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Original Article

Non-Static Plane Symmetric Space Time with Constant Deceleration Parameter in $f(R,T)$ Theory of Gravity

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Abstract *- In this study, we intend to investigate non-static plane symmetric spacetime in a framework of f(R,T) gravity, in an attempt to explain the accelerated expansion of the universe. Using the constant deceleration parameter approach, we obtain an exact solution of the cosmological model. The paper analyses some physical parameters such as the Hubble parameter, the equation of state, energy density, and the expansion scalar, which give information about the evolution of the universe. According to our findings, the deceleration parameter remains constant at* $q = -0.45$*, which means the constant acceleration of the universe. The model shows a transition from a rapid early expansion phase to a more moderate, yet still accelerating phase, aligning well with observational data. Moreover, we examine the deviations from the equation of state parameter, demonstrating a progressive increase over cosmic time, highlighting the dynamic nature of dark energy. The outcomes recommend that the proposed model is a viable alternative to standard cosmological theories and presents a deeper understanding of the underlying mechanics driving cosmic acceleration.*

Keywords *- Non static plane symmetric space-time, f(R,T) gravity, Cosmology, Constant deceleration parameter.*

1. Introduction

Observations of many astronomical bodies [1-5] have reported that the universe is now facing a period of increased growth. Although General Relativity (GR) has been successful in various aspects, it single-handedly fails to provide the source of this acceleration. This has elicited controversies, especially considering the Type Ia Supernovae (SNeIa) data, which are the foundation of most of the hypotheses of the expanding universe. Due to these failures, a number of theoretical frameworks were formulated. A notable suggestion is to propose a hypothetical type of energy, so-called dark energy, into GR. This idea has been well supported by both the theoretical analyses and observational datasets [6-8].

Instead, the gravitational framework has been altered, giving birth to Modified Theories of Gravity. These methods entail reformulation or generalization of the Einstein-Hilbert action to include the phenomena that cannot be well described by the standard GR. These changes tend to bring new geometric or dynamical structures, providing new insights into the dynamics of gravitational interactions and the large-scale dynamics of the universe.

Among the first, as well as most prominent, of such alternatives is $f(R)$ gravity theory. This approach, which has been presented by Buchdahl [9], involves substituting linear "Ricci scalar R" in action with "non-linear function $f(R)$ ". His original work showed that a variation in the structure of the gravitational Lagrangian could have significant effects when it came to the dynamics of cosmology, and therefore, it could be possible to eliminate the tension that could not be explained by classical GR. One of the seminal works in this direction was made by Starobinsky [10], who suggested the $R + R^2$ model. The given higher-order curvature term naturally leads to an inflationary period in the early universe, which efficiently addresses problems of flatness and the horizon. This model is now referred to as Starobinsky inflation, since its predictions are very similar to those measured in the nearly scale-invariant spectrum of "primordial fluctuations" in "CMB (Cosmic Microwave Background)".

$f(T)$ Gravity is another significant branch of modified gravity, in which "torsion scalar T" is used instead of Ricci scalar as the key geometric quantity. In this regard, Shekh *et al*. [11] introduced a model $f(T)$ which is designed to reproduce the late-

time behavior of standard ΛCDM cosmology, and describes observed acceleration by torsion and not the curvature. They used a redshift-dependent deceleration parameter $q(z)$ in their formulation, they were consistent with a number of cosmological observations. Building on this, Shekh *et al.* [12] further explored Tsallis $f(T)$gravity by reconstructing a transition scale factor (TSF) to capture the universe's shift from decelerated to accelerated expansion. Employing MCMC (Markov Chain Monte Carlo) simulations, they constrained model parameters utilising Hubble parameter measurements, supernova data, and baryon acoustic oscillation (BAO) datasets. The results revealed that these models mimic quintessence-like behavior at the present epoch $(z = 0)$ and gradually converge toward ΛCDM characteristics in the distant past $(z = -1)$.
Another promising alteration is $f(R,T)$ gravity, where "gravitational Lagrangian" depends on "Ricci scalar R", also on trace of "stress-energy tensor T". Harko *et al.* [13] introduced this framework and formulated the corresponding field equations, demonstrating how a matter content influences the gravitational dynamics. Their work also explored specific solutions, including those involving scalar fields, with implications for cosmology. Chirde and Shekh [14] applied this theory to "non-static plane symmetric" spacetime occupied with a perfect fluid, deriving the state parameter equation and analyzing its role in cosmic acceleration. Their outcomes show that $f(R,T)$ gravity could effectively describe observed late-time accelerated expansion.

In recent progress, Moraes [15] went further in developing the induced matter approach into a five-dimensional $f(R,T)$ gravity framework, where he came up with exact cosmological solutions. In his analysis, he found that "non-conservation" of "energy-momentum tensor" is inherent in $f(R,T)$ gravity, allowing dynamical reduction of extra dimension. This mechanism is suggested as one of the possible explanations of the acceleration of the expansion of our four-dimensional observable universe.

On this framework, Moraes *et al.* [16] explored one specific type of $f(R,T)$ in which the dependence on the trace T is of a polynomial form. Their results indicate that such a formulation is capable of generating a natural transition between decelerated and accelerated cosmic growth, and the same is consistent with the observations. In a related manner, Singh *et al.* [17] investigated a "Friedmann–Lemaître–Robertson–Walker (FLRW)" model within the same modified gravity, as well as studied cases where different spatial curvature $(k = 0, \pm 1)$ could occur. They demonstrated that one can incorporate into the model a smooth transition between various stages of expansion of the universe by a parameterized scale factor.
All these research papers give evidence that $f(R,T)$ gravity is an intriguing substitute for dark energy, and it can explain the current accelerated expansion without requiring the assumption of some exotic components. It is on the basis of such considerations that the present work is carried out within the context of "$f(R,T)$ gravity".

2. Formation of $f(R,T)$ Gravity

We begin by considering a revised theory of gravity suggested by Harko *et al.* (2011), where action takes the form.

$$s = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m \sqrt{-g}\, d^4x, \tag{1}$$

with $f(R,T)$ representing a generic function dependent on "Ricci scalar R" alongside "trace T" of matter "energy-momentum tensor" T_{ij}, while L_m is Lagrangian describing a matter content.
"Energy-momentum tensor" associated with matter is described through a relation.

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}, \tag{2}$$

alongside its trace by $T = g^{ij}T_{ij}$ respectively, which can also be rewritten as

$$T_{ij} = g_{ij}\, L_m - 2\,\frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

If we assume L_m to be a function solely of the metric tensor g^{ij}, not its derivatives.

To obtain field equations corresponding to the $f(R,T)$ framework, we perform a variation of the action on the metric tensor, g^{ij}, resulting in

$$f(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij}, \tag{4}$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}, \quad (5)$$

Where $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ and ∇_i is "covariant derivative". If one chooses a function of the form $f(R,T) \equiv \varphi(R)$, then the above relation becomes the field equation of the $f(R)$ gravity theory.

When modelling matter as a perfect fluid with "energy density ρ", "pressure p", as well as "four-velocity u^i" there is ambiguity in selecting the matter Lagrangian.

For simplicity, we consider a form of "energy-momentum tensor" as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \quad (6)$$

and matter "Lagrangian" could be denoted as $L_m = -p$

also we have $u^i u_i = 1$. (7)

Using equation (5), we get a "variation of stress-energy tensor" of a "perfect fluid" is

$$\theta_{ij} = -2T_{ij} - p g_{ij}. \quad (8)$$

Furthermore, in some cases, field equations are also affected by a specific form of θ_{ij}, which depends on the physical features of matter. Choice of $f(R,T)$ impacts the resulting dynamics. A frequently studied model is

$$f(R,T) = R + 2F(T), \quad (9)$$

as 1st choice, here $F(T)$ is "arbitrary function of trace" of "stress-energy tensor of matter". By inserting this expression into the general field equations, we derive.

$$R_{ij} - \frac{1}{2}R g_{ij} = 8\pi T_{ij} - 2F'(T)T_{ij} - 2F'(T)\theta_{ij} + F(T)g_{ij}, \quad (10)$$

Here, prime indicates differentiation for argument. Using equation (8), above equation (10) becomes

$$R_{ij} - \frac{1}{2}R g_{ij} = 8\pi T_{ij} + 2F'(T)T_{ij} + [2pF'(T) + F(T)]g_{ij}. \quad (11)$$

3. Field equations and their solution

Let us consider "Riemannian spacetime" defined by "line element"

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2), \quad (12)$$

Where r, θ, z are usual "cylindrical polar coordinates", along with h & s, are functions of t alone. This line element's "plane symmetry" is well recognized.

The gravitational field equation is

$$G_{ij} = R_{ij} - \frac{1}{2}R\, g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij}, \quad (13)$$

"Energy-momentum tensor" for "anisotropic dark energy" is represented by

$$T_j^i = diag[\rho, -p_x, -p_y, -p_z]$$
$$= diag[1, -w_x, -w_y, -w_z], \quad (14)$$

Where ρ is "energy density of fluid" & p_x, p_y, p_z are pressure along x, y, z axes correspondingly.

"Energy momentum tensor" could be computed as

$$T_j^i = diag[1,, -(w+\delta), -(w+\delta), -w]\rho. \quad (15)$$

To simplify, we select $w_x = w$ and a "skewness parameter", δ denotes deviations from w on x & y axes, correspondingly. Then the field equation (13) could be denoted as

$$R_j^i - \frac{1}{2} R\, \delta_j^i = 8\,\pi\, T_j^i - 2f'(T)\, T_j^i + [\, 2p\, f(T) + f(T)]\, \delta_j^i. \tag{16}$$

We now select a function $f(T)$ as a trace of "matter's stress energy tensor" so that, $f(T) = \mu\, T$ here, μ is an "arbitrary constant". Using equations (13) and (15), the field equation (16) can be written as

$$e^{-2h}\left(2\ddot{h} + \dot{h}^2 + \frac{2\dot{h}\,\dot{s}}{s} + \frac{\ddot{s}}{s}\right) = (8\pi + 2\mu)(\omega + \delta)\rho - \mu(1 - 3\omega - 2\delta)\rho - 2\mu p, \tag{17}$$

$$e^{-2h}\left(2\ddot{h} + \dot{h}^2\right) = 8\pi(\omega)\rho - \mu(1 - 3\omega - 2\delta)\rho - 2\mu p, \tag{18}$$

$$e^{-2h}\left(\frac{2\dot{h}\dot{s}}{s} + 3\dot{h}^2\right) = -(8\pi + 2\mu)\rho - \mu(1 - 3\omega - 2\delta)\rho - 2\mu p. \tag{19}$$

Here, the dot represents the derivative for t.
Equations (17) to (19), representing Einstein's field equations, constitute an interdependent set of nonlinear differential relations. Deriving viable and physically relevant solutions from this system is crucial for their utilization in astrophysical and cosmological investigations. In pursuit of such solutions, we proceed by introducing:

- A relationship between the metric potentials given by
  $$e^h = \beta s^n \quad , \tag{20}$$
- The ω is assumed to have a direct proportional relationship with the δ, expressed as
  $$w + \delta = 0 \quad . \tag{21}$$

With the above conditions, terms of "physical parameters", "energy density", "equation of state parameter", alongside deviation from the equation of state parameter, respectively, are obtained as"

$$\rho = \left(\frac{1}{(8\pi + 2\mu)}\right)\left\{2\ddot{h} - 2\dot{h}^2 + \frac{\ddot{s}}{s}\right\} e^{-2h} \tag{22}$$

$$\omega = \left(\frac{-1}{(8\pi + 2\mu)\rho}\right)\left\{2\frac{\dot{h}\,\dot{s}}{s} - \frac{\ddot{s}}{s}\right\} e^{-2h} \tag{23}$$

$$\delta = \left(\frac{1}{(8\pi + 2\mu)\rho}\right)\left\{2\frac{\dot{h}\,\dot{s}}{s} - \frac{\ddot{s}}{s}\right\} e^{-2h} \tag{24}$$

4. Solution of the Field Equations

Cosmological models are often categorized based on how the "Hubble parameter" and "deceleration parameter" evolve over time. These parameters can exhibit sign changes during cosmic evolution, indicating transitions between distinct phases of the universe. A central goal in cosmology is to trace this temporal behaviour and uncover the mechanisms driving it. In scenarios where "Hubble parameter" stays constant, "deceleration parameter" maintains a steady value of -1, which is characteristic of models such as "de Sitter and steady-state universes". Classification of such models typically hinges on whether they represent accelerating or decelerating, expanding or contracting cosmic behaviours. Current observational evidence strongly favours a model of the universe undergoing accelerated expansion.
As proposed by Berman [18,19], a specific relationship connects the "deceleration parameter" q with a scale factor a, given by the expression:

$$q = \frac{a\ddot{a}}{\dot{a}^2} \tag{25}$$

Here a represents "scale factor" and q denotes "deceleration parameter".

The sign of q determines the inflationary nature of the model. A negative q suggests an inflationary phase, while recent "Type Ia supernovae" observations represent that the universe is presently accelerating, with q estimated within the range $-1 \le q \le 0$. "Deceleration parameter" remains constant if "Hubble parameter" H is linked to "scale factor" as follows:

$$H = ba^{-m} = bV^{-m/3} \tag{26}$$

Where b and m are constants.

From equation (25), the parameter for deceleration is provided by:

$$q = -1 + m \tag{27}$$

This equation provides a constant value for q, which could be “positive or negative”. “Positive value” corresponds to “decelerating universe”, while a “negative value” denotes an “accelerating model”. By solving equation (26), we get

$$a = (a_1 t + a_2)^{\frac{1}{(1+q)}} \tag{28}$$

From equation (28), it is evident that the average “scale factor” of the model depends on cosmic time. When $q > -1$, the scale factor increases over time, whereas for $q < -1$, it decreases. However, the scale factor is undefined at $q = -1$. Additionally, it is seen that for $q > -1$, these parameters begin at a constant value, except at a specific point. $t_s = -\frac{a_2}{a_1}$. At this point, the scale factor starts from 0, indicating the presence of a singularity (point-type) [20] at t_s.
Hence, the metric potentials are obtained as

$$s = (a_1 t + a_2)^{\frac{1}{(1+q)(4n+1)}} \tag{29}$$

$$h = \frac{n}{(1+4n)(1+q)} \mathrm{Log}(a_1 t + a_2) \tag{30}$$

The given Figure illustrates the evolution of metric potentials $s(t)$ & $h(t)$ in a non-static plane-symmetric spacetime as functions of cosmic time t, both showing a monotonically increasing behavior (See Figure 1). This suggests an expanding spacetime geometry, where distances grow over time. The close proximity of the two curves indicates nearly isotropic expansion, though a slight difference between them hints at some anisotropic effects. In a cosmological context, such behavior is typically associated with the large-scale evolution of the universe, influenced by matter, radiation, or dark energy. The smooth and continuous growth of the metric potentials implies a gradual and steady expansion without abrupt transitions. This expansion could indicate a universe evolving towards isotropy, where any initial anisotropies diminish over time due to cosmic evolution.

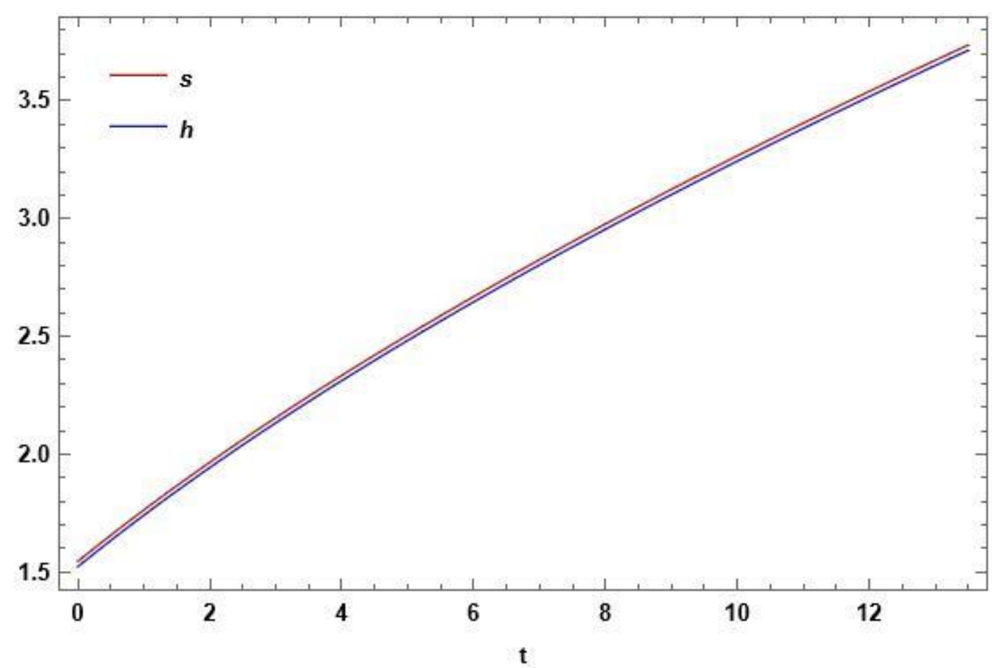

Fig. 1 The behaviour of the metric potentials of the universe versus time (t).

To extend this interpretation, the increasing nature of the metric potentials aligns with general relativistic models of cosmological evolution, such as Bianchi-type anisotropic models or plane-symmetric solutions that describe the early universe. If spacetime were static, the metric potentials would remain constant; their variation with time signifies dynamic geometry, possibly driven by evolving "energy-momentum tensor". Presence of anisotropy, as suggested by slight deviation between $s(t)$ and $h(t)$, could be due to directional dependencies in pressure or shear effects. Such models are essential in understanding deviations from the standard isotropic and homogeneous cosmologies, shedding light on early universe conditions and possible remnants of primordial anisotropies.

With these metric potentials, "physical" and the "kinematical parameters" are obtained as the "energy density" of the derived model and are gained as

$$\rho = \frac{{a_1}^2(q+2n(3+5n+3q+4nq))(a_1 t+a_2)^{-2-\frac{2n}{1+4n+q+4nq}}}{2(1+4n)^2(1+q)^2(4\pi+\mu)} \tag{31}$$

The graph depicting the energy density (ρ) in a "non-static plane-symmetric space time" reveals a rapid decline from an initially high value, asymptotically approaching zero as time (t) progresses. This behavior is clearly seen in Figure 2 and the characteristic of an expanding universe, where the same amount of energy is distributed over an increasingly larger volume, leading to a dilution effect. This aligns with the "Big Bang model", which posits that the universe initiated from a hot, dense state and has been growing as well as cooling ever since. The smooth and continuous decay of energy density suggests a gradual and consistent expansion, devoid of abrupt changes.

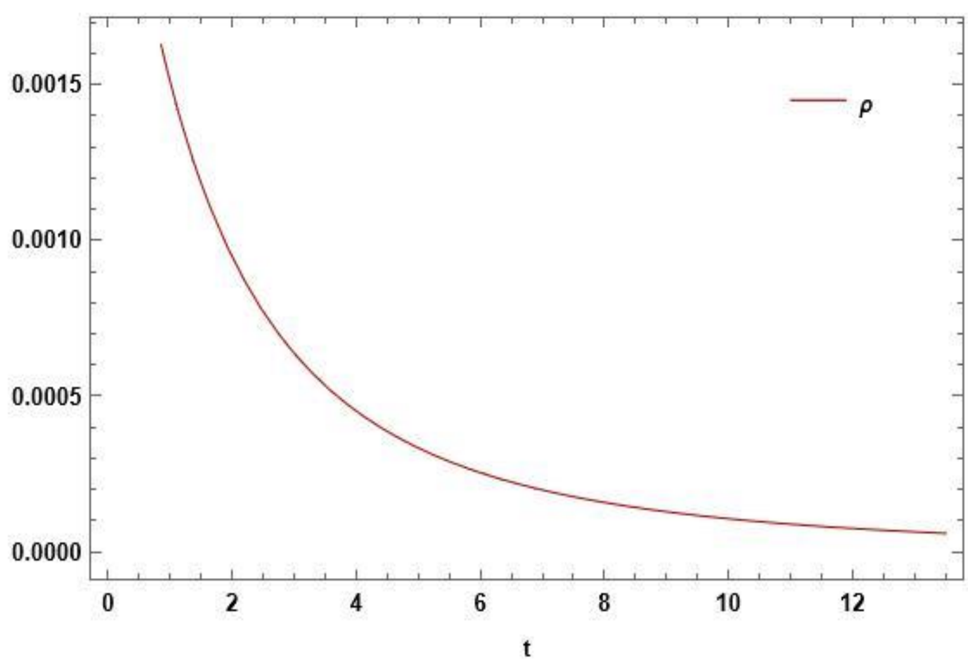

Fig. 2 Behaviour of energy density (ρ) of the universe versus time (t).

Although this specific graph does not make any direct consideration of dark energy, a thorough cosmological model would have to factor in the effect of dark energy, since it is believed that the accelerated expansion that is being experienced in the universe today is a resultant effect of dark energy. Moreover, the fact that there are no oscillations or variations in the energy density implies that the universe is relatively "homogeneous and isotropic", at least in the domain characterized by this plane-symmetric space time. Such a simplified model can serve to give interesting insights into the generally evolving energy content of the universe, but would probably need additional refinements to correspond to the actualities of the real cosmos, such as the structures that form and the distribution of matter.

The parameter of the equation of state is represented as

$$\omega = \frac{(q+n(6+4q))(a_1 t+a_2)^{\frac{2n}{1+4n+q+4nq}}}{q+2n(3+5n+3q+4nq)}. \tag{32}$$

Graph 3, which illustrates the behaviour of the equation of “state parameter” (ω) in a “non-static plane-symmetric space time”, shows an interesting as well as possibly disturbing image of the universe’s future. The parameter, ω, that connects pressure as well as energy density of “dark energy”, is dynamically changing; it has a smooth negative evolution that depends on “cosmic time (t)”. At first, the value of ω is definitely greater than -1, which makes it fall in the context of quintessence-like “dark energy”. Such a form of dark energy, though accelerating the expansion, does so at a rate that is manageable. Nevertheless, ω continues to decrease as time goes by, and at some point, it passes the critical threshold of -1. This "phantom divide line" is the border between quintessence and a more dreadful type of dark energy called phantom dark energy.

When ω falls below -1, the universe transitions to a period of super-accelerated expansion. Growth in this case not only increases but also at an increasing rate. Consequences of “phantom dark energy” are dire, and they might cause a "Big Rip" scenario. In this catastrophic destiny, the growth becomes violent to the extent that it will eventually rip apart all the structures that are held together, including galaxies and stars, to atoms and even space time itself.

The negative decrease seen in the value of ω indicates that “dark energy,” which is causing expansion of this space-time, is not a constant cosmological constant but a dynamic entity. The adverse pressure that drives rapid growth increases with time. This begs very fundamental questions on the nature and origin of the “dark energy” and the eventual fate of the universe.

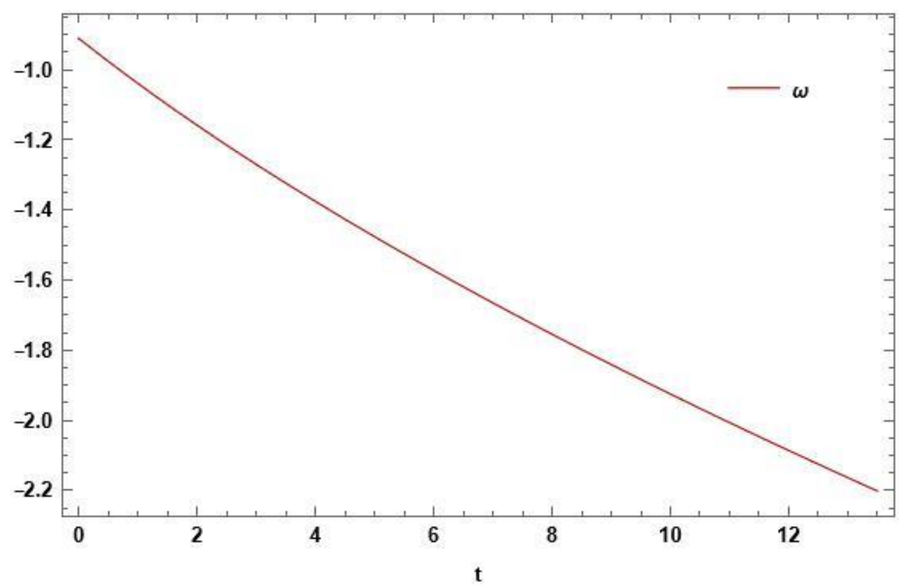

Fig. 3 The behaviour of the state parameter (ω) of the universe versus time (t).

Although this graph can offer a persuasive insight into the potential development of dark energy and its implications, it should be borne in mind that it is merely a simplified model. The actual universe is far more complicated, and our knowledge of dark energy is still developing. However, this visualization is a graphic illustration of the absolute strength of the dark energy and the overwhelming impact it has on the cosmic landscape. It emphasizes the need to conduct more studies to unveil the secrets of “dark energy” and ascertain the real direction of growth of our universe.

Deviation from the equation of state parameter is

$$\delta = -\frac{(q+n(6+4q))(a_1 t+a_2)^{\frac{2n}{1+4n+q+4nq}}}{q+2n(3+5n+3q+4nq)} \tag{33}$$

Our analysis of the deviation (δ) of a standard cosmological parameter in a dark energy scenario revealed a uniform positive and increasing trend in keeping with the cosmic time evolution (See Figure 4). Starting at a rough value of 0.9 at the early epoch, this deviation shows that there was an early departure from a standard cosmological parameter. As the universe developed, the deviation increased slowly, peaking at about 1.6 at some time in the middle. When it got to the current period, the value was close to 2.2, which indicates an increased diversification in terms of the expected behaviour. This smooth increase emphasizes the dynamic nature of the parameter, which means that it was not fixed in cosmic times, and it did not have a concrete value.

Moreover, the identified deviation indicates the potential for non-homogeneity of the universe development. In an absolutely homogeneous isotropic universe, one would expect a uniform parameter, but the point of divergence is directional, and therefore, there could be a non-uniform distribution. This non-uniformity can be due to a lot of different things and can be a consequence of effects of the phenomena in the early universe, asymmetric expansion patterns, or the effect of interaction between “dark energy” and other elements that form the universe. The existence of this non-uniformity throughout the universe’s history is a good chance to explore the main peculiarities of the dark energy and its contribution to the formation of large-scale structures. Investigation of non-uniform behavior has an essential role in the development of “cosmological models” and the identification of the accuracy of different “theories of gravity”.

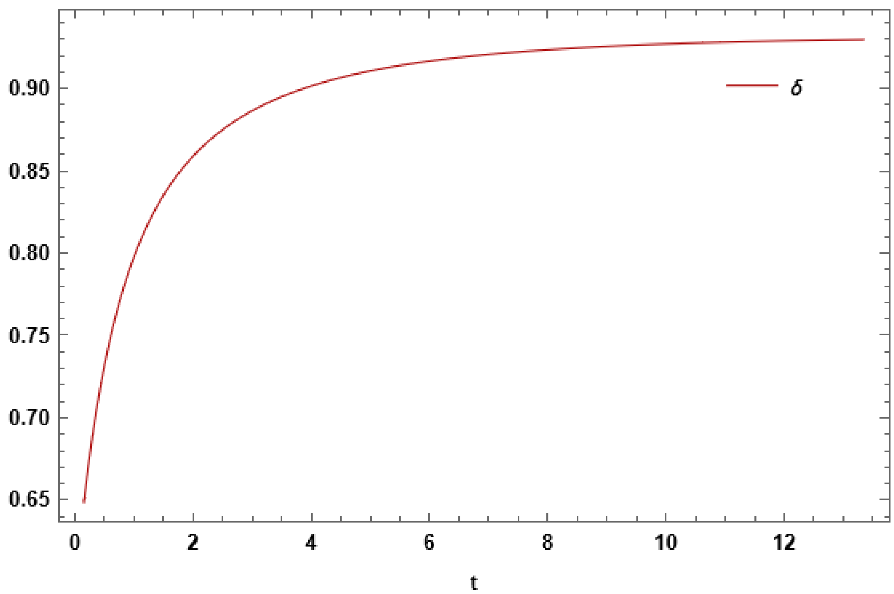

Fig. 4 Behaviour of the deviation (δ) of the universe versus time (t)

Under the assumption of a constant “deceleration parameter”, which we have used to build our proposed cosmological scenario, the “deceleration parameter q” is kept at a constant value of -0.45, and hence, the model parameter choice of $m = 0.55$. This constant negative value of q implies the universe, which is continually gaining speed in the course of cosmic time. As naive as it is, the constant deceleration parameter approach is a strong analytical tool. It allows us to get precise solutions of equations of “gravitational field”, and this allows us to better comprehend the dynamics of expansion of the universe without the complexity of having time-dependent behaviour of deceleration. Although this model is not likely to account exhaustively for that detailed evolution implied by current observational data, it provides a conceptual foundation and a starting point against which more detailed models can be compared. The time evolution of “Hubble parameter” as well as “expansion scalar θ” is shown in graphical representation (See fig. 5). Both quantities exhibit monotonic decrease as cosmic time increases, showing that the universe is experiencing acceleration in its expansion, but the expansion rate reduces as time passes. In the beginning, H and θ are large, representing the rapid expansion of the universe. With time, both parameters slowly decrease and asymptotically approach a constant value, indicating a passage to a steady-state expansion in the far future. This implies that the universe is moving out of a phase of fast growth into moderate or faster-but-still-accelerating growth, as is predicted by the current negative constant value of “deceleration parameter” $q = -0.45$.

Findings of current study, where the real-life scenario of a “non-static plane symmetric space time” was examined within a framework of $f(R,T)$ gravity having constant “deceleration parameter” is in concurrence with number of other studies in the recent past, which have been dedicated to the dynamics of accelerated expansion of universe. For example, Shekh *et al.* [21] studied a late-time $f(T)$ gravity model with a redshift-dependent deceleration parameter and showed that it is consistent with observational data, which is quite similar to the constant q approach used here. Likewise, Shekh *et al.* [22] in their reconstruction of Tsallis $f(T)$ gravity using transition scale factors and MCMC constraints had evolving EoS parameters that crossed the phantom divide, an effect which the present model sees in its dynamic ω. Değer *et al.* [23] constructed FLRW-based $f(R,T)$ cosmologies that admit decelerated and accelerated phases through parametric forms of the scale factor, which is similar to what has been done in this article, a methodology analogous to the Berman-type variation. Moreover, Moraes and collaborators [24] showed that polynomial $f(R,T)$ models naturally yield late-time acceleration, paralleling the asymptotically stable expansion behavior demonstrated here. Recent findings by Beesham and Jokweni [25] further confirm that anisotropic spacetimes in $f(R,T)$ gravity can replicate realistic cosmic histories without invoking dark energy, supporting this article's emphasis on matter-geometry coupling effects. Compared to these recent works, the current study's use of a plane symmetric non-static configuration offers an alternative geometric framework that upholds the broader viability of $f(R,T)$ models describing "late-time cosmic acceleration".

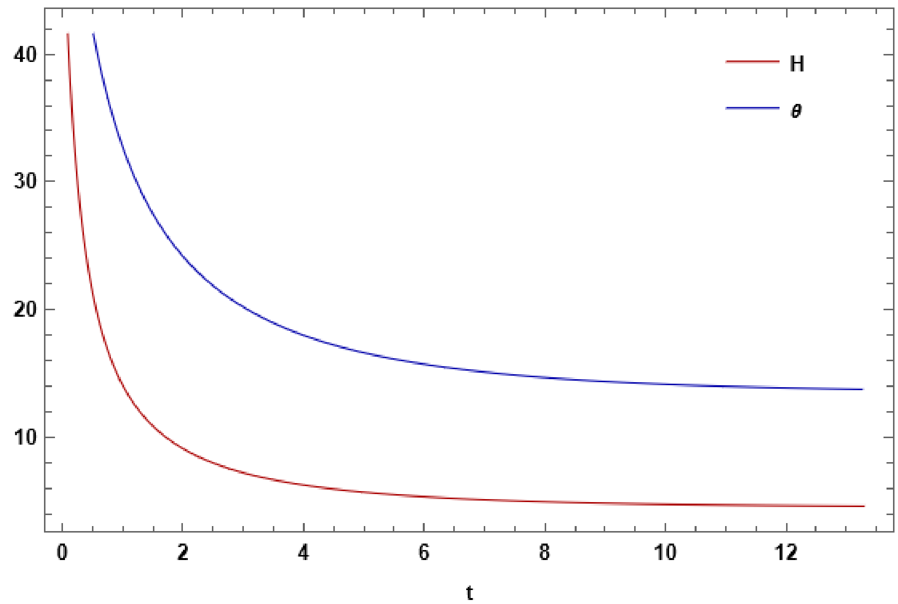

Fig. 5 Behaviour of Hubble parameter (H) and expansion scalar (θ) of the universe versus time t.

5. Results and Conclusion

In the current paper, we assume a cosmological model that is not static but formulated as a "plane-symmetric model" in the framework of "$f(R,T)$ gravity" based on the assumption of a constant parameter of deceleration. Evidences suggest that the universe undergoes a long-term accelerated growth with a definite deceleration parameter $q = -0.45$. Such a simplified picture presents a model that is analytically tractable and that exhibits salient aspects of cosmic evolution.

According to our analysis of the physical behavior of the model, the "Hubble parameter", as well as the expansion scalar, will decrease in time, which indicates a shift between the initial phase of rapid expansion and some more constant, but still accelerating, cosmic evolution. The energy density is displayed that decays asymptotically, which is in line with the expectations of an expanding universe where matter is diluted more. Moreover, the parameter of the Equation of State (EoS) is dynamic; it changes with cosmic time. It is initially at the quintessence regime and then slowly traverses the phantom divide, which is an indicator of a more accelerated phase. Such time dependence of the EoS parameter is an illustration of the dynamical behavior of "dark energy" and a confirmation of the notion of non-constancy of its influence.

Compared to the existing observational data, it may be noted that the model agrees with the contemporary cosmological constraints, and hence it can be considered as an alternative explanation of "late-time acceleration of the universe". Such results are important because they give the possibility of having new explanations of the cosmological phenomena through modified gravity theories like $f(R,T)$. In the future, this framework can be refined by using more detailed data sets of observations and experimenting with the different versions of the $f(R,T)$ function to come up with a better understanding of how the universe evolved.

References

[1] S. Perlmutter et al., "Measurements of the Cosmological Parameters W and L from the First Seven Supernovae at z ≥ 0.35," Astrophys. J. 483(2), 565-581, 1997. [CrossRef] [Google Scholar] [Publisher Link]

[2] Adam G. Riess et al., "New Hubble Space Telescope Discoveries of Type Ia supernovae at z ≥ 1 Narrowing Constraints on the Early Behavior of Dark Energy," *The Astrophysical Journal*, vol. 659, no. 1, pp. 98-121, 2007. [CrossRef] [Google Scholar] [Publisher Link]

[3] D.N. Spergel et al., "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters," *The Astrophysical Journal Supplement Series*, vol. 148, no. 1, pp. 175-194, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[4] Ed. Hawkins et al., "The 2dF Galaxy Redshift Survey: Correlation Functions, Peculiar Velocities and the Matter Density of the Universe," *Monthly Notices of the Royal Astronomical Society*, vol. 346, no. 1, pp. 78-96, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[5] D.J.Eisenstein *et al*., "Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies," *The Astrophysical Journal*, vol. 633, pp. 560-574, 2005. [CrossRef] [Google Scholar] [Publisher Link]

[6] Varun Sahni, "Dark Matter and Dark Energy," *arXiv:astro-ph/0403324*, pp. 1-40, 2022. [CrossRef] [Google Scholar] [Publisher Link]

[7] T. Padmanabhan, "Dark Energy and Gravity," *General Relativity and Gravitation*, vol. 40, pp. 529-564, 2008. [CrossRef] [Google Scholar] [Publisher Link]

[8] R.R. Caldwell, "A Phantom Menace? Cosmological Consequences of a Dark Energy Component with Super-Negative Equation of State," *Physics Letters B*, vol. 545, no. 1-2, pp. 23-29, 2002. [CrossRef] [Google Scholar] [Publisher Link]

[9] H.A. Buchdahl, "Non-Linear Lagrangians and Cosmological Theory," *Monthly Notices of the Royal Astronomical Society*, vol. 150, no. 1, pp. 1-8, 1970. [CrossRef] [Google Scholar] [Publisher Link]

[10] A.A. Starobinsky, "A New Type of Isotropic Cosmological Models without Singularity," *Physics Letters B*, vol. 91, no. 1, pp. 99-102, 1980. [CrossRef] [Google Scholar] [Publisher Link]

[11] S.H. Shekh et al., "Late Times ΛCDM *f(T)* Gravity Model with Parameterized *q(z)*," *Modern Physics Letters A*, vol. 39, no. 23, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[12] S.H. Shekh et al., "Observational Constraints on Transit Reconstructed Tsallis *f(T)* Gravity," *International Journal of Geometric Methods in Modern Physics*, vol. 20, no. 12, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[13] Tiberiu Harko et al., "*f(R, T)* Gravity," *Physical Review D*, vol. 84, no. 2, pp. 1-11, 2011. [CrossRef] [Google Scholar] [Publisher Link]

[14] V. R. Chirde, and S.H. Shekh, "Dark Energy Cosmological Model in a Modified Theory of Gravity," *Astrophysics*, vol. 58, no. 1, pp. 106-119, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[15] P.H.R.S. Moraes, "Cosmology from Induced Matter Model Applied to 5D *f(R, T)* Theory," *Astrophysics and Space Science*, vol. 352, pp. 273-279, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[16] Vijay Singh, Siwaphiwe Jokweni, and Aroonkumar Beesham, "FLRW Transit Cosmological Model in *f(R, T)* Gravity," *Universe*, vol. 10, no. 7, pp. 1-18, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[17] P.H.R.S Moraes, G. Ribeiro, and R.A.C Correa, "*A Transition from a Decelerated to an Accelerated Phase of the Universe Expansion from the Simplest Non-Trivial Polynomial Function of T in the f (R, T) Formalism*," *Astrophysics and Space Science*, vol. 361, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[18] M.S. Berman, "A Special Law of Variation for Hubble's Parameter," *IINuovoCimentoB*, vol. 74, pp. 182-186, 1983. [CrossRef] [Google Scholar] [Publisher Link]

[19] Marcelo Samuel Berman, and Fernando de Mello Gomide "Cosmological Models with Constant Deceleration Parameter," *General Relativity and Gravitation*, vol. 20, pp. 191-198, 1988. [CrossRef] [Google Scholar] [Publisher Link]

[20] Rishi Kumar Tiwari et al., "Bianchi Type I Cosmological Model in f(R,T) Gravity †," *Physical Sciences Forum*, vol. 2, no. 1, pp. 1-6, 2021. Not Found [CrossRef] [Google Scholar] [Publisher Link]

[21] Salim Harun Shekh et al., "Observational Constraints on $\mathscr{F}(\mathcal{T},\mathcal{TG})$ Gravity with Hubble's Parametrization," *Symmetry*, vol. 15, no. 2, pp. 1-13, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[22] M. Koussour et al., "*Quintessence Universe and Cosmic Acceleration in f (Q, T) Gravity*," *International Journal of Modern Physics D*, vol. 31, no. 16, pp. 1-11, 2022. [CrossRef] [Google Scholar] [Publisher Link]

[23] Değer Sofuoğlu, and Aroonkumar Beesham, "f(R,T) Gravity and Constant Jerk Parameter in FLRW Spacetime, *Physical Sciences Forum*, vol. 7, no. 1, pp. 1-6, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[24] P.H.R.S. Moraes, R.A.C. Correa, and G. Ribeiro, "The Starobinsky Model within the f(R,T) Formalism as a Cosmological Model," *arXiv Preprint*, pp. 1-7, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[25] Vijay Singh, Siwaphiwe Jokweni, and Aroonkumar Beesham "Plane Symmetric Cosmological Model with Strange Quark Matter in f(R,T) Gravity," *Universe*, vol. 9, no. 9, pp. 1-16, 2023. [CrossRef] [Google Scholar] [Publisher Link]