 23, 26], ... All of these research directions focus on the goal of forecasting stock fluctuations, finding the most beneficial way for investors. Normal distribution and lognormal distribution are widely used in financial mathematics. These two distributions are commonly used in the process of stock price analysis. The daily return rate of a stock can have a normal distribution. Then, the stock price movement can be described by the graph of the lognormal distribution. A very important random process used to build a stock price model is the Brownian motion process. Brownian motion is widely used in physics, economics, engineering... In finance, there have been many studies on the application of Brownian motion to forecast stock prices [1, 2, 22], ...Studies have shown that the geometric Brownian motion model is a useful model in stock price analysis and forecasting. This is a simple forecasting model but has quite high accuracy with short-term forecasting time.

The Vietnamese stock market has undergone nearly 25 years of formation and development. Although there have been remarkable developments in terms of capitalization and number of listed assets, the Vietnamese stock market is still considered a small, emerging market, very attractive but also potentially risky. Therefore, accurate forecasting of stock prices is very important for investors to minimize risks and develop appropriate investment strategies. In this report, the author presents a stock price model and uses geometric Brownian motion to simulate, analyze and forecast stock prices in the near future. The empirical analysis is conducted on stocks in the VN30 portfolio in the Vietnamese stock market.

2. Continuous-Time Stock Price Model

Let S_t be the stock price at time t, and dS_t be the amount of stock price change in an arbitrarily small time interval $[t, t + dt]$. We can assume that the relative change in stock price is $\frac{dS_t}{S_t}$, which is proportional to the length of time dt with a certain proportionality factor μ, that means,

$\frac{dS_t}{S_t} \approx \mu dt.$

In addition, there are random factors in the market that affect stock prices. These random factors create a type of random noise, represented by the random derivative dB_t of a Brownian motion B_t with a certain proportionality factor σ. Therefore, it can be assumed that the stock price satisfies the following stochastic differential equation:

$$dS_t = \mu S_t\, dt + \sigma S_t\, dB_t. \tag{1}$$

Equation (1) shows that, the random motion will be larger if σ is larger. Therefore, σ is also called the volatility of the stock price.

2.1. Theorem ([7, section 3])

The solution of the stochastic differential Equation (1) is a stochastic process,

$$S_t = S_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t}. \tag{2}$$

Thus, the stock price satisfies Equation (1) as determined by (2). The price process S_t is a geometric Brownian motion. The expectation of S_t is [1]:

$$E(S_t) = S_0 e^{\mu t}, \tag{3}$$

Where S_0 is the initial price value of the stock.

Equation (2) can be rewritten as:

$$\ln S_t = \ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma B_t. \tag{4}$$

The random variable $\ln S_t$ has a normal distribution with expectation $\ln S_0 + (\mu - \frac{\sigma^2}{2})t$ and variance $\sigma^2 t$.

3. Estimating the Parameters of the Model

We can estimate the parameters μ and σ of the geometric Brownian motion based on the historical price data of the stocks. Suppose S_i is the closing price on the i-th trading day ($i = 0,1,2,\dots,n$), Δt is the length of the time interval between two consecutive transactions in a year. Especially if we call N the number of days that the stock market is open in a year, then,

$$\Delta t = \frac{1}{N}.$$

From the data on the closing prices of the stocks, we calculate the daily return rate series $\{R_i\}$ for each corporate security according to the formula,

$$R_i = \ln\frac{S_i}{S_{i-1}}\ , i = 1,2,\dots,n.$$

Let

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i.$$

The estimates of the displacement μ and volatility σ of the stock price can be written by [20],

$$\mu = \frac{\bar{R}}{\Delta t} + \frac{\sigma^2}{2},$$

$$\sigma = \frac{1}{\sqrt{\Delta t}} . \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2}. \tag{5}$$

Since $\ln S_t$ follows the normal distribution with the expectation $\ln S_0 + (\mu - \frac{\sigma^2}{2})t$ and variance $\sigma^2 t$, with 95% confidence, the confidence interval of the value of $\ln S_t$ is,

$$\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t - 1.96\sigma\sqrt{t} \leq \ln S_t \leq \ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t + 1.96\sigma\sqrt{t}. \quad (6)$$

From Equation (6), we have the confidence interval of the stock price following the lognormal distribution with 95% confidence is,

$$e^{\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t - 1.96\sigma\sqrt{t}} \leq S_t \leq e^{\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t + 1.96\sigma\sqrt{t}}. \quad (7)$$

4. Empirical Analysis

The study uses daily closing price data of stocks in the VN30 index in the Vietnamese stock market. Closing price data is collected from January 3, 2023 to March 29, 2024. According to the presented stock price model, the daily return rate series of stocks must follow a normal distribution. The author randomly selected 20 stock codes: ACB, BCM, BID, BVH, CTG, FPT, GAS, GVR, HDB, HPG, MBB, PLX, SAB, SSB, SSI, TCB, TPB, VCB, VHM, VIC.

Using the closing price data of these stocks from January 3, 2023, to December 29, 2023, including 249 values for each stock code, calculate the daily return series for each stock code and use density charts and QQ charts to test the normal distribution assumption for the daily return series. From the analysis of the charts, it is possible to accept 5 stocks with daily return series following the normal distribution, which are stocks BID, BVH, GAS, TCB, and VCB. The data processing in this report is performed using R and Matlab software.

Table 1. Company stocks that satisfy the model's assumptions

No.	Stock code	Company name
1	BID	Vietnam Joint Stock Commercial Bank for Investment and Development
2	BVH	Bao Viet Group
3	GAS	Vietnam Gas Corporation - Joint Stock Company
4	TCB	Vietnam Technological and Commercial Joint Stock Bank
5	VCB	Vietnam Joint Stock Commercial Bank for Foreign Trade

The number of days the stock market is open in 2023 is 249 days, and so,

$$\Delta t = \frac{1}{249}.$$

From the daily return series in 2023, find the estimates of the stock price movement μ and volatility σ using formula (5). The results are given in Table 2. The author will use these estimates to forecast the stock price in the first three months of 2024.

Table 2. Stock price movement and volatility

Stock code	Movement μ	Volatility σ
BID	0.0904	0.2764
BVH	−0.1722	0.1854
GAS	−0.2986	0.2551
TCB	0.1834	0.2673
VCB	0.1601	0.2918

To simulate the stock price in the first quarter of 2024, the author uses Matlab software to build an algorithm based on the equation:

$$F_t = F_{t-1} e^{(\mu - \frac{\sigma^2}{2})\Delta t + \sigma\varepsilon\sqrt{\Delta t}},$$

Where F_t is the forecast price at time $t, F_0 = S_0$ is the current stock price, ε is a random number generated from the normal distribution $N(0,1)$. For each stock code, the author simulates 1000 random paths for the forecast price. The forecast price simulations are shown by the blue line, and the actual stock price chart is shown by the red line.

The corresponding images of the forecast price and the actual price can be observed in Figure 1.

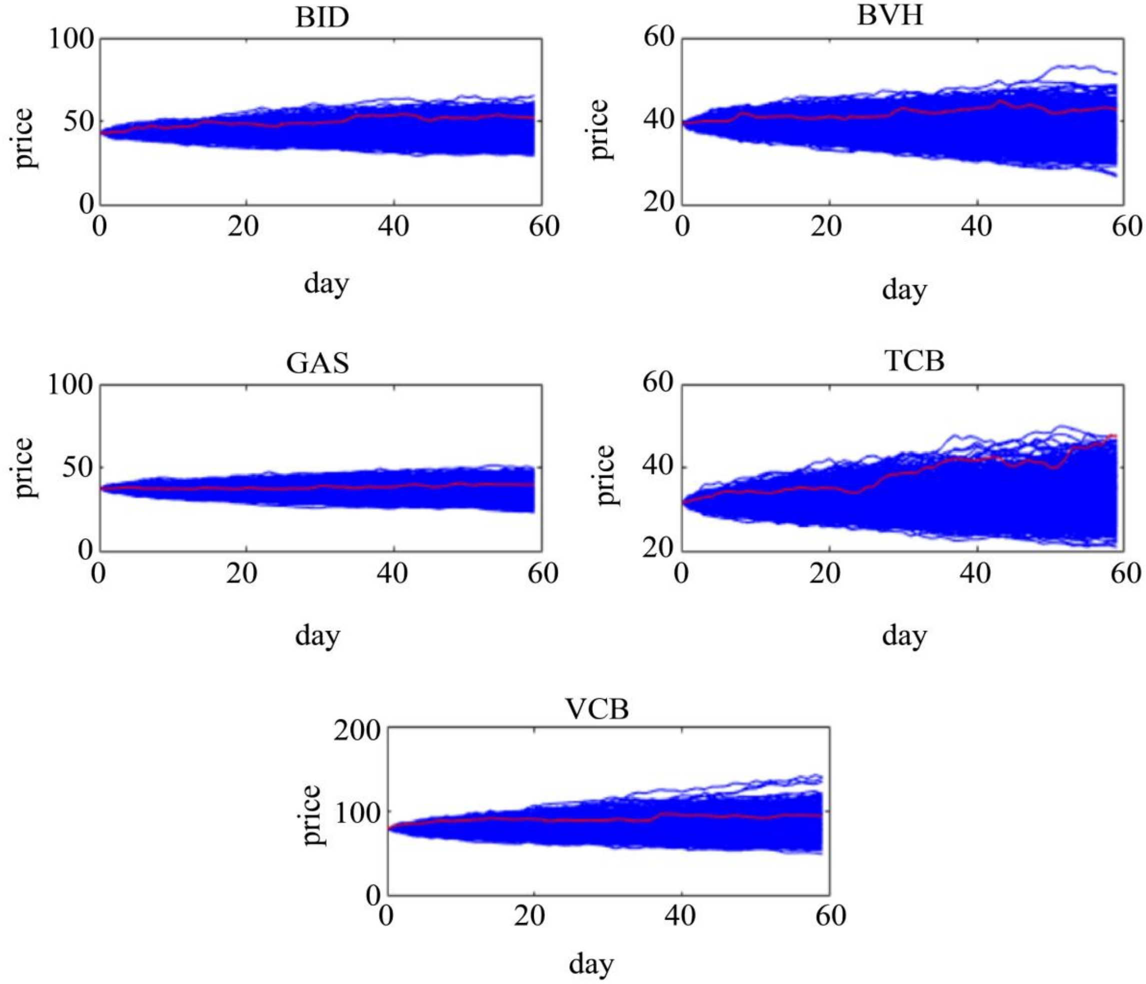

Fig. 1 Simulate forecasted and actual prices of stocks

From the results of the above simulation, we can see that the actual prices of the stocks are all within the simulated price range. To analyze more clearly the accuracy of the forecasting model, we will find the expected stock price and the confidence interval of the stock price after one day, one week, two weeks, three weeks, one month, two months and three months with 95% confidence. Using formulas (3) and (6), we can calculate these results and present them in the following tables:

Table 3. Expected price, confidence interval, actual price of stock after 1 day

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	43.42	(41.95; 44.92)	43
BVH	39.47	(38.57; 40.39)	39.6
GAS	75.41	(73.05; 77.82)	75.6
TCB	31.82	(30.78; 32.89)	32.1
VCB	80.35	(77.48; 83.3)	83.5

Table 4. Expected price, confidence interval, actual price of stock after 1 week

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	43.48	(40.24; 46.9)	45.6
BVH	39.36	(37.38; 41.43)	39.2
GAS	75.05	(69.88; 80.5)	76.6
TCB	31.92	(29.62; 34.35)	32.1
VCB	80.56	(74.24; 87.27)	83.2

Table 5. Expected price, confidence interval, actual price of stock after 2 weeks

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	43.56	(39.02; 48.47)	43.55
BVH	39.23	(36.45; 42.16)	39.26
GAS	74.6	(67.42; 82.34)	75.46
TCB	32.03	(28.81; 35.52)	34.1
VCB	80.82	(71.96; 90.46)	83.45

Table 6. Expected price, confidence interval, actual price of stock after 3 weeks

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	43.64	(38.13; 49.71)	49.8
BVH	39.04	(35.73; 42.69)	41.6
GAS	74.16	(65.48; 83.66)	76.6
TCB	32.15	(28.22; 36.48)	34.1
VCB	81.08	(70.3; 93.03)	92.5

Table 7. Expected price, confidence interval, actual price of stock after 1 month

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	43.75	(37.13; 51.2)	47.7
BVH	38.91	(34.88; 43.27)	40.6
GAS	73.54	(63.22; 85.06)	75.4
TCB	32.32	(27.58; 37.63)	34.15
VCB	81.44	(68.47; 96.14)	88.5

Table 8. Expected price, confidence interval, actual price of stock after 2 months

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	44.0	(35.42; 54.03)	53
BVH	38.48	(33.1; 44.22)	42.6
GAS	72.15	(59.08; 87.25)	77.6
TCB	32.7	(26.51; 39.89)	42.1
VCB	82.28	(65.41; 102.18)	97.5

Table 9. Expected price, confidence interval, actual price of stock after 3 months

Stock Code	Expected Price	Confidence Interval	Actual Price
BID	44.34	(33.77; 57.17)	53
BVH	37.93	(31.66; 45.07)	42.6
GAS	70.36	(54.77; 89.02)	80.6
TCB	33.21	(25.53; 42.47)	47.1
VCB	83.39	(62.53; 109.0)	94.5

With the results found in the tables above, we can see that the actual prices of each stock are mostly within the 95% confidence intervals. The error between the forecast price and the actual price is very small when the forecast period is one day. However, the longer the forecast period, the larger the error between the actual price and the expected price of the stocks. Therefore, the geometric Brownian motion model should only be applied to short-term forecast periods, less than one month.

5. Conclusion

This paper presented the stock price model and the method of stock price forecasting using the geometric Brownian motion model, assuming that stock prices follow the logarithmic normal distribution law. From historical closing price data, analyze the daily return rate of stocks, test the series of returns following the normal distribution law, and find the shift and volatility of stock prices. From there, simulate the forecast price, find the expected price, and the confidence interval of stock prices in the next three months. The empirical analysis was performed on five stocks in the VN30 portfolio in the Vietnamese stock market. In general, the empirical results show that the expected forecast price of stocks is close to the actual price for an investment period of no more than one month. When using the geometric Brownian motion model to simulate stock prices, only the first simulated price is based on the actual price, while the subsequent simulated prices are based on the previous simulated price, so the accuracy of the forecast will decrease over time. However, the simulation results are still effective in short-term forecasting and the geometric Brownian motion model is still a useful tool in stock price forecasting analysis.

References

[1] Abdelmoula Dmouj, "*Stock Price Modelling: Theory and Practice*," Vrije Universiteit Faculty of Sciences Amsterdam, The Netherlands, pp. 1-40, 2006. [Google Scholar] [Publisher Link]

[2] Siti Nazifah Zainol Abidin, and Maheran Mohd Jaffar, "Forecasting Share Price of Small Size Companies in Bursa Malaysia Using Geometric Brownian Motion," *Journal International, Applied Mathematics & Information Sciences*, vol. 8, no. 1, pp. 107-112, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[3] Joshua Achiam et al., "Constrained Policy Optimization," *Proceedings of the 34th International Conference on Machine Learning*, pp. 22-31, 2017. [Google Scholar] [Publisher Link]

[4] Andrea Lonza, *Reinforcement Learning Algorithms with Python: Learn, Understand, and Develop Smart Algorithms for Addressing AI Challenges*, Packt Publishing, pp. 1-366, 2019. [Google Scholar] [Publisher Link]

[5] Imre Csiszár, and János Körner, *Information Theory: Coding Theorems for Discrete Memoryless Systems*, Academic Press, pp. 1-452, 1981. [Google Scholar] [Publisher Link]

[6] Divya Aggarwal, "Random Walk Model and Asymmetric Effect in Korean Composite Stock Price Index," *Afro-Asian Journal of Finance and Accounting*, vol. 8, no. 1, pp. 85-104, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[7] Dang Hung Thang, *Advanced Probability*, Hanoi National University Publishing House, 2013.

[8] G. Bormetti et al., "A Backward Monte Carlo Approach To Exotic Option Pricing," *European Journal of Applied Mathematics*, vol. 29, no. 1, pp. 146-187, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[9] Hongyang Yang et al., "Deep Reinforcement Learning for Automated Stock Trading: An Ensemble Strategy," *Proceedings of the First ACM International Conference on AI in Finance*, New York New York, pp. 1-8, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[10] Sham M. Kakade, "A Natural Policy Gradient," *Proceedings of the 15th International Conference on Neural Information Processing Systems: Natural and Synthetic*, Vancouver British Columbia Canada, pp. 1531-1538, 2001. [Google Scholar] [Publisher Link]

[11] Karl Cobbe et al., "Phasic Policy Gradient," *Arxiv*, pp. 1-17, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[12] Laura Graesser, and Wah Loon Keng, *Foundations of Deep Reinforcement Learning: Theoryand Practice in Python*, 1sted., Addison-Wesley Professional, pp. 1-416, 2019. [Google Scholar] [Publisher Link]

[13] Maxim Lapan, *Deep Reinforcement Learning Hands-On: Apply Modern RL Methodsto Practical Problems of Chatbots, Robotics, Discrete Optimization, Web Automation, and More*, 2nd ed., Packt Publishing Ltd, pp. 1-826, 2020. [Google Scholar] [Publisher Link]

[14] Nguyen Duy Tien, *Probability Models and Applications, Part I: Markov Chains and Applications*, Hanoi National University Publishing House, 2001. [Google Scholar]

[15] Qing Wang et al., "Divergence-Augmented Policy Optimization," *Advances in Neural Information Processing Systems*, vol. 32, pp. 1-12, 2019. [Google Scholar] [Publisher Link]

[16] John Schulman et al., "Trust Region Policy Optimization," *Arxiv*, pp. 1-16, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[17] John Schulman et al., "Proximal Policy Optimization Algorithms," *Arxiv*, pp. 1-12, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[18] S.O.N. Agwuegbo, A.P. Adewole, and A.N. Maduegbuna, "A Random Walk Model for Stock Market Prices," *Journal of Mathematics and Statistics*, vol. 6, no. 3, pp. 342-346, 2010. [Google Scholar] [Publisher Link]

[19] Richard S. Sutton, and Andrew G. Barto, *Reinforcement Learning: An Introduction*, 2nd ed., The MIT Press, pp. 1-552, 2018. [Google Scholar] [Publisher Link]

[20] Tran Hung Thao, *Introduction to Financial Mathematics*, Science and Technology Publishing House, 2009.

[21] Wenhang Bao, and Xiao-yang Liu, "Multi-Agent Deep Reinforcement Learning for Liquidation Strategy Analysis," *Arxiv*, pp. 1-9, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[22] W. Farida Agustini, Ika Restu Affianti, and Endah R.M. Putri, "Stock Price Prediction Using Geometric Brownian Motion," *Journal of Physics: Conference Series*, vol. 974, pp. 1-11, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[23] Xiao-Yang Liu et al., "FinRL: A Deep Reinforcement Learning Library for Automated Stock Trading in Quantitative Finance," *Arxiv*, pp. 1-12, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[24] Yuhui Wang et al., "Trust Region-Guided Proximal Policy Optimization," *Arxiv*, pp. 1-22, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[25] Yuhui Wang et al., "Truly Proximal Policy Optimization," *Arxiv*, pp. 1-29, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[26] Zhuoran Xiong et al., "Practical Deep Reinforcement Learning Approach for Stock Trading," *Arxiv*, pp. 1-7, 2018. [CrossRef] [Google Scholar] [Publisher Link]