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Research Article

A Study of Fuzzy Number and its Properties

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Abstract - *A fuzzy number is an important concept for handling vague, imprecise and ambiguous cases in daily life. We propose to study different types of fuzzy numbers with examples and some results on fuzzy numbers. In biological science, the growth/decay of bacteria is a very important factor in knowing the symptoms/diseases in humans and animals. The decagonal fuzzy number has proved important in studying bacteria, particularly the growth rate in bacteria, which consists of 10 points, so it is not easy to handle this situation by applying triangular or trapezoidal fuzzy numbers. Therefore, we plan to study decagonal fuzzy numbers in detail here.*

Keywords - *Decagonal, Interval, Weight, α- cut etc..*

1. Introduction

Most of the developments in mathematics are rooted in Western logic(two-valued logic) and Eastern logic. More algebraic ideas and theories have been established with specific (true as1/false as 0) results. There are various examples of groups, such as $(z,+),(Q,+),(Q,.),(R,+),(R,.),(C,+),(C,.),(Z_n,+n),(Z_n,X_n)$, Permutation group (Pn). The classical set theory defines the Dihedral group (Dn) based on Western logic. In development with Eastern logic, in 1965, Lotfi A. Zadeh introduced the Fuzzy set theory [8]. In continuation with concepts in Fuzzy set theory, Pashinathan [4] and Dr. S. Chandrasekaran [2] define Fuzzy number, Triangular fuzzy number, Trapezoidal fuzzy number, Pentagonal fuzzy number, Hexagonal fuzzy number, Decagonal fuzzy number etc in different notations. We propose to study in detail a family of decagonal fuzzy numbers and some results regarding decagonal fuzzy numbers. We shall denote the family of decagonal fuzzy numbers by the symbol $F_{A'_D}$.

2. Decagonal Fuzzy Number

A decagonal fuzzy number is defined as

$$A_D = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}), a_i \in R$$

and membership function of A_D is given below.

$$\mu_{A_D}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(\frac{x-a_3}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ 1, & a_5 \le x \le a_6 \\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(\frac{a_7-x}{a_7-a_6}\right), & a_6 \le x \le a_7 \\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{a_9-x}{a_9-a_8}\right), & a_8 \le x \le a_9 \\ \frac{1}{2}\left(\frac{a_{10}-x}{a_{10}-a_9}\right), & a_9 \le x \le a_{10} \\ 0, & x > a_{10} \end{cases}$$

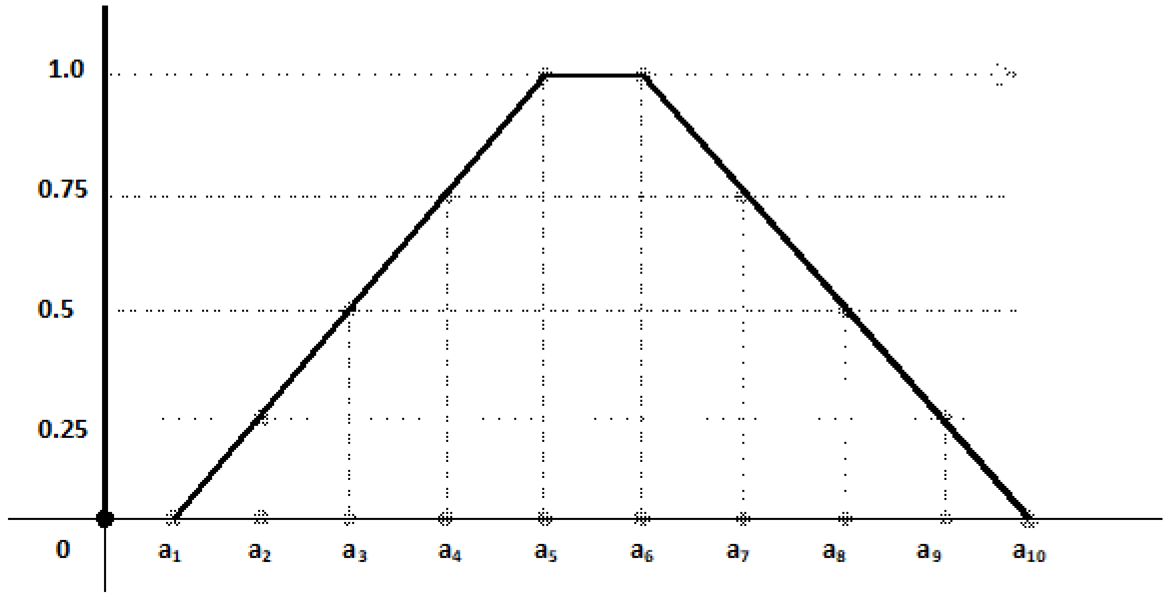

Fig. 1 (A_D)

3. Material and Methods

Let $F_{A'_D}$ be a family of decagonal fuzzy numbers.
Existence of Inverse -----E_{I_n}
Existence of Identity -----E_{I_d}
Operation is associative----$\oplus_a$

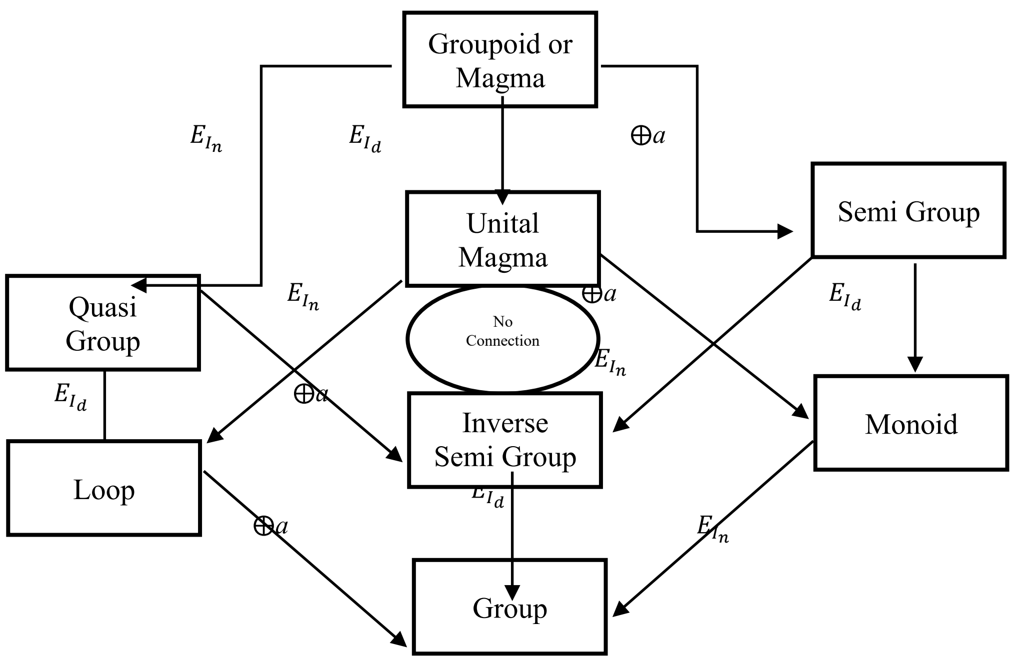

Fig. 2

From the above figure, it is clear that there are multiple approach paths between groupoid and group. In this paper, we select the path such as

$Groupoid \rightarrow$ $Semi\ Group \rightarrow$ $Monoid \rightarrow$ $G\ roup$
(G_1) (G_2) (G_3) (G_4)

For this, first of all, we define addition as the binary operation on $F_{A'_D} = \{A'_D, A''_D, \ldots\}$ that is $(F_{A'_D}, +)$ considered as an algebraic structure satisfying the following properties.

G_1: Addition of two decagonal fuzzy numbers

Let $A'_D = (a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7, a'_8, a'_9, a'_{10})$

and $A''_D = (a''_1, a''_2, a''_3, a''_4, a''_5, a''_6, a''_7, a''_8, a''_9, a''_{10})$

be any two decagonal fuzzy numbers.

The addition of A'_D and A''_D provided by the method given by Dr. S.chandrasekaran [2] such as

$A'_D + A''_D = (a'_1 + a''_1, a'_2 + a''_2, a'_3 + a''_3, a'_4 + a''_4, a'_5 + a''_5, a'_6 + a''_6, a'_7 + a''_7, a'_8 + a''_8, a'_9 + a''_9, a'_{10} + a''_{10})$

Which is again a decagonal fuzzy number. Consequently, we say that. $F_{A'_D}$ is closed w.r.t addition and thus $(F_{A'_D}, +)$ is a groupoid.

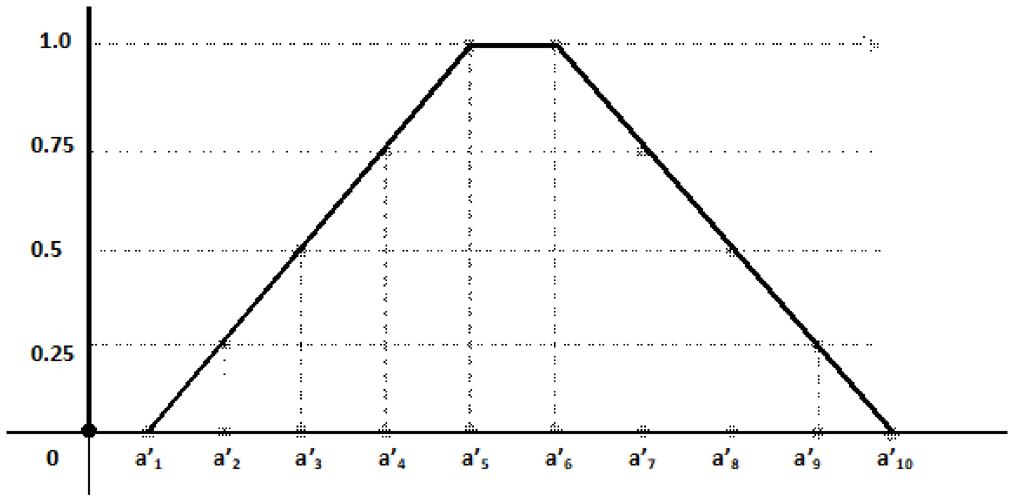

Fig. 3 (A'_D)

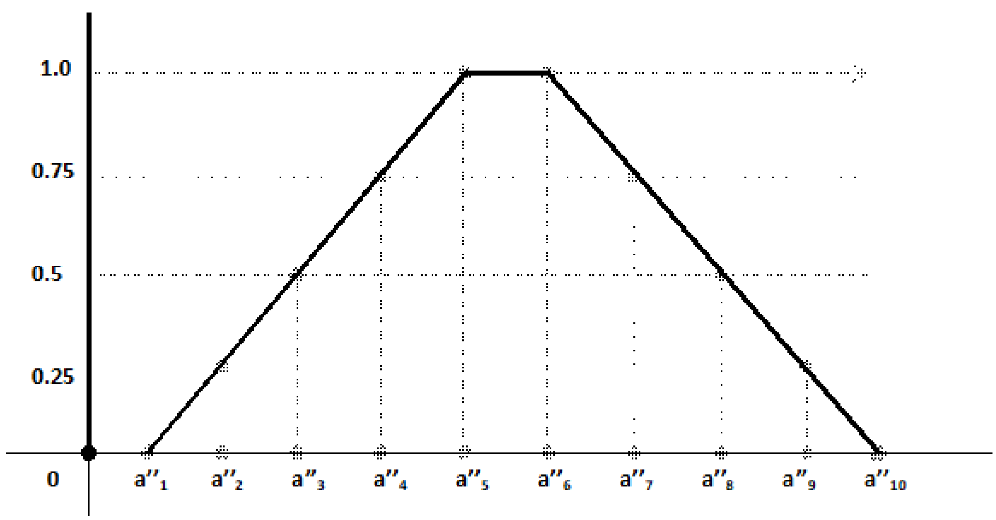

Fig. 4 (A''_D)

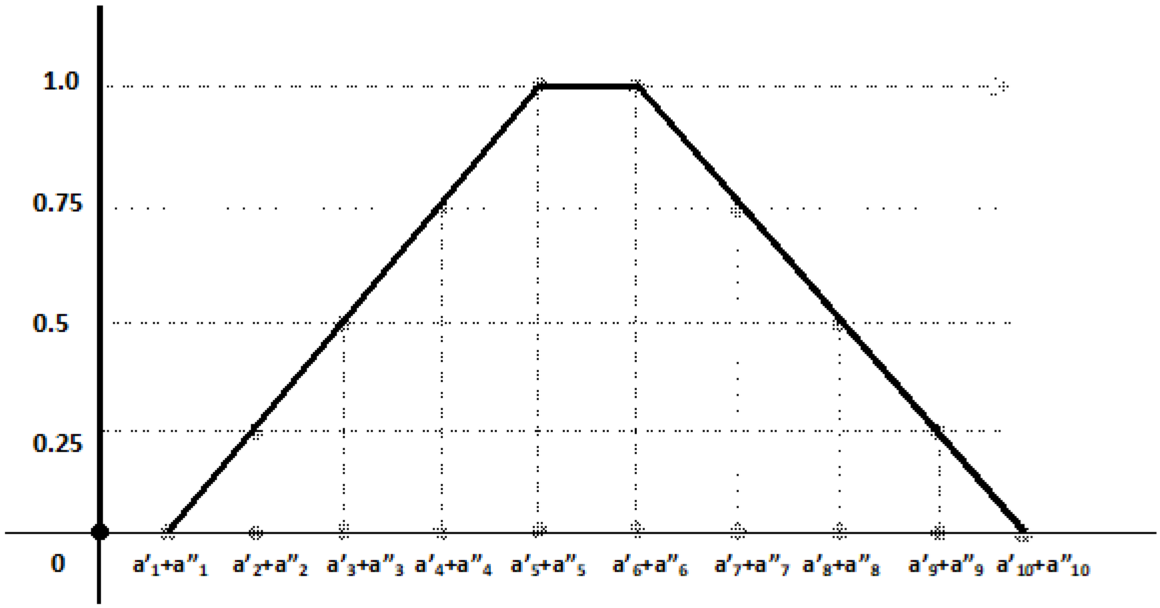

Fig. 5 ($A'_D + A''_D$)

$G_2(Associative\ law)$:

As we know, adding two decagonal fuzzy numbers is again a decagonal fuzzy number, and it is free from grouping subject to the order of bracketing, giving the same results that establish the associative property of decagonal fuzzy numbers.

$(A_D + B_D) + C_D = A_D + (B_D + C_D)$, Where $A_D, B_D, C_D \in F_{A'_D}$

Therefore, associative law holds in $F_{A'_D}$ w.r. to addition.

$G_3(Existence\ of\ identity)$:

Let $\Theta'_D = (0,0,0,0,0,0,0,0,0,0)$ be a decagonal fuzzy number with each component *0* such that

$$A'_D + \Theta'_D = (a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7, a'_8, a'_9, a'_{10}) + (0,0,0,0,0,0,0,0,0,0)$$
$$= (a'_1 + 0, a'_2 + 0, a'_3 + 0, a'_4 + 0, a'_5 + 0, a'_6 + 0, a'_7 + 0, a'_8 + 0, a'_9 + 0, a'_{10} + 0)$$
$$= (a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7, a'_8, a'_9, a'_{10})$$

$\Rightarrow A'_D + \Theta'_D = A'_D.\ \forall A'_D \in F_{A'_D}$

Similarly

$$\Theta'_D + A'_D = A'_D, \forall A'_D \in F_{A'_D}$$

Consequently $F_{A'_D}$ has an additive identity element

G_4:

By Dr. S. Chandrasekaran [2], a symmetric image of a decagonal fuzzy number

$A'_D = (a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7, a'_8, a'_9, a'_{10})$ is calculated as

$$-A'_D = (-a'_{10}, -a'_9, -a'_8, -a'_7, -a'_6, -a'_5, -a'_4, -a'_3, -a'_2, -a'_1)$$

Which is also a decagonal fuzzy number. We observed that the symmetric image of any decagonal fuzzy number is the inverse of that fuzzy number.

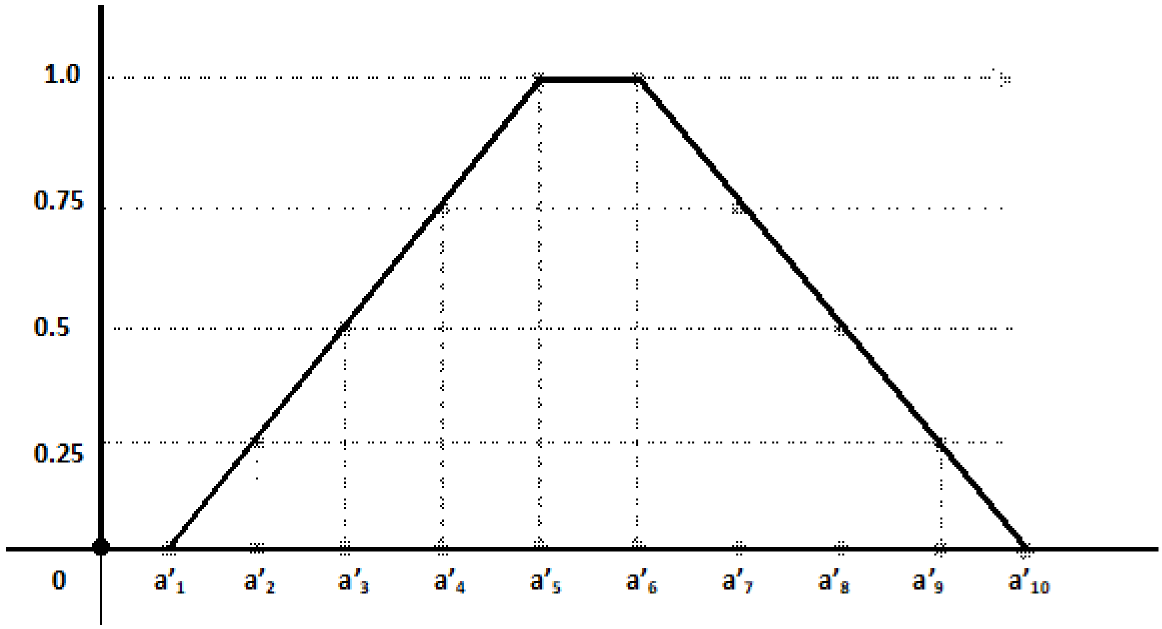

Fig. 6 (A'_D)

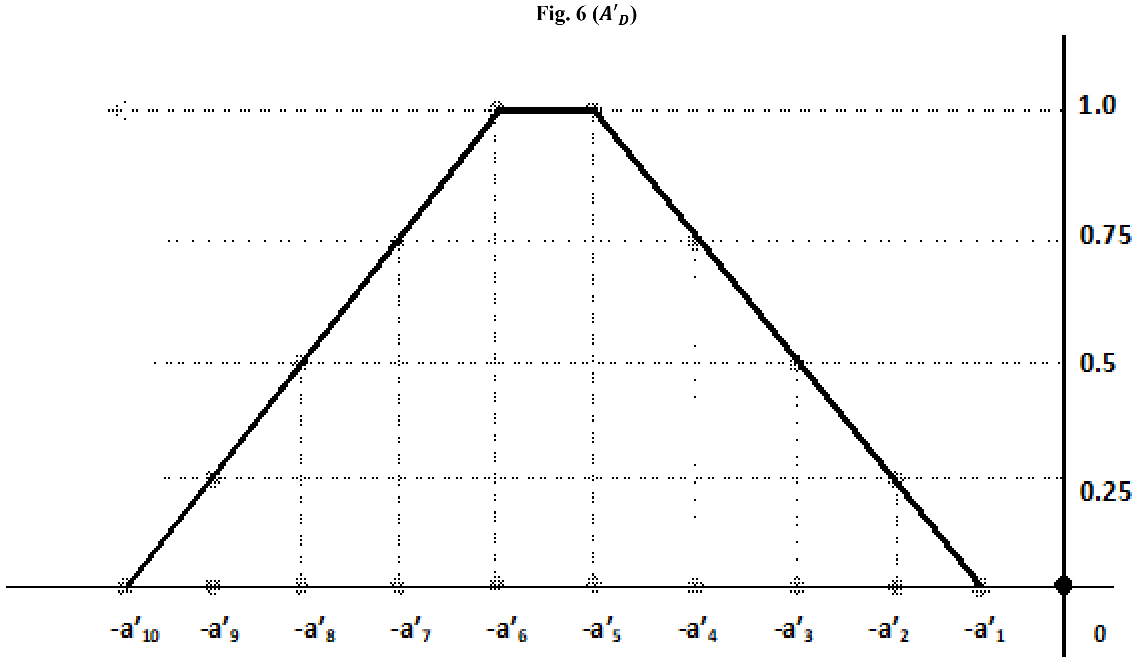

Fig. 7 ($-A'_D$)

4. Numerical Example

Let $A'_D = (2.3, 2.5, 2.7, 2.9, 3.1, 3.3, 3.5, 3.7, 3.9, 4.1)$ with length 2 units

$B'_D = (5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4, 6.6, 6.8, 7.0)$ with length 2 units

And $C'_D = (6.1, 6.3, 6.5, 6.7, 6.9, 7.1, 7.3, 7.5, 7.7, 7.9)$ be three decagonal fuzzy numbers. These are seen from the following figures, and let $F_{A'_D}$ be the set of all decagonal fuzzy numbers

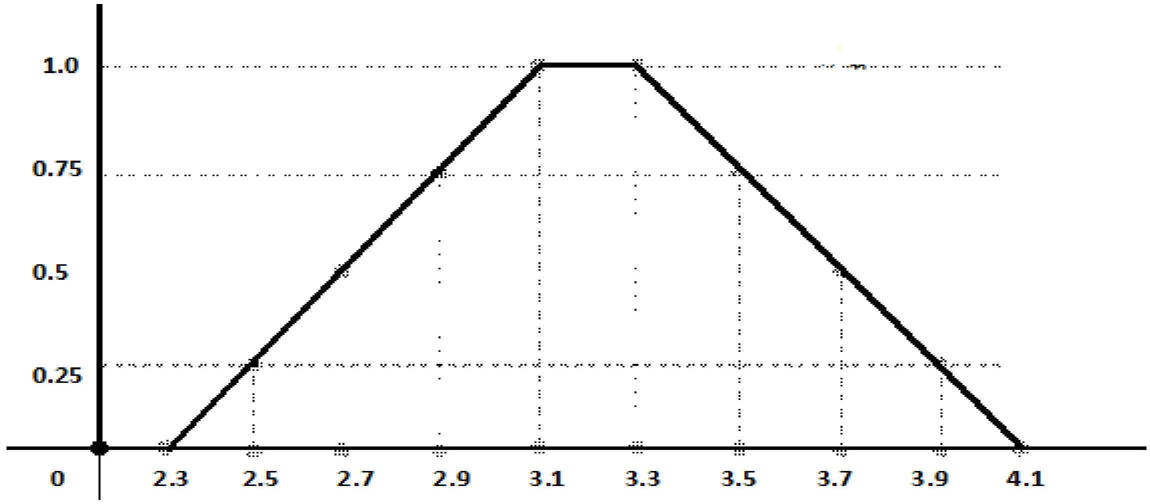

Fig. 8 (A'_D)

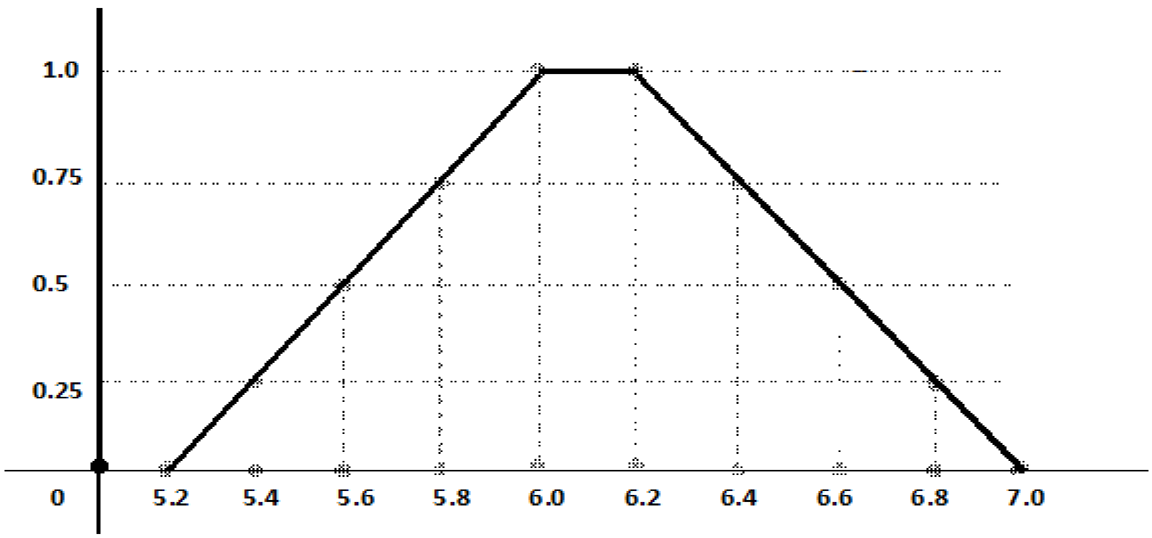

Fig. 9 (B'_D)

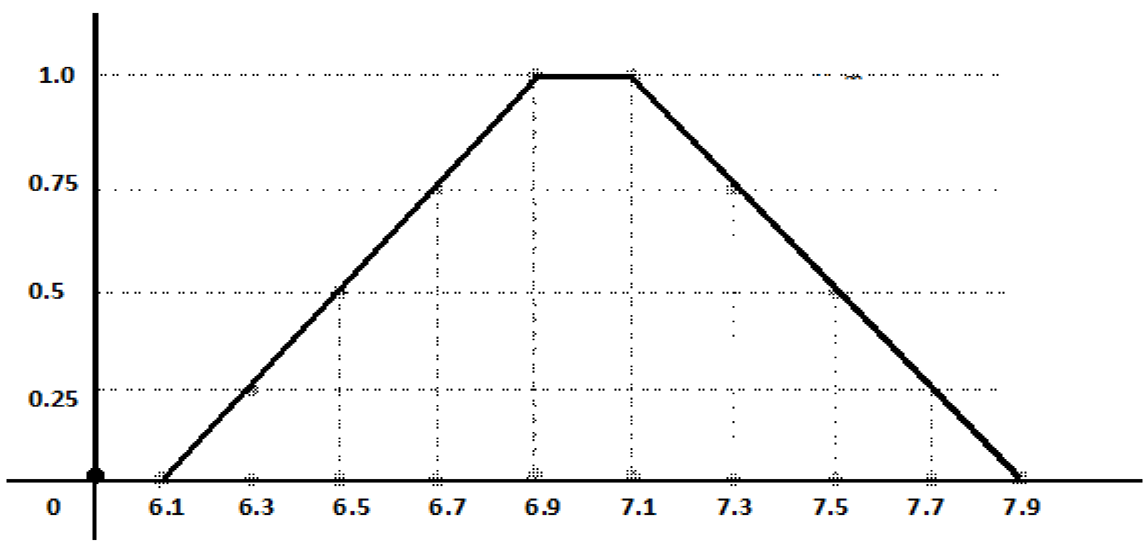

Fig. 10 (C'_D)

G_1: Addition of the above two decagonal fuzzy numbers A'_D *and* B'_D

$A'_D + B'_D = (2.3 + 5.2, 2.5 + 5.4, 2.7 + 5.6, 2.9 + 5.8, 3.1 + 6.0, 3.3 + 6.2, 3.5 + 6.4, 3.7 + 6.6, 3.9 + 6.8, 4.1 + 7.0)$

$A'_D + B'_D = (7.5, 7.9, 8.3, 8.7, 9.1, 9.5, 9.9, 10.3, 10.7, 11.1)$

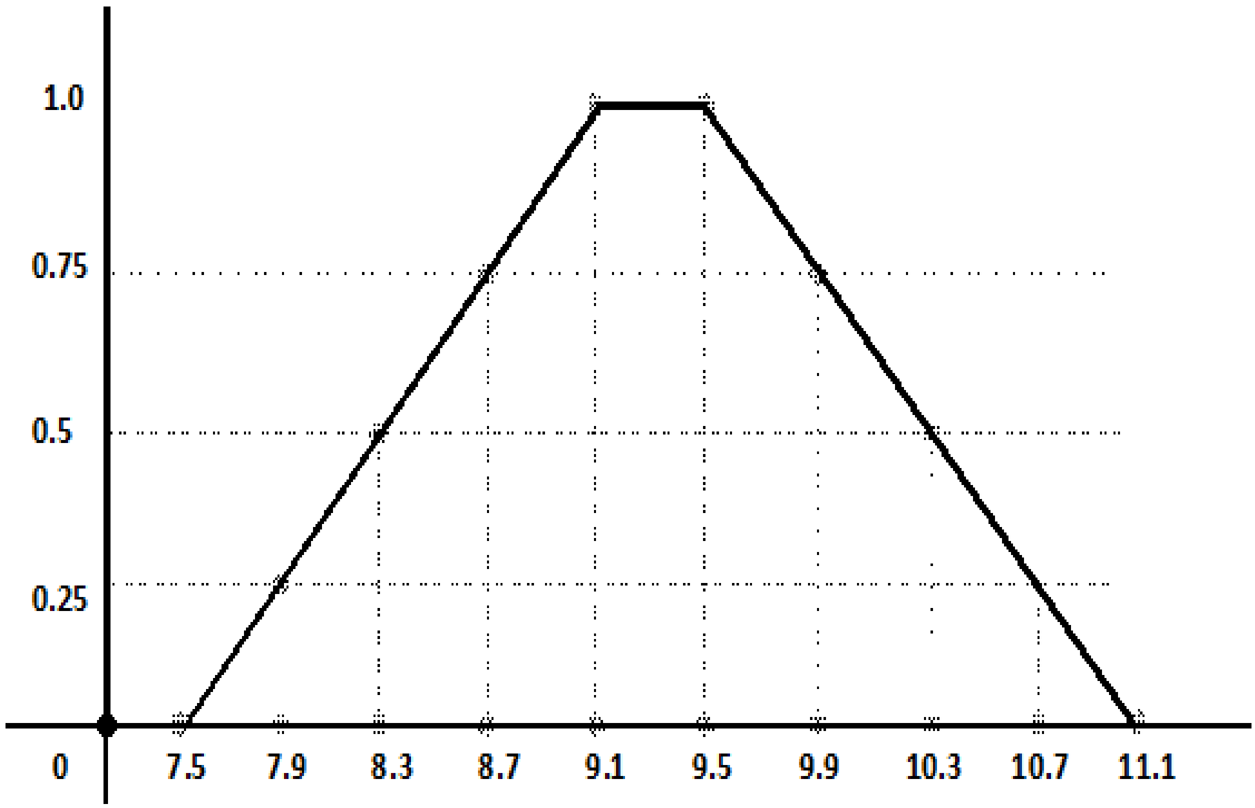

Fig. 11 (A'_D+B'_D)

G_2:
Associative law holds in $F_{A'_D}$ w.r.t addition, which is shown theoretically before.

G_3:
Since, $\theta'_D = (0,0,0,0,0,0,0,0,0,0)$ is a decagonal fuzzy number with 0 unit such that

$A'_D + \theta'_D = (2.3, 2.5, 2.7, 2.9, 3.1, 3.3, 3.5, 3.7, 3.9, 4.1) + (0,0,0,0,0,0,0,0,0,0)$
$= (2.3, 2.5, 2.7, 2.9, 3.1, 3.3, 3.5, 3.7, 3.9, 4.1)$
$= A'_D$

i.e. $A'_D + \theta'_D = A'_D = \theta'_D + A'_D$

similarly, $B'_D + \theta'_D = B'_D = \theta'_D + B'_D$

$C'_D + \theta'_D = C'_D = \theta'_D + C'_D$

and for another, all such decagonal fuzzy number θ'_D cannot affect after the operation.

Consequently, $F_{A'_D}$ has an additive identity element θ'_D

$G_4(Group)$:

We write theoretically before that the symmetric image of any decagonal fuzzy number is the inverse of that fuzzy number. One example of the considered decagonal fuzzy number A'_D written below:

$-A'_D = (-4.1, -3.9, -3.7, -3.5, -3.3, -3.1, -2.9, -2.7, -2.5, -2.3)$

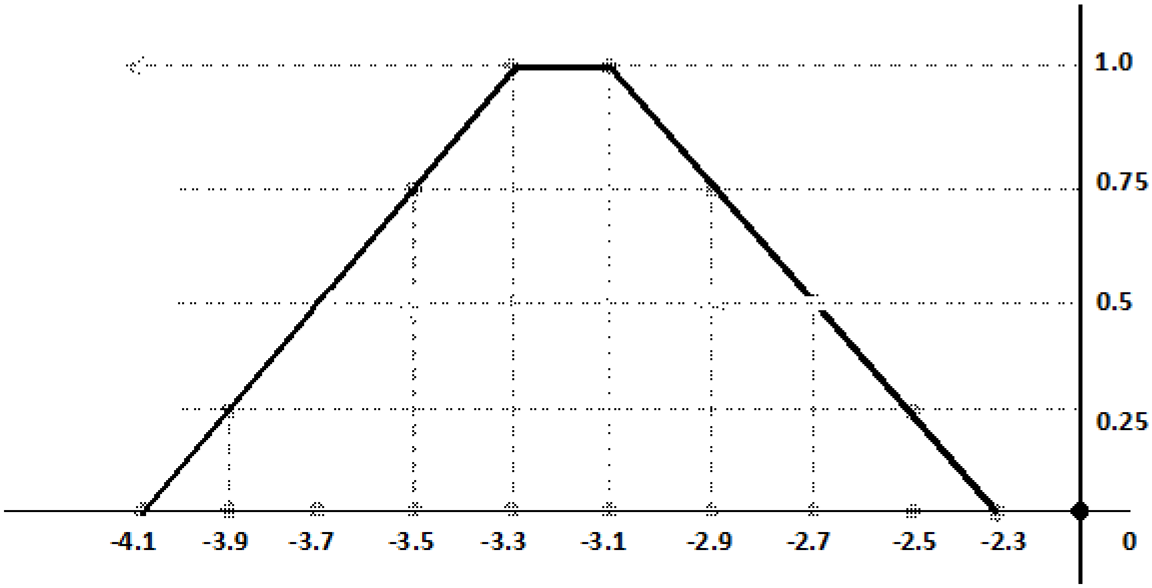

Fig. 12 ($-A'_D$)

5. Conclusion

Decagonal fuzzy numbers are studied in detail and a more explanatory form. It is established that the set of decagonal fuzzy numbers forms a group under the binary operation addition, which is defined as the ordinary sum of two decagonal fuzzy numbers component-wise.

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