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Original Article

Mamadu-Adomain Decomposition Method for Time-Fractional Wave Equation

Ebimene James Mamadu[1], Jude Chukwuyem Nwankwo[2], Omoregbe Charles Osahon[3], Vincent Odiaka[4], Onos Destiny Erhiakporeh[5], Ebikonbo-Owei Anthony Mamadu[1*,6], Henrietta Ify Ojarikre[1], Jonathan Tsetimi[1], Ignatius Nkonyeasua Njoseh[1], Ewoma Justice Aluya[1]

[1]*Department of Mathematics, Delta State University, Abraka, Nigeria.*
[2]*Department of Mathematics, University of Delta, Agbor, Delta State, Nigeria.*
[3]*Department of Mathematics, University of PortHarcourt, PortHarcourt, Rivers State, Nigeria.*
[4]*Department of Computer Science, Nottingham Trent University, 50 Shakespeare Street Nottingham, NG1 4FQ, UK.*
[5]*Department of Applied Computer Science and Artificial Intelligence, University of Bradford, UK.*
[6]*Department of Mathematics, Micheal and Cecilia Ibru University, Agbarha-Otor, Delta State, Nigeria.*

Corresponding Author : emamadu@delsu.edu.ng

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Abstract *- The present study explores the Mamadu-Adomain Decomposition Method (MADM) for solving fractional wave equations. The method is formulated by coupling the Mamadu Transform and the Adomain Decomposition Methods. The Mamadu Transforms handles the non-integer order derivatives, especially those defined in the Caputo Fractional derivative. The method is an analytical-numerical approach powered by power series decomposition techniques without requiring the standard traditional discretization methods to generate accurate and reliable solutions. Numerical evidences are presented in 2D and 3D plots to show convergence. The results suggest that the MADM offers significant advantages, including reduced complexity and adaptability to initial functions. Also, the method offers a balance between analytical insight and computational feasibility.*

Keywords *- Mamadu Transform, A domain decomposition method, Wave equation, Caputo fractional derivative.*

1. Introduction

Many complex engineering models and biological systems with memory and hereditary properties are best described using fractional differential equations. Unlike the classical differential equations, fractional derivatives offer a more adaptable mathematical framework for describing various anomalous diffusion and wave propagation phenomena [1], [2]. The fractional-order behavior of many real-world systems, including viscoelastic materials, control systems, financial markets, and fluid dynamics, is effectively resolved using analytical and numerical methods [3], [4].

The Adomain Decomposition Method (ADM), which decomposes the nonlinear operators using Adomain polynomials into infinite series, is one of the best methods for solving fractional differential equations. The method provides a step-by-step approach to finding close analytical solutions without simplifying or changing the equations to linear ones for both linear and nonlinear fractional problems. However, many types of fractional differential models, particularly those with memory-dependent features and unusual behaviors, are hard for the traditional ADM to manage. We developed the Mamadu-Adomain Decomposition Method (MADM) to address these challenges.

The Mamadu Transform has been recently introduced as a powerful tool for solving fractional and integral equations, particularly in the context of special function expansions and numerical interpolation [7]. Its combination with ADM leads to MADM, an improved decomposition method capable of handling complex fractional differential equations.

Numerous numerical schemes have been developed in recent studies to investigate the accuracy and stability of fractional differential equations. Huang et al. [8] investigated some numerical approximations that can be unconditionally stable using the implicit finite difference techniques. Tretyakov and Zhang [9] reduced stochastic fractional wave equations to solvable systems of ordinary differential equations by using the method of lines. A wavelength-based method was presented by Slattery et al [10] to efficiently manage singularities and steep gradients. These developments underline the necessity of reliable and effective computational strategies, such as MADM, which combines decomposition methods with fractional transformations to improve numerical stability and convergence.

Thus, the study introduces a new computational method for solving fractional differential equations, which advances the expanding subject of fractional calculus.

2. Preliminaries and Definitions

2.1. Riemann-Liouville Fractional Integral

For a function $u(t)$, the Riemann-Liouville fractional integral of order $\alpha > 0$ is defined as

$$I^{\alpha}u(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}u(s)ds, \qquad \alpha > 0,$$

Where $\Gamma(\cdot)$ is the Gamma function.

2.2. Caputo Fractional Derivative

For a function, $u(t)$, the Caputo fractional derivative with fractional order $\alpha \epsilon (n-1, n)$ is defined as

$$\mathrm{D}_t^{\alpha}u(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1}u^{(n)}(s)ds, \qquad n-1 < \alpha < n.$$

The Caputo fractional derivative is advantageous for solving different differential equations with memory effect and physical interpretation. It can incorporate initial conditions in problems as classical integer-order derivatives.

2.3. Mamadu Transform

The Mamadu Transform (M.T) is an integral transform defined as

$$M[u(t)] = \int_0^{\infty} (1+t^2)e^{-st}u(t)dt,$$

Where $s > 0$ is a transform parameter, e^{-st} ensures exponential decay for convergence, and $(1+t^2)$ is a weight function. The inverse Mamadu Transform is given by

$$u(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} M[u](s)(1+t^2)e^{-st}ds,$$

Where γ is a real constant ensuring convergence.

A few properties of Mamadu Transform are summarized below as follows:

- $M[1](s) = \frac{1}{s} + \frac{1}{s^3}$
- $M[t](s) = \frac{1}{s^2} + \frac{6}{s^4}$
- $M[e^{at}](s) = \frac{1}{s-a} + \frac{1}{(s-a)^2}$
- $M[D^{\alpha}u(t)](s) = s^{\alpha}M[u(t)](s) - \sum_{k=0}^{m-1} s^{\alpha-k-1}u^{(k)}(0), m = 1,2,3, \ldots$
- $M\left[E_{\alpha,\beta}(t^{\alpha})\right](s) = \frac{s^{\alpha-\beta}}{s^{\alpha-1}},$

Where $E_{\alpha,\beta}(u(t))$ is the Mittag-Leffler function of $u(t)$ with fractional orders α and β.

3. Main Result

Mamadu-Adomain Decomposition Method (MADM) can be formulated by merging the Mamadu Transform and the Adomain Decomposition Method (ADM). This enhances ADM's efficiency in resolving fractional differential equations, especially those defined in the Caputo fractional derivatives.

Now, consider a general fractional differential equation of the form.

$$D^{\alpha}u(x) + Nu(x) = f(x),\ 0 < \alpha < 1, \tag{3.1}$$

where, D^{α} Is the Caputo fractional derivative of order $\alpha, Nu(x)$ represents nonlinear terms and $f(x)$ is a known function.

The Mamadu Transform is applied to both sides of (3.1) to obtain,

$$M[D^{\alpha}u(x)] + M[Nu(x)] = M[f(x)]. \tag{3.2}$$

Using the Mamadu Transform property for Caputo derivative, (3.2) becomes,

$$s^{\alpha}M[u(s)] - \sum_{k=0}^{n-1} s^{\alpha-k-1}u^{(k)}(0) + M[Nu(x)] = M[f(x)]. \tag{3.3}$$

$$\Rightarrow M[u(x)] = \frac{M[f(x)] + \sum_{k=0}^{n-1} s^{\alpha-k-1}u^{(k)}(0) - M[Nu(x)]}{s^{\alpha}}. \tag{3.4}$$

Let the solution (assumed) be decomposed as

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{3.5}$$

For the nonlinear term, we use Adomain polynomials

$$Nu(x) = \sum_{n=0}^{\infty} A_n, \tag{3.6}$$

where

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n} N\left(\sum_{i=0}^{\infty} \lambda_i u^i\right)_{]\lambda=0}. \tag{3.6a}$$

Substitute (3.5) and (3.6) into (3.4), we have,

$$M\left[\sum_{n=0}^{\infty} u_n(x)\right] = \frac{M[f(x)] + \sum_{k=0}^{n-1} s^{\alpha-k-1}u^{(k)}(0) - \sum_{n=0}^{\infty} A_n}{s^{\alpha}} \tag{3.8}$$

This leads to the relation given as

$$M[u_0(x)] = \begin{cases} \dfrac{M[f(x)] + \sum_{k=0}^{n-1} s^{\alpha-k-1}u^{(k)}(0)}{s^{\alpha}}, \\ -\dfrac{M[A_n]}{s^{\alpha}},\ n \geq 0 \end{cases} \tag{3.9}$$

To obtain the term $u_n(x)$ for approximating $u(x)$, we apply the inverse Mamadu Transform. Let us recall the inverse Mamadu Transform given as

$$u_n(x,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} M[u_n(x,s)](1+t^2)e^{-st}ds. \tag{3.9a}$$

Thus, the zeroth-order term approximation is given as,

$$u_0(x,t) = M^{-1}\left[\frac{M[f(x,t) + \sum_{k=0}^{n-1} s^{\alpha-k-1}u^{(k)}(x,0)}{s^{\alpha}}\right].$$

$$\Longrightarrow u_0(x,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{M[f(x,t) + \sum_{k=0}^{n-1} s^{\alpha-k-1}u^{(k)}(x,0)}{s^{\alpha}}(1+t^2)e^{-st}ds, \tag{3.10}$$

and subsequent approximations can be computed using the recurrence relation,

$$u_{n+1}(x,t) = M^{-1}\left[-\frac{A_n(x,t)}{s^\alpha}\right], \qquad n \geq 0.$$

$$\Rightarrow u_{n+1}(x,t) = -\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{M[A_n(x,t)]}{s^\alpha}(1+t^2)e^{-st}ds, \tag{3.11}$$

which can explicitly be computed for a specific function $f(x,t)$.

Hence, the approximate solution is given as

$$u(x,t) \approx \sum_{n=0}^{\infty} u_n(x,t).$$

As an illustration, let's consider the time-fractional wave equation defined by the Caputo derivative of the form,

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t), \ 0 < \alpha \leq 1, \tag{3.12}$$

with initial conditions

$$u(x,0) = g(x), \qquad \frac{\partial u}{\partial t}(x,0) = h(x).$$

By (3.3) and (3.4), we have (3.12) as

$$M[u(x,t)] = \frac{s^{\alpha-1}g(x) + s^{\alpha-2}h(x) + M[f(x,t) + M\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right]}{s^\alpha}, \tag{3.13}$$

Using (3.5) and (3.6), we have

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t),$$

and

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \sum_{n=0}^{\infty} A_n(x,t),$$

Where $A_n(x,t)$ are adomain polynomials defined by (3.6a).

Using (3.8), we have the recurrence relation as

$$\begin{cases} M[u_0(x,t)] = \frac{s^{\alpha-1}g(x) + s^{\alpha-2}h(x) + M[f(x,t)]}{s^\alpha}, \\ M[u_{n+1}(x,t)] = -\frac{M[A_n(x,t)]}{s^\alpha}, \ n \geq 0 \end{cases} \tag{3.14}$$

Using the definition of inverse Mamadu Transform (3.9a) on (3.14), we have the zero. $u_0(x,t)$ and higher $u_{n+1}(x,t)$ terms recurrence relation given as,

$$u_0(x,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\left(\frac{s^{\alpha-1}g(x) + s^{\alpha_2}h(x) + M[f(x,t)]}{s^\alpha}\right)(1+t^2)e^{-st}ds. \tag{3.15}$$

Using the properties of inverse Mamadu transform

$$M^{-1}\left(\frac{s^{\alpha-1}}{s^{\alpha}}\right)=t^{\alpha-1}, \qquad M^{-1}\left(\frac{s^{\alpha-2}}{s^{\alpha}}\right)=t^{\alpha-2},$$

On (3.15), we have,

$$u_0(x,t)=(1+t^2)[g(x)t^{\alpha-1}+h(x)t^{\alpha-2}+I^{\alpha}f(x,t)]. \tag{3.16}$$

Similarly,

$$u_{n+1}(x,t)=-\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\left(\frac{A_n(x,t)}{s^{\alpha}}\right)(1+t^2)e^{-st}ds.$$

$$\Longrightarrow u_{n+1}(x,t)=-(1+t^2)I^{\alpha}A_n(x,t) \tag{3.17}$$

For small t, we have *n=0.*

$$A_0(x,t)=\frac{\partial^2 u_0(x,t)}{\partial x^2}=\frac{\partial^2}{\partial x^2}\left((1+t^2)[g(x)t^{\alpha-1}+h(x)t^{\alpha-2}+I^{\alpha}f(x,t)]\right)$$

$$=(1+t^2)\left(g''^{(x)t^{\alpha-1}}+h''^{(x)t^{\alpha-2}}+\tfrac{\partial^2}{\partial x^2}I^{\alpha}f(x,t)\right).$$

$$\Rightarrow u_1(x,t)=-(1+t^2)^2 I^{\alpha}\left[g''(x)I^{\alpha}t^{\alpha-1}+h(x)I^{\alpha}t^{\alpha-2}+I^{\alpha}\left(\tfrac{\partial^2}{\partial x^2}I^{\alpha}f(x,t)\right)\right].$$

Let compute $u_2(x,t)$ and then generate the nth order approximation.

Now,

$$A_1=\tfrac{\partial^2 u_1(x,t)}{\partial x^2}=-(1+t^2)^2 I^{\alpha}\left[g^{(4)}(x)t^{\alpha-1}+h^{(4)}(x)t^{\alpha-2}+\tfrac{\partial^4}{\partial x^4}I^{\alpha}f(x,t)\right].$$

Using (3.17) $n=1$, we have ,

$$u_2(x,t)=-(1+t^2)^3\left[g^{(4)}(x)I^{2\alpha}t^{\alpha-1}+h^{(4)}(x)I^{2\alpha}t^{\alpha-2}+I^{\alpha}\left(\tfrac{\partial^4}{\partial x^4}I^{\alpha}f(x,t)\right)\right].$$

Thus, to the nth order approximation, we have

$$u_n(x,t)=(-)^n(1+t^2)^{n+1}\left[g^{(2n)}(x)I^{n\alpha}t^{\alpha-1}+h^{(2n)}(x)I^{n\alpha}t^{\alpha-2}+I^{n\alpha}\left(\tfrac{\partial^{(2n)}}{\partial x^{(2n)}}I^{(n-1)\alpha}f(x,t)\right)\right]. \tag{3.18}$$

The approximate solution can be obtained by summing all terms from $u_0(x,t)$ to $u_n(x,t)$, that is,

$$u(x,t)\approx\textstyle\sum_{n=0}^{N}u_n(x,t). \tag{3.18a}$$

To establish the convergence of MADM for solving the fractional wave equation (3.12), we present the following relevant theorems and their proofs.

4. Convergence Analysis

To establish the convergence of MADM for solving the fractional wave equation, (3.12), we present the following relevant theorems and their proofs.

Theorem 4.1 (Absolute and Uniform Convergence)

Let $g(x)$ and $h(x)$ be analytic functions on $x \in [a,b]$, and α satisfy $0 < \alpha < 1$. Then, the series

$$U_N(x,t) = \sum_{n=0}^{N} u_n(x,t),$$

is uniformly convergent for all $t \in [0,1]$ for any finite $T > 0$.

Proof. Taking absolute bounds on $u_n(x,t)$ from the recurrence relation (3.18), we have,

$$|u_n(x,t)| = |(1+t^2)^{n+1}| \left| g^{(2n)}(x) I^{n\alpha} t^{\alpha-1} + h^{(2n)}(x) I^{n\alpha} t^{\alpha-2} + I^{n\alpha}\left(\frac{\partial^{(2n)}}{\partial x^{(2n)}} I^{(n-1)\alpha} f(x,t)\right)\right|.$$

Using growth bound on derivatives for analytic functions, we have,

$$\left|g^{(2n)}(x)\right| \le k_1^{(2n)}(2n)!, \left|h^{(2n)}(x)\right| \le k_2^{(2n)}(2n)!, \left|\frac{\partial^{(2n)}}{\partial x^{(2n)}} f(x,t)\right| \le k_3^{(2n)}(2n)!,$$

Where $k_1, k_2, k_3 > 0$ are constants.

Also, for large n , the asymptotic form of the Gamma function gives

$$\Gamma((n+1)\alpha) \approx \sqrt{2\pi(n+1)\alpha}\left(\frac{(n+1)\alpha}{e}\right)^{(n+1)\alpha}.$$

Thus, for small t, the term

$$\frac{t^{(n+1)\alpha-1}}{\Gamma((n+1)\alpha)},$$

Decays super-exponentially for large n. Hence, there exists $M > 0$ such that.

$$|u_n(x,t)| \le M(1+t^2)^{n+1} \frac{(2n)!}{\Gamma((n+1)\alpha)}.$$

Using Stirling's approximation,

$$(2n)! \approx \sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n},$$

Which grows super-exponentially, while $\Gamma((n+1)\alpha)$ grow sub-exponentially. Therefore,

$$(1+t^2)^{n+1} \frac{(2n)!}{\Gamma((n+1)\alpha)} \to 0 \; as \; n \to \infty,$$

ensuring absolute convergence.

To prove uniform convergence, we apply the Weierstrass M-test, where we must find a major sequence M_n such that

$$|u_n(x,t)| \le M_n,$$

where $\sum_{n=0}^{\infty} M_n$ is convergent.

Since,

$$|u_n(x,t)| \leq M(1+t^2)^{n+1}\frac{(2n)!}{\Gamma((n+1)\alpha)},$$

and

$$\sum_{n=0}^{\infty}\frac{(2n)!}{\Gamma((n+1)\alpha)}(1+t^2)^{n+1},$$

It is known to converge for $0<\alpha<1$; the Weierstrass M-test guarantees uniform convergence.

Theorem 4.2 (Error Estimate)
The truncation error after N terms satisfies

$$|R_N(x,t) = |u(x,t)-\sum_{n=0}^{N}u_n(x,t)| \leq K(1+t^2)^{n+1}\frac{(2N)!}{\Gamma((N+1)\alpha)},$$

Where $K>0$ is a constant.

Proof.

Let

$$R_N(x,t)=\sum_{n=N+1}^{\infty}u_n(x,t).$$

Since,

$$|u_n(x,t)| \leq K(1+t^2)^{n+1}\frac{(2n)!}{\Gamma((n+1)\alpha)},$$

We sum over n from $(N+1)$ to ∞, and using asymptotics, we have that.

$$(1+t^2)^{n+1}\frac{(2N)!}{\Gamma((N+2)\alpha)} \to 0 \ as\ N\to\infty.$$

Thus, the truncation error vanishes exponentially.

5. Stability Analysis
For the MADM series solution,

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t),$$

Stability means that for any perturbation $\in$, the difference between the perturbed and unperturbed solution remains bounded as $t\to\infty$.

We consider the following theorems.

Theorem 5.1

Let $u(x,t)$ Be the approximation using MADM for (3.12). Suppose $g(x)$ and $h(x)$ are in the Sobolev space $H^k(\Omega)$ for some integer K . Then, for sufficiently smooth initial conditions, the MADM approximate solution is satisfied.

$$||u(x,t)||_{L_2} \leq Ke^{-\lambda t}||u(x,0)||_{L_2},$$

For some positive constants $K,\lambda>0,$ ensuring exponential decay and stability in L_2-norms.

Proof.

Let the perturbed solution be defined as

$$\tilde{u}(x,t) = \sum_{n=0}^{\infty} \tilde{u}_n(x,t),$$

Where each term satisfies

$$\tilde{u}_n(x,t) = u_n(x,t) + \epsilon_n(x,t).$$

Now,

$$\epsilon(x,t) = \tilde{u}(x,t) - u(x,t) = \sum_{n=0}^{\infty} \epsilon_n(x,t).$$

From the relation (3.14), we have

$$M[\epsilon_n(x,t)] = \frac{M[A_{n-1}(x,t) + B_{n-1}(x,t)]}{s^{\alpha}},$$

Where $B_{n-1}(x,t)$ represents the error term. Applying the Inverse Mamadu Transform (3.17), we have

$$\epsilon_n(x,t) = -\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \left(\frac{B_{n-1}(x,t)}{s^{\alpha}}\right)(1+t^2)e^{-st}ds.$$

Since the inverse Mamadu transform operator is bounded, we have,

$$||\epsilon_n(x,t)||_{L_2} \le Ke^{-\lambda t}||\epsilon_0(x,t)||_{L_2}.$$

Summing over all n , we get,

$$||\epsilon(x,t)||_{L_2} \le Ke^{-\lambda t}||\epsilon(x,0)||_{L_2}.$$

Thus, the error decays exponentially, ensuring stability.

Theorem 5.2
For the MADM approximate solution to be stable, there exists a constant $Q > 0$ such that.

$$\sup_{x\epsilon\Omega, t>0} |u(x,t)| \le Q < \infty.$$

Proof.

For the MADM series,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t),$$

Each term is bounded by

$$|u_n(x,t)| \le K_n \frac{t^{(n+1)\alpha-1}}{\Gamma((n+1)\alpha)}.$$

Since $\Gamma(\cdot)$ grows faster than any polynomial, for large n,

$$\frac{t^{(n+1)\alpha-1}}{\Gamma((n+1)\alpha)} \to 0.$$

Thus, there exists a uniform bound.

$$|u(x,t)| \le \sum_{n=0}^{\infty} k_n < \infty,$$

Which implies boundedness and, consequently, stability.

Theorem 5.3

Let $u(x,t)$ and $v(x,t)$ There will be two solutions to the fractional wave equation (3.12) using MADM, corresponding to two different initial conditions. $g_1(x), h_1(x)$ and $g_2(x), h_2(x)$. Then, there exists a constant $K > 0$ such that.

$$\left|\left|u(x,t) - v(x,t)\right|\right|_{L_2} \le K\left|\left|u(x,0) - v(x,0)\right|\right|_{L_2}.$$

Proof. The difference $w(x,t) = u(x,t) - v(x,t)$ satisfies

$$M[w_n(x,t)] = \frac{-M[A_{n-1}(x,t) - B_{n-1}(x,t)]}{s^{\alpha}},$$

Taking the inverse Mamadu transform, we have,

$$||w_n(x,t)||_{L_2} \le K_n||w_0(x,t)||_{L_2}.$$

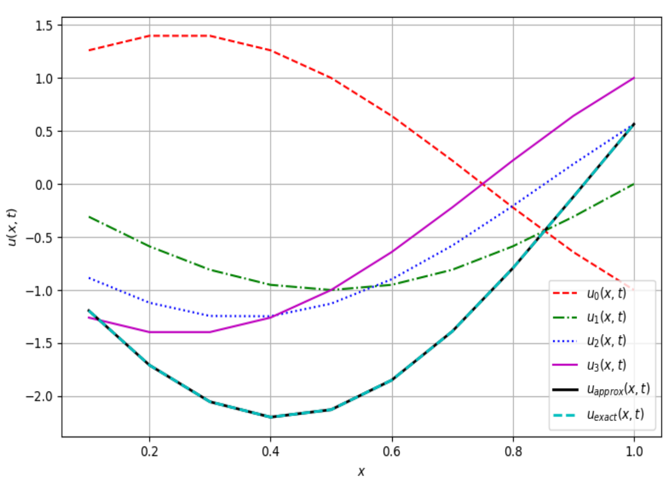
Summing over n,

$$||w(x,t)||_{L_2} \le K||w(x,0)||_{L_2}.$$

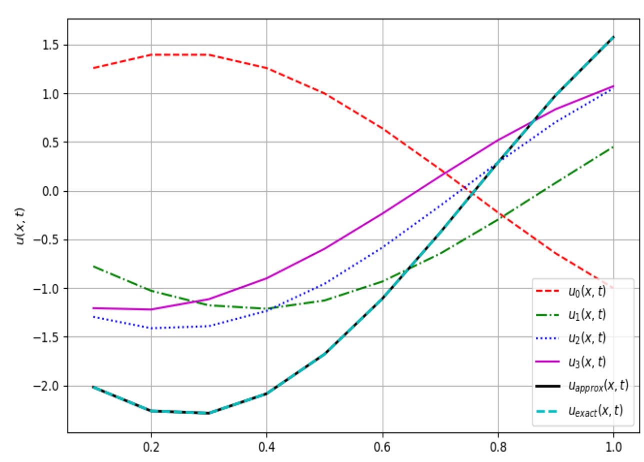
Thus, small changes in the initial conditions result in proportionally small changes in the solution, ensuring Lipschitz stability.

6. Numerical Illustrations

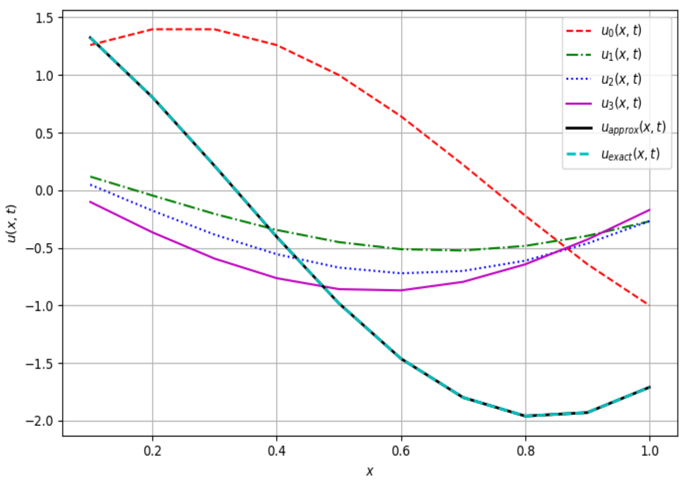
This section presents a numerical simulation of the MADM for the given fractional wave equation (3.12). We compute the first four terms. $u_0, u_1, u_2, u_3,$ and obtain the approximate solution using (3.18a). For the sake of clarity, we consider (3.12) for different initial functions, forcing terms, and parameters (α, t and $x \in [0.1, 1.0]$) with graphical simulations obtained using PYTHON script. The script computes the solution using the MADM and plots the numerical results with a comparison made with the analytic solution, as shown in the figures below.



(a) $\alpha = 0.5$

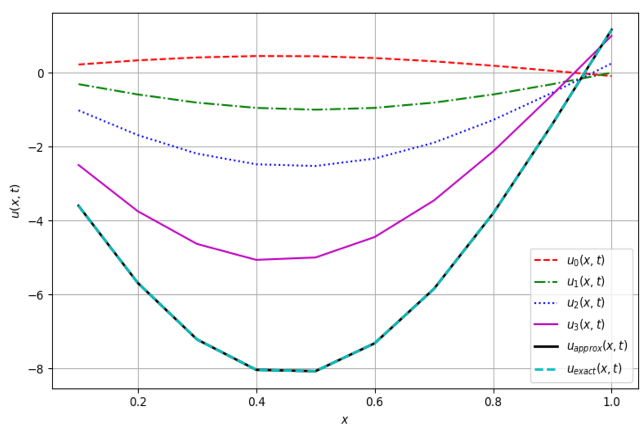


(b) $\alpha = 0.7$

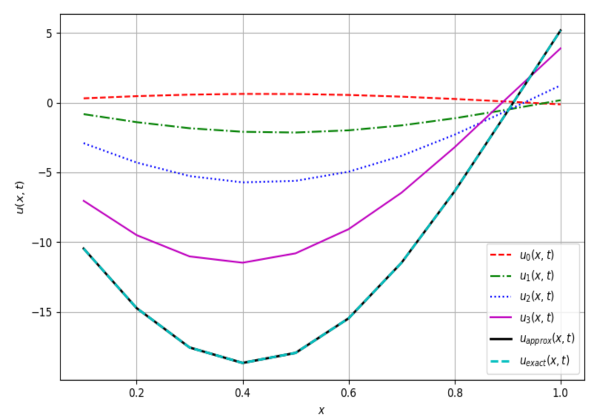


(c) $\alpha = 0.2$

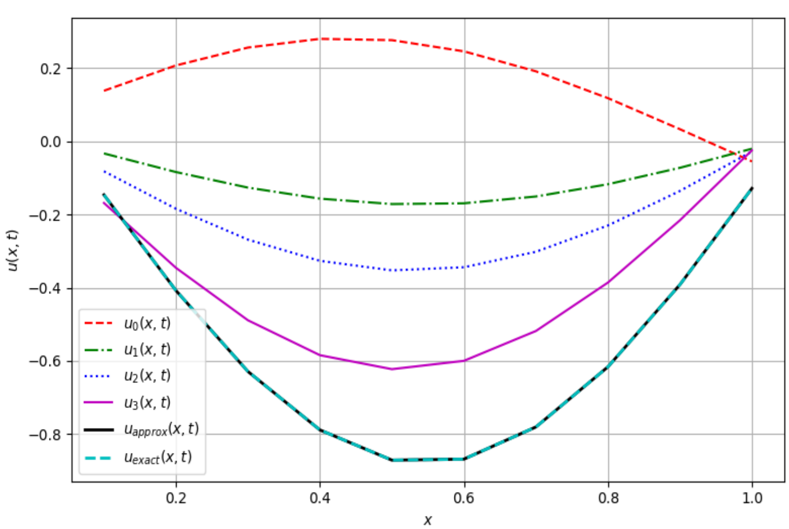
Fig. 1 Comparison between MADM and Exact solutions for $t = 1, f(x,t) = e^{-x}t^2, g(x) = \cos \pi x, h(x) = \sin \pi x$



(a) $\alpha = 0.5$

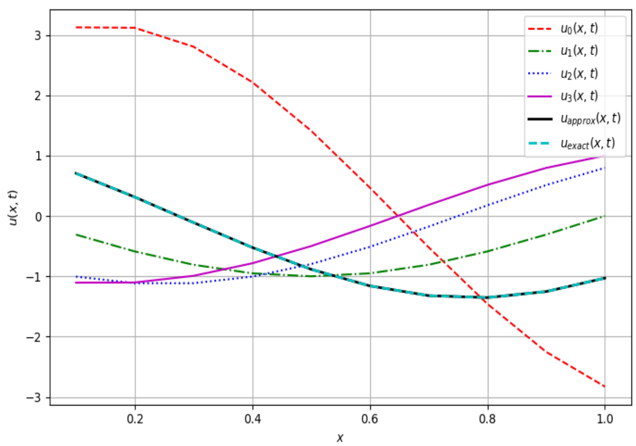


(b) $\alpha = 0.7$

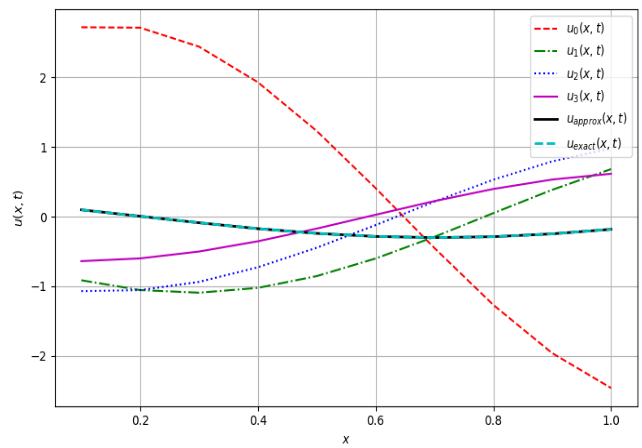


(c) $\alpha = 0.2$

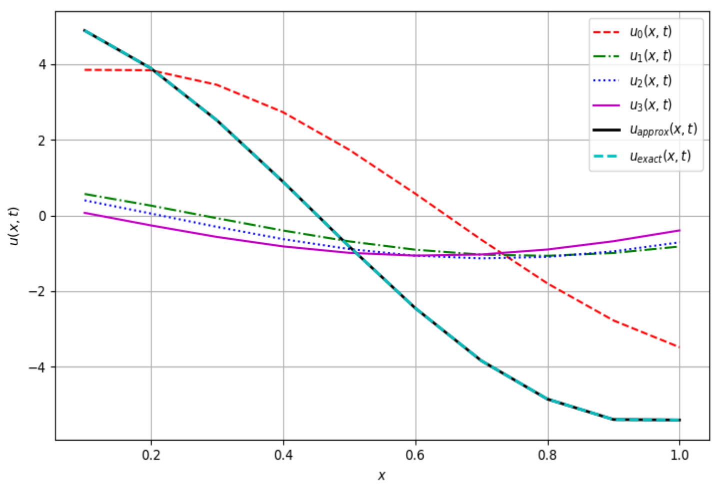
Fig. 2 Comparison between MADM and Exact solutions for $t = 5, f(x,t) = e^{-x}t^2, g(x) = \cos \pi x, h(x) = \sin \pi x$



(a) $\alpha = 0.5$

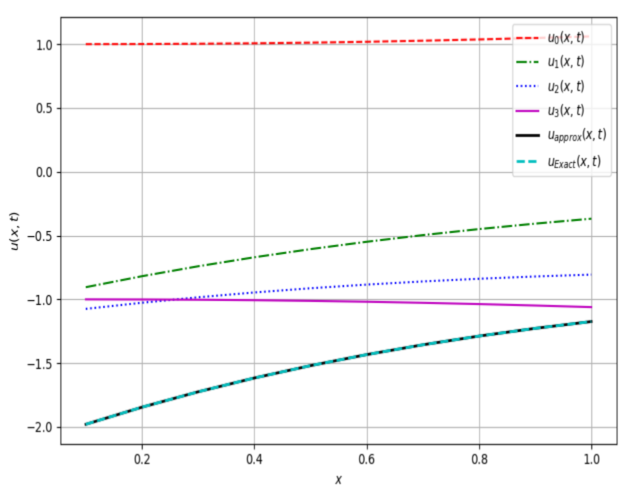


(b) $\alpha = 0.7$

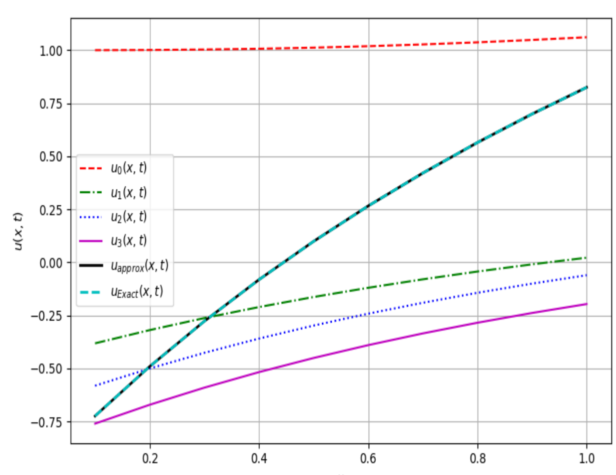


(c) $\alpha = 0.2$

Fig. 3 Comparison between MADM and Exact solutions for $t = 0.5, f(x,t) = e^{-x}t^2, g(x) = \cos \pi x, h(x) = \sin \pi x$

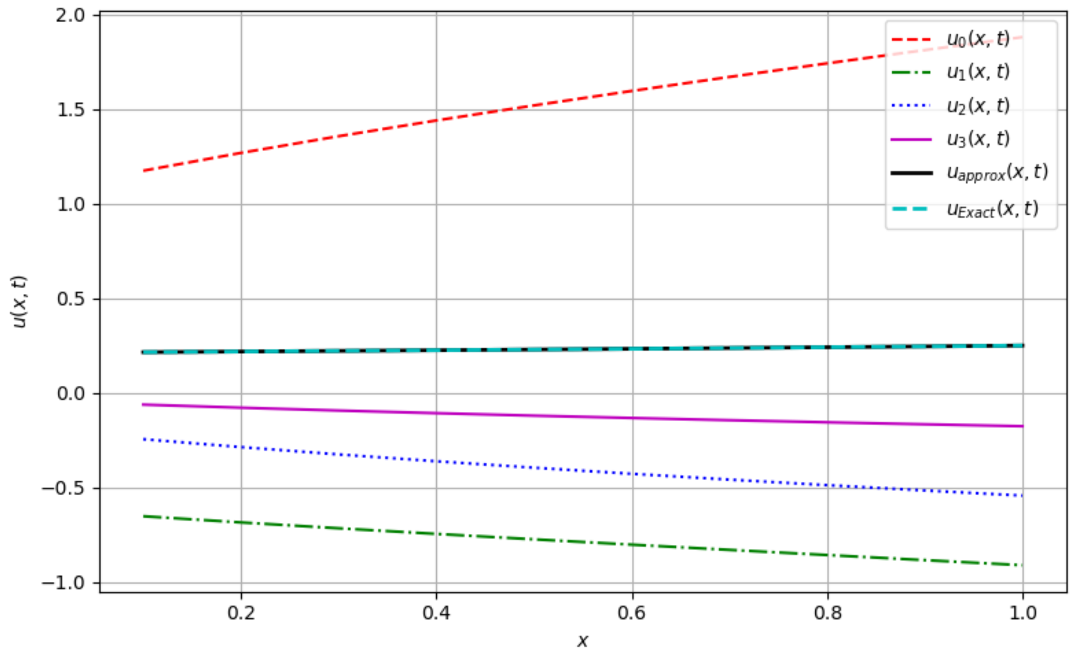


(a) $\alpha = 0.5$

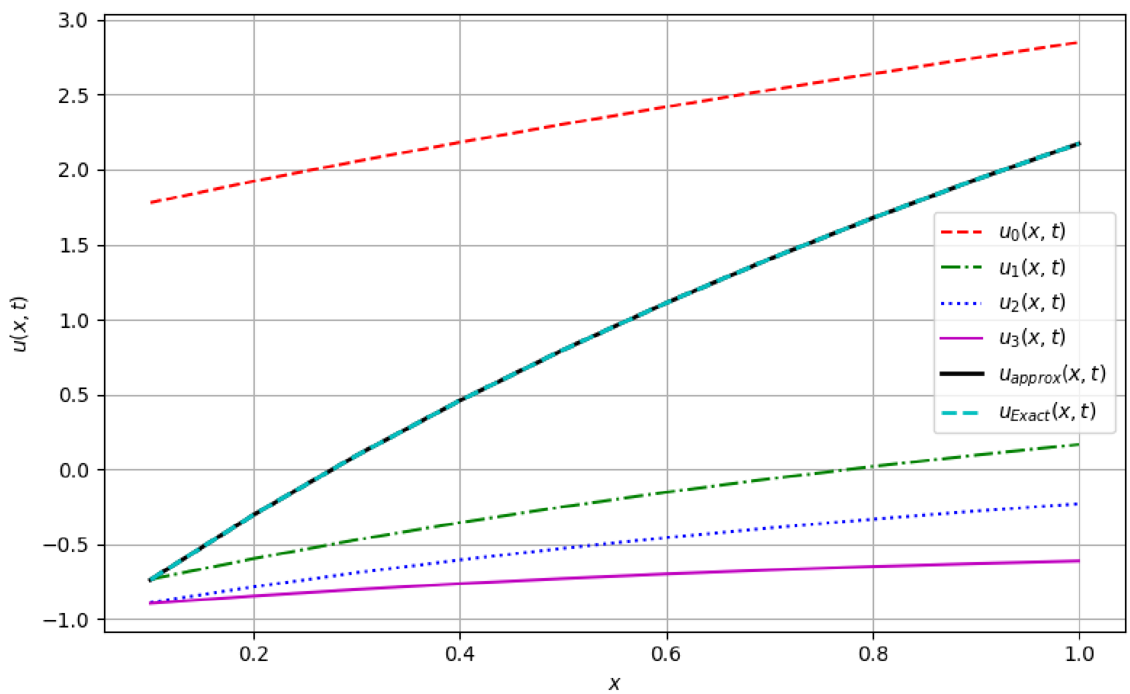


(b) $\alpha = 0.2$

Fig. 4 Comparison between MADM and Exact solutions for $t = 1, f(x,t) = sinxcost, g(x) = e^{-x}, h(x) = log(1 + x)$

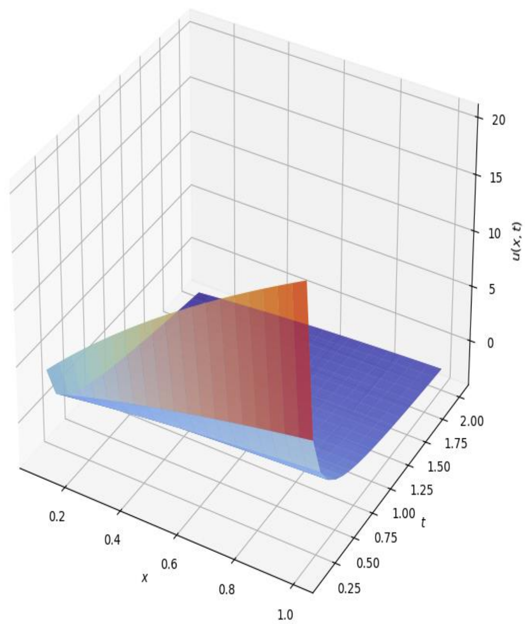


(a) $\alpha = 0.9$

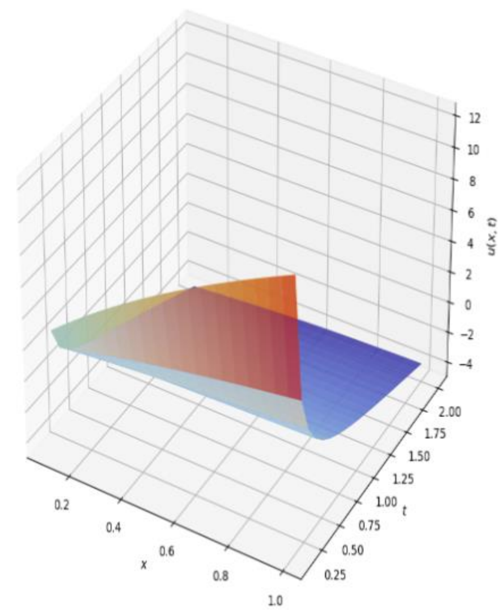


(b) $\alpha = 0.3$

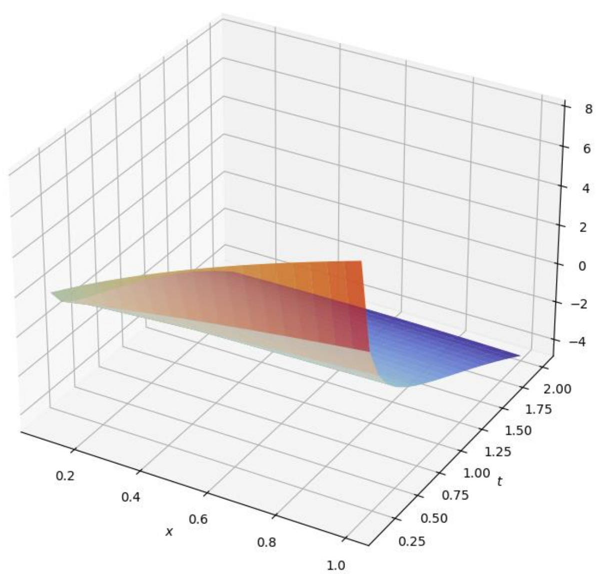
Fig. 5 Comparison between MADM and Exact solutions for $t = 0.5, f(x,t) = sinxcost, g(x) = e^{-x}, h(x) = log(1 + x)$



(a) $\alpha = 0.3$



(b) $\alpha = 0.7$



(c) $\alpha = 0.9$

Fig. 6 MADM Graphical simulation in 3D for $f(x,t) = sinxcost, g(x) = e^{-x}, h(x) = log(1+x)$

7. Discussion of Results

The MADM has been applied to solve the fractional wave equation (3.12). The method allows us to express the solution as an infinite series, summing up the first few terms to approximate the analytic solution. Including higher-order terms improves the approximate solution's precision, ensuring rapid convergence. The convergence rate depends on the problem's characteristics and the fractional order α. The fractional order α introduces a memory effect. The 2D plots show individual terms. u_0, u_1, u_2, u_3 contribute to the approximate solution. Also, the plots, as shown in Figures $1-5$, with varying initial functions, forcing terms and parameters (α, t, x) indicate that even a few terms in the expansion provide a good approximation. The 3D surface plot, as shown in Figure 6, illustrates the approximate solution evolution over both space x and time t.

8. Conclusion

The Mamadu-Adomain Decomposition Method (MADM) has proven to be an efficient and effective numerical technique for solving fractional wave equations. The method is an analytical-numerical approach powered by the power series decomposition technique to generate reliable and accurate results without relying on traditional discretization methods. The method offers significant advantages, including reduced computational complexity and adaptability to various initial functions. The method balances computational and analytical insight feasibility and can resolve many mathematical models in fractional differential equations.

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