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Original Article

Formulating a Deterministic Model for Suppressing the Effects of Fuel-Subsidy Removal on the Nigerian Economy

Ayotokun Kolade Emmanuel[1], Adeosun Mabel Eruore[2], Bakare Olasubomi Abdulateef[3]

[1,2]*Department of Mathematics & Statistics, Osun State College of Technology, Esa-Oke, Osun State, Nigeria.*
[3]*Department of Mathematics, University of Lagos, Akoka, Lagos State, Nigeria.*

[2]*Corresponding Author : adeosunme@oscotechesaoke.edu.ng*

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Abstract - *This study develops a deterministic SEIR model to evaluate and mitigate the impact of fuel subsidy removal on the Nigerian economy. It derives a system of nine non-linear differential equations driven by assumed interactions among economic sectors. The model analyzes fuel subsidy removal, free and prevalent steady states and obtains the economic distress reproduction number, which was found to be greater than one, indicating ongoing economic distress. The sensitivity indices show that the rates at which major sectors are affected due to the increase in fuel price need to be reduced to the greatest extent. Descriptive data of the fuel prices for a four-year period covering the pre- and post-fuel subsidy removal show higher mean, range, and variance in the post-fuel subsidy removal era. Numerical simulations reveal a sharp rise in distress across sectors, followed by a slow and non-uniform recovery rate. The sector comprising individual households that consume foodstuffs experiences greater disruption than the commuters and utilizers of manufactured goods. Also, the recovery path remains unstable, pointing to long-term structural effects. The sensitivity analysis result suggests that adjusting certain parameters can minimize the economic distress number. The study concludes that fuel subsidy removal causes immediate, widespread disruption across Nigeria's economy, with non-uniform recovery rates. It stresses the urgent need for targeted interventions to support the most vulnerable sectors.*

Keywords *- Deterministic models, Economic sectors, Financial contagion, Fuel subsidy, SEIR FS removal modeling.*

1. Introduction

Fuel subsidy (FS) is a form of financial assistance supplied by a nation's government to minimize the cost of fuel for consumers. These are designed to keep the price of fuel artificially low for consumers and to protect them from the effects of rising fuel prices. In Nigeria, the first FS was introduced in the 1970s ([1]) as a means to alleviate the impact of rising global fuel prices on the Nigerian population. Over the years, Nigeria's economy has become heavily dependent on oil revenues, as global oil prices have fluctuated, and the government has faced challenges in managing its fiscal deficit, leading to the expansion of subsidies on various goods and services. The government unexpectedly ended FS in 2012 ([2, 3]). The FS removal sparked large protests, coercing the government to reinstate the FS, which led to the return of the FS in 2012. This change was followed by a drastic increase in FS payments, reaching $4 trillion (approximately $ 6.088 billion) in 2022, accounting for 23% of Nigeria's national budget of 17.126 trillion naira. As a result, Nigeria's FS could no longer be sustained in 2023, and the government embarked on a total phased-out of FS in June 2023 ([3, 4]). The removal of partial and total fuel subsidies from the oil and gas sector has been found to have had severe and multiple direct effects on other sectors of the Nigerian economy ([13-15]). These critical sectors of the Nigerian economy encompass the agricultural [30] and transportation sectors.

Real-life situations (like the case of the removal of FS from the oil and gas sector and its multiplier effects on critical economic sectors, mentioned above) can be represented by deterministic models, which explain real-life scenarios with differential equations, showing the rate of spread (rate of impact) from one compartment (sector) to the other. These are primarily used in mathematical epidemiology to model the spread of infectious diseases within a population. About a particular infection under study, individuals in a given population can be divided into compartments based on their status concerning the infection. Deterministic and stochastic models ([5]) are two broad types of models that have been found to be useful in the study of infectious disease spread. Several works ([6-9]) on epidemic deterministic models have been extensively reported in the extant literature.

Many authors ([10-12]) have explored the applications of deterministic models not only on disease contagion but also to financial or economic contagion. The current financial crisis in Nigeria, resulting from the surge in fuel prices, is likened to a pandemic that has, in turn, affected other economic sectors. A single infected entity induced both COVID-19 contagions and financial crisis contagions. In the case of COVID-19 contagion, the infected individual is *an **individual*** incubating the virus, which is spread through physical contact with the infected individual, resulting in thousands of deaths. In the case of financial contagion, the entity is typically a *financial institution* or an *economic sector* that is affected by specific policies or irregularities, which can in turn impact other sectors. These can lead to job losses, high inflation, human capital flight, and millions of ruined shareholders, among other consequences.

Researchers ([11-15]) have extensively explored precautionary measures that the Nigerian government can implement to mitigate the impacts of the FS removal on the economy. Recent studies ([16-17, 20]) have demonstrated that reinvesting the funds generated from subsidy removal in various economic sectors can significantly enhance economic activities and promote sustainable development. These findings were further supported by [13], who noted that compensational and reinvestment plans will provide a very bright light at the end of the economic tunnel. Some other literature have investigated the adverse impacts of FS removal on the Nigerian economy, these include the following works: [28] proposed a model to study the dynamics of fuel subsidy in which the removal of FS affects Nigerian economy income, oil pirating groups, commodity markets and the consumer purchasing power; thereby, creating a four compartmental model. The formulated model used a time delay to represent the oil theft control, thereby obtaining three steady states. These are subsidy-free, pirate-free, and critical steady-states. The conditions for their existence were determined and analyzed. Numerical methods were used to verify the analytical results, and graphical representations of the dynamics under these states were given. The work in [28] suggests some criteria for the realization of fuel subsidy removal, specifically, the obstruction of oil theft. An assessment of the effects of FS removal on the transportation sector in Nigeria was carried out by the authors in [29]. Through a descriptive survey of 70 commuter transporters, the research found that fuel subsidies had positive impacts on the system, such as improved performance. However, eliminating subsidies led to negative consequences, including decreased revenue, fewer long-distance vehicles, and reduced vehicle maintenance.

It can be vividly stated that the above-mentioned have modelled the effects of FS removal and suggested precautionary measures. However, *the contagious rate of total FS removal by a deterministic (epidemiological) approach in other economic sectors, as seen in the Nigerian case, which is a significant factor, has not been captured.* In order to bridge the gap in the existing literature, the study has formulated a deterministic (Susceptible-Exposed-Infected-Recovered (SEIR)) FS removal model to investigate the contagious effects of total FS removal on the Nigerian economy. Specifically, methods derived from nonlinear dynamical systems of first-order differential Equations (ODEs), statistics, and numerical analysis were used to study the models rigorously.

The formulated model equations were solved simultaneously at the equilibrium point to derive the FS removal effect-free and endemic equilibrium states. To achieve the point of extinction of the FS removal effect, the basic reproduction number (*Economic distress reproductive number)* which is the average number of infected firms in all the sectors produced by this effect on the oil and gas industry, was computed, assuming that the firms in the oil and gas sector are directly susceptible to the effect of FS removal. This method was applied using the next-generation matrix method ([20]). Secondary data were collected and statistically analyzed from regulatory bodies, and the model's parameters were calibrated using numerical methods. The results obtained in this work facilitated the allocation and implementation of the proposed control measures, thereby minimizing the effects of FS removal and maximizing the utilization of available resources. The model measured its effect on five primary sectors: agricultural, mining and quarrying, manufacturing, construction, and services.

The novelty of this work is articulated in various ways. First, the study provides answers to questions concerning the efficiency of measures that the government can implement to mitigate the impact of removing fuel subsidies on the Nigerian economy. Answers to these questions are crucial for policymakers to execute the proposed plan efficiently, maximizing allocations and minimizing economic impacts. Second, to the best of our knowledge, this study is different from other studies ([13, 15]) since it was the first to determine the control measures for suppressing the effects of total FS removal in major economic sectors in Nigeria, especially in the short-term and long-term implementation of these proposed measures. Moreover, the work differs from existing epidemiological studies ([6-9]), as the total FS removal-contagion in the Nigerian economy was modeled using a deterministic epidemiological model for the first time. The existing literature ([13]) has not considered the effects of total FS removal on the interplay between major sectors. Moreover, most of the existing works are restricted to the effects of FS removal on specific sectors over time ([2, 15]). To the best of our knowledge, the study is the first to give a generalization of the above concept.

The rest of the paper is structured into three sections. The following section presents the primary materials for this work, which include the premium motor spirit price (fuel price) data, descriptive statistics of the fuel price for four years, the formulated model, and its analytical solutions. Numerical solutions are presented in section three, while the discussions, concluding remarks, and recommendations are given in section four.

2. Materials and Models

This section outlines the primary materials for this study, including the data, methods, and models formulated and analyzed. Descriptive statistics of the fuel price for four years, the formulated model, and its analytical solutions are given in this section.

2.1. Descriptive Statistics of PMS Price before and after Fuel Subsidy Removal in Nigeria

The Premium Motor Spirit (PMS) price data, commonly referred to as fuel (petrol) price data, was obtained from the National Bureau of Statistics (NBS) via the online microdata catalogue available on the web platform: https://microdata.nigerianstat.gov.ng/index.php (see [27]). The sample period covers a four-year (approximately) period, from Wednesday, September 1, 2021, to Tuesday, April 1, 2025, which represents a total of 56 monthly observed fuel prices taken from the 37 states (including FCT, Abuja) of Nigeria. The descriptive statistics of the PMS price data, presented in two distinct periods — before and after the FS removal in Nigeria — are provided in Table 1 below.

Table 1. Descriptive statistics of the Nigerian PMS price before and after FS removal

Descriptive	PMS Price before FS removal	PMS Price after FS removal
m	361.03	859.6087
$SE\ \overline{M}$	39.1881	55.45264
$\|M\|$	238	701
$\hat{M}$	238	701
SD	225.1185	265.9415
V	50678.34	70724.89
Kr	-1.63354	-1.4906
Sk	0.590058	0.612789
JB	336	716
$LCL\ \overline{M}$	165	546
$UCL\ \overline{M}$	701	1262
$\langle\Sigma\rangle$	11914	19771
N	33	23

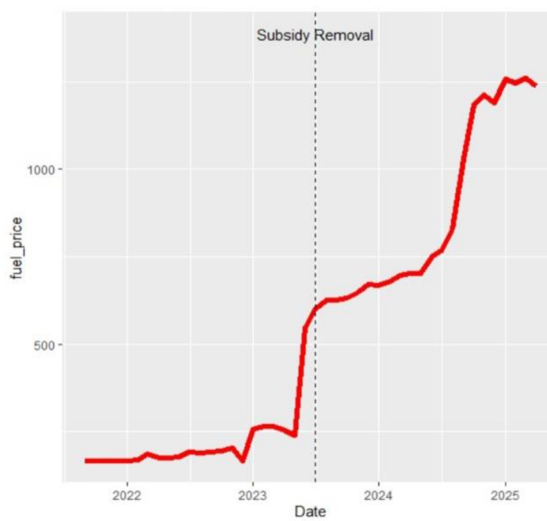

Fig. 1 Plots showing the fuel price before and after FS removal in Nigeria

In the Table 1 above, m = mean; $SE\ \overline{M}$ = standard error; $\|M\|$ = median; $\hat{M}$ = mode; SD = standard deviation; V = sample variance; Kr = kurtosis; Sk = skewness; JB = range; $LCL\ \overline{M}$ = minimum; $UCL\ \overline{M}$ = maximum; $\langle\Sigma\rangle$ = sum; N = count.
Figure 1 gives the plot of the fuel price data obtained from [26] for the stipulated period. It can be seen from the graph that there is a sudden spike in fuel prices (from over two thousand naira to over five hundred naira per litre of PMS) following the announcement of the FS removal on May 29, 2023 (National Bureau of Statistics, 2025; [2]).

2.2. SEIR-FS Removal Deterministic Model Formulation

This paper proposes a deterministic SEIR-FS removal model to study the effect of fuel subsidy removal on the Nigerian economy. The following is a general description of the four central compartments, with a focus on the financial contagion caused by the removal of fuel subsidies. The sub-compartments are outlined in Table 2 below:

1. Susceptible (S): This compartment represents the sectors or individuals who are potentially vulnerable to the negative effects of removing fuel subsidies. These could include industries that heavily rely on fuel, low-income households, and businesses with limited pricing power. ·
2. Exposed (E): This compartment represents firms or individuals that have been exposed to the initial shock of fuel subsidy removal but have not yet experienced significant adverse impacts, which might include businesses that are starting to see rising costs or consumers who are adjusting to higher prices. Examples are Manufacturing, Agriculture, and Transportation
3. Infected (I): This compartment represents firms and individuals who are actively experiencing the negative consequences of fuel subsidy removal, including businesses facing reduced profits, job losses, or increased operational costs.
4. Recovered (R): This compartment represents sectors or individuals who have adapted to the new economic conditions and are no longer significantly affected by the removal of FS. This group may include businesses that have implemented cost-saving measures or consumers who have found ways to reduce their fuel consumption.

The entire Nigerian economic sector was divided into nine (9) mutually exclusive sub-compartments, comprising many firms in the oil and gas sector, which include international, national, and indigenous oil companies; banking sector and individuals, comprising commercial, Merchant, development, Microfinance banks, and individuals. The exposed class comprises of Agriculture sector and individuals, these are the crop production firms, livestock production firms, Fisheries and Forestry representing $E_1(t)$, Manufacturing, Construction sector transportation sectors and Individuals; Food and Beverages, chemicals, pharmaceuticals and cement industries, building, civil Engineering and Real Estate development for $E_2(t)$; and the Road, Rail, Maritime, Aviation and Inland waterways Transport systems representing $E_3(t)$.

The infected class are $I_1(t)$, $I_2(t)$ and $I_3(t)$ sub-compartments. These are the group of individuals that consume foods, represented by $I_3(t)$ the group of individuals that utilizes manufactured goods, represented by $I_2(t)$, and the group of individuals that commute from place to place, represented by $I_3(t)$. The recovery class comprises all individuals and firms that recover from the effects of the FS removal after some precautionary measures have been applied.

The parameters in the SEIR-FS removal optimal control model are described as follows: the rate at which the number of oil firms ($S_1(t)$) increases (that is, more domestic refineries are built), due to the change in fuel pump price η. Also, the banks are affected by the increased inflation rate and credit risk θ when the banks embark on borrowing and lending activities with the oil companies, and the inadequacy of these firms in repaying due to increased expenses caused by the increased running cost of fuel.

The rate at which food production in the agricultural sector increases is the same as the rate of investment in renewable energy (an alternative means to reduce fuel consumption) as a result of the fuel price in the manufacturing and construction sectors α. The operating systems in the transportation firms $E_3(t)$ increase at the rate δ, the prices of foodstuffs obtained from the agricultural sector increase at the rate, and the rate of price increment in the manufactured goods and services increases.

In contrast, the rate of increment in transportation costs increases. Finally the recovery parameters ρ, φ and λ are respectively the rate at which consumers $I_1(t)$ of foodstuffs that have been affected by the hike in fuel price, adjust their purchasing habits; the rate at which users $I_2(t)$ of manufactured goods opt for cheaper ones, and the individuals $I_3(t)$ that commutes from place to place reduce or adjust their routes to mitigate the impact of higher fuel cost. The compartments $I_1(t)$ $I_2(t)$ $I_3(t)$ also increase due to their direct contact rates ω, ξ and ζ with the oil and gas sector. Tables 1 and 2 give a summary of the variables and parameters described above.

Table 1. State variables of the SEIR-FS removal model

Variables	Description
$S_1(t)$	number of individual firms in the oil and gas sector at the time t
$S_2(t)$	number of individual banks at time t
$E_1(t)$	Number of individual agricultural firms in operation that depend heavily on fuel usage at time t
$E_2(t)$	Number of individual manufacturing and construction industries that depend heavily on fuel usage at time t
$E_3(t)$	number of individual transportation systems at a time t
$I_1(t)$	The number of individual households that consume foodstuffs that have been affected by the hike in fuel prices.
$I_2(t)$	The number of individuals in a household who utilize manufactured goods
$I_3(t)$	The number of individuals who commute from place to place via the various transportation systems
$R(t)$	The number of individuals who recover from the effects of the FS removal after some precautionary measures have been applied

Table 2. Parameter values of the SEIR-FS removal model

Parameters	Description
η	The rate at which new firms enter the oil and gas sector (economic expansion) is driven by FS removal.
θ	The rate at which banks support firms in the oil and gas sector.
μ	Natural exit rate due to business closure, economic decline, or the death of firms.
μ_1	The rate at which the transportation system goes into extinction due to the FS removal
β	The rate at which agricultural firms are affected by fuel price hikes.
α	The rate at which manufacturing and construction firms are affected by fuel price hikes.
ϕ	The rate at which firms in the oil and gas sector transition to banking due to fuel subsidy removal.
δ	The rate at which transportation systems are affected by fuel price hikes.
ω	The rate at which food consumers feel the impact of increased fuel costs.
ς	The rate at which consumers of manufactured goods feel the impact of increased fuel costs.
ζ	The rate at which individuals who commute are affected by fuel price hikes, leading to increased transportation costs
ξ	The rate at which exposed agricultural firms transition into the affected category.
π	The rate at which exposed manufacturing/construction firms transition into the affected category.
ε	The rate at which transportation systems transit into financial distress or closure due to fuel price increases
ρ	Recovery rate of households consuming foodstuffs after implementing economic relief measures.
φ	Recovery rate of manufactured goods' consumers, after adaptation measures.
λ	Recovery rate of transport users after adaptation measures.

2.3. SEIR-FS Removal Model Assumption

The following assumptions are made for the SEIR FS –removal model:

- The financial contagion originated from the oil and gas sector ($S_1(t)$) The ripple effect is shown using PMS's historical prices.
- All firms in the economic sectors recover after the removal of the fuel subsidy.
- All the individual firms and individual households form the total population in the context of the Nigerian economy.
- That all individual firms and individual households in the economic sectors possess the tendency to go into economic extinction naturally at the same rate μ.

Based on the description of the SEIR FS- removal model and the assumptions given above, the schematic flow diagram for the SEIR FS removal compartmental model is shown in Figure 2 below.

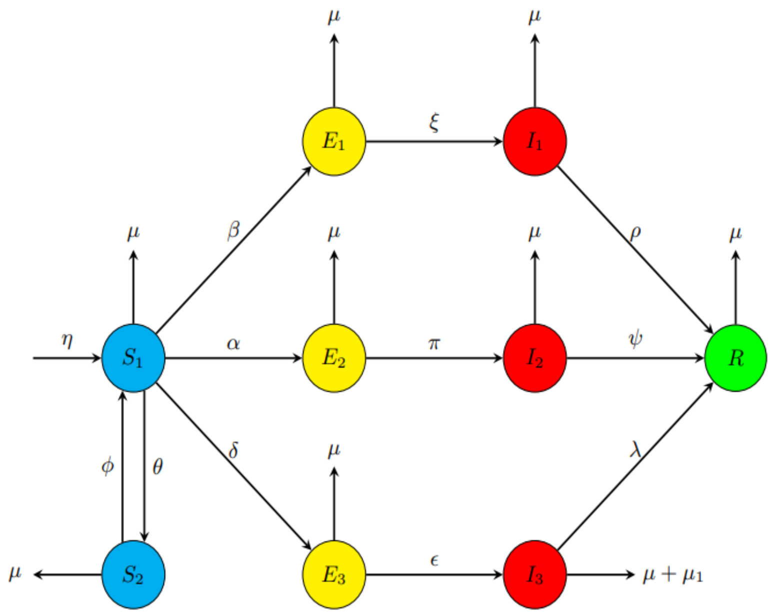

Fig. 2 SEIR- FS Removal Model's compartmental flow diagram

Thus, the rate of change of $S_1, S_2, E_1, E_2, E_3, I_1, I_2, I_3$ and R are defined by a deterministic model with equations as follows:

With initial conditions:

$$S_1(0) = S_0^1, S_2(0) = S_0^2, E_1(0) = E_0^1, E_2(0) = E_0^2, E_3(0) = E_0^3, I_1(0) = I_0^1, I_2(0) = I_0^2, I_3(0) = I_0^3, R(0) = R_0$$

Based on the assumption (iii) above, the total economic population function is given as:

$$N(t) = S_1(t) + S_2(t) + E_1(t) + E_2(t) + E_3(t) + I_1(t) + I_2(t) + I_3(t) + R(t) \tag{2}$$

Hence, economic dynamics resulting from fuel subsidy removal can be described as:

$$\dot{N}(t) = \dot{S}_1(t) + \dot{S}_2(t) + \dot{E}_1(t) + \dot{E}_2(t) + \dot{E}_3(t) + \dot{I}_1(t) + \dot{I}_2(t) + \dot{I}_3(t) + \dot{R}(t) \tag{3}$$

Putting Equation (1) into Equation (3) gives:

$$\begin{aligned}\dot{N}(t) = {} & \eta + \theta S_2 - \mu S_1 - (\beta E_1 + \alpha E_2 + \delta E_3) S_1 - (\omega I_1 + \varsigma I_2 + \zeta I_3) S_1 - \phi S_1 - \phi S_1 + \phi S_1 - (\theta + \mu) S_2 \\ & + \beta S_1 E_1 - (\mu + \xi) E_1 + \alpha S_1 E_2 - (\mu + \pi) E_2 + \delta S_1 E_3 - (\mu + \varepsilon) E_3 + \omega S_1 I_1 + \xi E_1 - (\mu + \rho) I_1 \\ & + \varsigma S_1 I_2 + \pi E_2 - (\mu + \varphi) I_2 + \zeta S_1 I_3 + \varepsilon E_3 - (\mu + \lambda + \mu_1) I_3 + \rho I_1 + \varphi I_2 + \lambda I_3 - \mu R\end{aligned}$$

Upon simplification,

$$\dot{N}(t) = \eta + N\mu - \mu_1 I_3 \tag{4}$$

Equation (4) gives the economic population dynamics.

2.4. Feasibility, Boundedness and Positivity of the SEIR-FS Removal Model

This section determines the Feasibility, positivity and boundedness of the SEIR FS-removal model.

2.4.1. Feasibility and Boundedness of the SEIR-FS Removal Model

Theorem 1 below establishes the Feasibility and boundedness of the model.

Theorem 1

Given that the simplified form of the economic population dynamics in Equation (4) holds, then every solution of the model in system (1) satisfying the initial conditions R_+^9 tends to and is contained in a compact set (Υ) . $t \to \infty$ The feasible solution, which lies within the model, is given by: $\Upsilon = \left\{ \left(X_t \right) \in R_+^9 : N_t \le \frac{\eta}{\mu} \right\}$ $X_t = S_{1,}, S_2, E_1, E_{2,}, E_3, I_1, I_2, I_3, R$

Proof

In the sequel, proof of the Feasibility and boundedness of the model will be given. From Equation (4),

$$N(t) = \eta - N\mu - \mu_1 I_3$$

In the absence of fuel subsidy removal, $(\mu_1 = 0)$, then

$$N(t) \le \eta - N\mu \quad (5)$$

Solving the above Equation (5) by the integrating factor method

$$N(t) + N\mu \le \eta \Rightarrow \frac{d(N(t)e^{\mu t})}{dt} \le \eta e^{\mu t} \quad (6)$$

By integrating both sides of the equation,

$$N(t) = \frac{\eta}{\mu} + N(0)e^{-\mu t} \quad (7)$$

This implies that the total number of economic participants $N(t)$ satisfies Equation (7), such that $t \dashrightarrow \infty$, $N(0)e^{-\mu t} \to 0$, given that $N(0)$ is a constant. Also,$0 \le N(t) \le \frac{\eta}{\mu}$ showing that $N(t)$ it is *bounded* in the region R_+^9 indicates that no economic compartment grows indefinitely. This also indicates that the feasible solution set of the model is given by $\Upsilon = \left\{ \left(X_t \right) \in R_+^9 : N_t \le \frac{\eta}{\mu} \right\}$ $X_t = S_{1,}, S_2, E_1, E_{2,}, E_3, I_1, I_2, I_3, R$

2.4.2. Positivity Solution of the State Variables in the SEIR FS-Removal Model

In the next theorem, the positivity of the state variables in model Equation (2) is established for all time $t \ge 0$.

Theorem 2

Let the initial values of the model Equation (1) be $\{\chi_0 \ge 0\} \in \Upsilon$ where $X_0 = \{S_0^1, S_0^2, E_0^1, E_0^2, E_0^3, I_0^1, I_0^2, I_0^3, R_0\}$. Then, the solution set X_t in model Equation (1) is positive for all. $t \ge 0$

Proof

To establish that the solution set X_t in the system Equation (1) is a positive state variable, then, for all $t \ge 0$, solutions that satisfy:

$$X_t \ge 0 \Rightarrow S_t^1 \ge 0, S_t^2 \ge 0, E_t^1 \ge 0, E_t^2 \ge 0, E_t^3 \ge 0, I_t^1 \ge 0, I_t^2 \ge 0, I_t^3 \ge 0, R \ge 0$$

First, the positivity from the initial condition is verified. Based on the initial conditions.:

$$S_1 = S_0^1 \ge 0, S_2 = S_0^2 \ge 0, E_1 = E_0^1 \ge 0, E_2 = E_0^2 \ge 0, E_3 = E_0^3 \ge 0, I_1 = I_0^1 \ge 0, I_2 = I_0^2 \ge 0, I_3 = I_0^3 \ge 0, R_0 = R \ge 0$$

The initial conditions $X_0 = \{S_0^1, S_0^2, E_0^1, E_0^2, E_0^3 I_0^1, I_0^2, I_0^3, R_0\}$are positive, i.e., the variables are non-negative $t = 0$. Second, the positivity of the model is established using the system of differential equations in (1). The first Equation in system (1) in terms of S_t is rewritten as:

$$\dot{S}_1(t) = \eta + \theta S_2 - \mu S_1 - (\beta E_1 + \alpha E_2 + \delta E_3)S_1 - (\omega I_1 + \varsigma I_2 + \zeta I_3)S_1 - \phi S_1 \quad (8)$$

Given that $S_1 = S_0^1 \geq 0$, a lower bound for S_1 is constructed as:

$$\dot{S}_1(t) + \left(\mu + \phi - (\beta E_1 + \alpha E_2 + \delta E_3) - (\omega I_1 + \varsigma I_2 + \zeta I_3)\right)S_1 = \eta + \theta S_2 \tag{9}$$

Let $\psi = \mu + \phi - (\beta E_1 + \alpha E_2 + \delta E_3) - (\omega I_1 + \varsigma I_2 + \zeta I_3)$then, $\psi$is non-negative, since it consists of positive parameters and non-negative state variables. Thus, from Equation (9):

$$\dot{S}_1(t) + \psi S_1 = \eta + \theta S_2 \tag{10}$$

Equation (10) is a linear first-order differential inequality. By Gronwall's inequality,

$$\dot{S}_1(t) + \psi S_1 \geq \eta \tag{11}$$

The solution of Equation (11) is obtained by the integrating factor method and by integrating from 0to t:

$$S_1(t) \geq S_0^1 e^{-\int_0^t \psi(s)ds} + \eta e^{-\int_0^t \psi(s)ds} \int_0^t e^{-\int_0^s \psi(u)du}\, dx$$

Since, $S_0^1 \geq 0$ and $\eta \geq 0$then, $S_1(t) \geq 0 \quad \forall t \geq 0$. Similarly, the above argument holds for the other state variables, showing their positivity properties.

Remark 2.1

The analysis given in Theorem 2 above shows that the SEIR-FS removal model is mathematically well-posed and realistic.

2.5. Steady State Solution of the SEIR-FS Removal Model

Here, the steady states of the SEIR FS-removal model are determined. In these states, the system reaches a steady state where the effect of the fuel subsidy removal on the Nigerian economic sectors grows or shrinks drastically. These states are determined under the following conditions of system (1):

$$\dot{S}_1(t) = \dot{S}_2(t) = \dot{E}_1(t) = \dot{E}_2(t) = \dot{E}_3(t) = \dot{I}_1(t) = \dot{I}_2(t) = \dot{I}_3(t) = \dot{R}(t) = 0$$

Next, the FS-removal free steady state and the FS-removal prevalent steady state are obtained. From Equation (2) of system (1), and for $\dot{S}_2(t) = 0$,

$$S_2 = \frac{\phi S_1}{(\theta + \mu)} \tag{12}$$

From Equations (3-5), for $\dot{E}_1(t) = \dot{E}_2(t) = \dot{E}_3(t) = 0$,

$$\left.\begin{aligned} E_1 &= \frac{\beta S_1}{(\mu+\xi)} E_1 \Rightarrow E_1 = 0 \\ E_2 &= \frac{\alpha S_1}{(\mu+\pi)} E_2 \Rightarrow E_2 = 0 \\ E_3 &= \frac{\delta S_1}{(\mu+\varepsilon)} E_3 \Rightarrow E_3 = 0 \end{aligned}\right\} \tag{13}$$

Similarly, for $\dot{I}_1(t) = \dot{I}_2(t) = \dot{I}_3(t) = 0$:

$$I_1 = \frac{\xi E_1}{\omega S_1 + (\mu + \rho)}, I_2 = \frac{\pi E_2}{\varsigma S_1 + (\mu + \varphi)}, I_3 = \frac{\varepsilon E_3}{\zeta S_1 + (\mu + \lambda + \mu_1) I_3} \tag{14}$$

For $\dot{R} = 0 \Rightarrow R(t) = \frac{\rho I_1 + \varphi I_2 + \lambda I_3}{\mu}$

2.5.1. FS removal Free Steady State

In the FS removal free steady state, the exposed and infected economic sectors are assumed to be zero because the economy at that time is assumed to be free of subsidy removal. That is, $E_1, E_2, E_3, I_1, I_2, I_3$ they are set to be zero. This implies

$$E_1^* = E_2^* = E_3^* = I_1^* = I_2^* = I_3^* = 0$$

Hence, the first equation in the system (1) is simplified to:

$$0 = \eta + \theta S_2 - \mu S_1 - \phi S_1 \tag{15}$$

Substituting Equation (12) into Equation (15) gives:

$$\eta + \theta\left(\frac{\phi S_1}{(\theta + \mu)}\right) - \mu S_1 - \phi S_1 = 0$$

Upon solving,

$$S_1^* = \frac{\eta}{\left(\theta\left(\frac{\phi}{(\theta+\mu)}\right) - \mu - \phi\right)} \tag{16}$$

Also, from Equation (12)

$$S_2^* = \frac{\eta\phi(\theta+\mu)}{\left(\theta\left(\frac{\phi}{(\theta+\mu)}\right) - \mu - \phi\right)} \tag{17}$$

And finally, from Equation (9) of system (1): $0 = -\mu R \Rightarrow R^* = 0$

Therefore, the *FS–removal free steady state* is

$$(S_1^*, S_2^*, E_1^*, E_2^*, E_3^*, I_1^*, I_2^*, I_3^*, R) = \left(\frac{\eta}{\left(\theta\left(\frac{\phi}{(\theta+\mu)}\right) - \mu - \phi\right)}, \frac{\eta\phi(\theta+\mu)}{\left(\theta\left(\frac{\phi}{(\theta+\mu)}\right) - \mu - \phi\right)}, 0,0,0,0,0,0,0\right) \tag{18}$$

2.5.2. FS Removal Prevalent Steady State

In the case of a disease contagion, the widespread state occurs when the disease spreads has grown drastically; such a state is termed *the endemic equilibrium state or endemic steady state*. Since a case of financial or economic contagion, particularly, a case of the effect of FS removal on the Nigerian economy, is being treated, this state is termed the *FS removal prevalent steady state.* In order to achieve the FS removal prevalent steady state in the nonlinear system of differential equation (1), the quasi steady state is applied by $E_1, E_2, E_3, I_1, I_2, I_3$ assuming they reach equilibrium faster than S_1, S_2. R For the exposed agriculture, transportation and manufacturing/construction individual firms and sectors,

$$\begin{aligned} \dot{E}_1(t) &= \beta S_1 E_1 - (\mu + \xi) E_1 = 0 \Rightarrow E_1^* = \frac{\mu+\xi}{\beta S_1} \\ \dot{E}_2(t) &= \alpha S_1 E_2 - (\mu + \pi) E_2 = 0 \Rightarrow E_2^* = \frac{\mu+\pi}{\alpha S_1} \\ \dot{E}_3(t) &= \delta S_1 E_3 - (\mu + \varepsilon) E_3 = 0 \Rightarrow E_3^* = \frac{\mu+\varepsilon}{\delta S_1} \end{aligned} \tag{19}$$

Similarly, the FS removal of prevalent steady states for the infected individuals and households is:

$$\dot{I}_1(t)=\xi E_1-(\mu+\rho)I_1=0\Rightarrow I_1^*=\frac{\xi E_1}{\mu+\rho}$$
$$\dot{I}_2(t)=\pi E_2-(\mu+\varphi)I_2=0_1^*\Rightarrow I_2^*=\frac{\pi E_2}{\mu+\varphi} \quad (20)$$
$$\dot{I}_3(t)=\varepsilon E_3-(\mu+\lambda+\mu_1)I_3=0\Rightarrow I_3^*=\frac{\varepsilon E_3}{\mu+\lambda+\mu_1}$$

Solving for S_2^* given that $\dot{S}_2(t) = 0 \Rightarrow \phi S_1 - (\theta + \mu)S_2 = 0 \Rightarrow S_2^* = \frac{\phi S_1}{\theta+\mu}$. The steady state value for the oil and gas sector is given as:

$$S_1^* = \frac{\eta}{\mu+\phi} \quad (21)$$

It follows from Equations (18-21) that the ***FS removal prevalent steady state*** is given as:

$$(S_1^*, S_2^*, E_1^*, E_2^*, E_3^*, I_1^*, I_2^*, I_3^*, R^*) = \left(\frac{\eta}{\mu+\phi}, \frac{\varphi\eta}{\Gamma}, \frac{\Delta}{\beta\eta}, \frac{\Theta}{\alpha\eta}, \frac{\Xi}{\delta\eta}, \frac{\xi\Delta}{\beta\eta(\mu+\rho)}, \frac{\pi\Theta}{\alpha\eta(\mu+\varphi)}, \frac{\varepsilon\Psi}{\delta\eta}, \Pi\right) \quad (22)$$

Where,

$$\Gamma = (\theta+\mu)(\mu+\phi), \Delta = (\mu+\xi)(\mu+\phi), \Theta = (\mu+\pi)(\mu+\phi), \Xi = (\mu+\varepsilon)(\mu+\phi), \Psi = (\mu+\xi)(\mu+\phi)(\mu+\lambda),$$
$$\Pi = \frac{\eta}{\mu}\left(\frac{\rho\xi\beta\eta}{(\mu+\rho)} + \frac{\phi\pi\alpha\eta}{(\mu+\theta)^3} + \frac{\lambda\varepsilon\delta\eta}{(\mu+\lambda)(\mu+\xi)(\mu+\varphi)}\right)$$

2.5.3. Economic Distress Reproductive Number (R_0^e)

The economic distress (resulting from fuel subsidy removal) reproductive number (R_0^e) is defined as the average number of individual firms or households that experience economic distress due to the removal of fuel subsidy, triggered by one economically affected individual firm/household. (R_0^e) Satisfies the following conditions in the different scenarios

1. If $R_0^e > 1$ this is the case, this implies that the economic distress increases due to fuel subsidy removal, resulting in higher cost of living, etc.
2. If $R_0^e < 1$ this is the case, this indicates that the economy has adapted, and the negative effect of FS removal will diminish over time.

In the SEIR FS-removal model, the economic distress resulting from the FS removal spreads through the infected compartments $E_1, E_2, E_3, I_1, I_2, I_3$. The next generation method shall be used to compute R_0^e.

$$R_0^e = max(FV^{-1}) \quad (23)$$

Where F is the new economic distress matrix

V is the transition matrix

R_0^e is the spectral radius (largest eigenvalue) of FV^{-1}

From the above, the economically distressed compartments are: $\tilde{Z} = (E_1, E_2, E_3, I_1, I_2, I_3)^T$. Used $F_{ij} = \frac{\partial z_i}{\partial z_j}$ to compute F:

$$F = \begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & f \end{pmatrix}$$

where, $a = \frac{\partial(\beta S_1 E_1)}{\partial E_1} = \beta S_1$, $b = \frac{\partial(\alpha S_1 E_2)}{\partial E_2} = \alpha S_1 c = \frac{\partial(\delta S_1 E_3)}{\partial E_3} = \delta S_1$, $d = \frac{\partial(\omega S_1 I_1)}{\partial I_1} = \omega S_1$,

$e = \frac{\partial(\varsigma S_1 I_2)}{\partial I_2} = \varsigma S_1$ and $f = \frac{\partial(\zeta S_1 I_3)}{\partial I_3} = \zeta S_1$

The matrix V that represents transitions out of economic distress due to recovery or movement between compartments is given as :

$$V_{ij} = \frac{\partial V_i}{\partial Z_j}$$

Where V_i are the negative terms in each equation? Using the model below:

$$\begin{aligned}
V_{E_1} &= (\mu + \xi)E_1 \\
V_{E_2} &= (\mu + \pi)E_2 \\
V_{E_3} &= (\mu + \varepsilon)E_3 \\
V_{I_1} &= (\mu + \rho)I_1 - \xi E_1 \\
V_{I_2} &= (\mu + \varphi)I_2 - \pi E_2 \\
V_{I_3} &= (\mu + \lambda + \mu_1)I_3 - \varepsilon E_2
\end{aligned}$$

Taking derivatives:

$$V = \begin{pmatrix} \mu+\xi & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu+\pi & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu+\varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu+\rho & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu+\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu+\lambda+\mu_1 \end{pmatrix}$$

In the matrix Vabove, the terms on the diagonal of the matrix are the recovery and exit rates, while the non-diagonal terms are the transitions between economic distress levels.

By using Equation (23), the economic distress reproductive number R_0^e is computed and obtained as:

$$R_0^e = max\left(\frac{\beta S_1^*}{\mu+\xi}, \frac{\alpha S_1^*}{\mu+\pi}, \frac{\delta S_1^*}{\mu+\varepsilon}, \frac{\omega S_1^*}{\mu+\rho}, \frac{\varsigma S_1^*}{\mu+\varphi}, \frac{\zeta S_1^*}{\mu+\lambda+\mu_1}\right) \quad (24)$$

Given that:

$$FV^{-1} = \begin{pmatrix} \frac{\beta S_1^*}{\mu+\xi} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta S_1^*}{\mu+\pi} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta S_1^*}{\mu+\varepsilon} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta S_1^*}{\mu+\rho} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta S_1^*}{\mu+\varphi} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta S_1^*}{\mu+\lambda+\mu_1} \end{pmatrix} \quad (25)$$

The value obtained R_0^e would determine whether the Nigerian economy recovers or remains distressed after fuel subsidy removal. Hence, from the eigenvalues in Equation (25)

$$R_0^e = max(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) \quad (26)$$

$$R_0^e = \max(732.60, 20000.00, 23333.33, 30000.00, 13333.33, 16666.66)$$

Where, at DFE S^* $= \frac{\eta(\theta+\mu)}{((\mu+\varphi)-\theta\varphi)(\mu+\varepsilon)} = 6666.67$

With the above result, it was discovered that λ_4 It is the largest eigenvalue, which is the reproductive number of people in economic distress. That is,

$$R_0^e = \frac{\delta\eta\,(\theta+\mu)}{[(\mu+\varphi)(\mu+\theta)-\theta\varphi)(\mu+\varepsilon)]} = 2.67 \quad (27)$$

2.6. Parameter Estimation

The parameter estimated values and their respective sources used in the SEIR-FS removal model are displayed in Table 4.

Table 4. Mean estimated parameters of the SEIR-FS removal model

Parameter	Mean Estimated Value	Source(s)
η	0.10	[25]
θ	0.07	[18-19]
μ	0.05	[24]
μ_1	0.13	[26]
β	0.15	Assumed
α	0.12	[23-25]
ϕ	0.50	Assumed
δ	0.51	Assumed
ω	0.13	[24]
ς	0.20	[23]
ζ	0.35	Assumed
ξ	0.13	[22]
π	0.10	Assumed
ε	0.22	Assumed
ρ	0.07	[22]
φ	0.05	[19]
λ	0.06	Assumed

2.7. Sensitivity Indices of Economic Distress Reproductive Number R_0^e

Given that the economic distress reproductive number, R_0^e In its explicit form as stated in equation (27), the analytical expression for its sensitivity to each parameter is obtained by applying the normalized forward sensitivity ([14]) given as:

$$\Upsilon_n^{R_0^e} = \frac{\partial R_0^e}{\partial x} \times \frac{x}{R_0^e}$$

Sensitivity indices R_0^e With regard to the following parameters:

$\eta, \theta, \mu, \delta, \varepsilon, \varphi$ were obtained and computed as follows:

$$\left.\begin{aligned}
\Upsilon_\theta^{R_0^e} &= \frac{\partial R_0^e}{\partial \theta} \times \frac{\theta}{R_0^e} = 0.17 \\
\Upsilon_\mu^{R_0^e} &= \frac{\partial R_0^e}{\partial \mu} \times \frac{\mu}{R_0^e} = -1.06 \\
\Upsilon_\delta^{R_0^e} &= \frac{\partial R_0^e}{\partial \delta} \times \frac{\delta}{R_0^e} = +1.00 \\
\Upsilon_\varepsilon^{R_0^e} &= \frac{\partial R_0^e}{\partial \varepsilon} \times \frac{\varepsilon}{R_0^e} = -0.81 \\
\Upsilon_\varphi^{R_0^e} &= \frac{\partial R_0^e}{\partial \varphi} \times \frac{\varphi}{R_0^e} = -0.29
\end{aligned}\right\} \quad (28)$$

The results given in equation (28) are summarized in Table 5 below, showing the sensitivity indices of the parameters that directly affect the value of the economic distress reproductive number.

Table 5. Sensitivity indices of economic distress, reproductive number R_0^e

Parameter	Description	Sensitivity Indices
η	The rate at which new firms enter the oil and gas sector (economic expansion) is driven by FS removal.	+1.00
θ	The rate at which banks support firms in the oil and gas sector	+0.17
μ	Natural exit rate due to business closure, economic decline, or the death of firms.	-1.06
δ	The rate at which transportation systems are affected by fuel price hikes.	+1.00
ε	The rate at which transportation systems transit into financial distress or closure due to fuel price increases	-0.81
φ	Recovery rate of manufactured goods' consumers, after adaptation measures.	-0.29

In Table 5, the positive signed indices indicate that the reproductive number of people with economic distress increases with an increase in the corresponding parameter. In contrast, the indices with negative values depict a decrease in the economic distress reproductive number when the corresponding parameters increase.

Therefore, R_0^e it tends to a minimal value if the indices with positive signs are reduced and if the indices with negative signs tend to zero. Next, the results of the sensitivity analysis of the varied parameters and their respective effects on the value of the economic distress reproductive number R_0^e are presented in Table 6.

Table 6. The result of the sensitivity analysis of the effect of varied parameters on R_0^e

Parameters	(η, R_0^e)		(θ, R_0^e)		(μ, R_0^e)		(δ, R_0^e)		(ε, R_0^e)		(φ, R_0^e)	
Case 1	0.00	0.00	0.00	1.89	0.00	7.24	0.1	0.52	0.0	14.4	0.05	2.67
Case 2	0.02	0.53	0.02	2.20	0.02	6.83	0.2	1.05	0.2	2.88	0.15	1.68
Case 3	0.04	1.07	0.04	2.43	0.04	3.37	0.3	1.57	0.4	1.60	0.25	1.23
Case 4	0.06	1.70	0.06	2.60	0.06	2.19	0.4	2.09	0.6	1.11	0.35	0.96
Case 5	0.08	2.13	0.08	2.73	0.08	1.59	0.5	2.61	0.8	0.85	0.45	0.80
Case 6	0.10	2.67	0.10	2.83	1.00	0.04	0.6	3.14	1.00	0.69	0.55	0.68

3. Numerical Results and Discussion

The SEIR FS-removal model equation. (1) was solved numerically using the Runge-Kutta scheme of the fourth order given in Matlab, subject to the assumed and mean estimated values of the parameters given in Table 3 and given the following initial parameters: $S_1(0) = 36$, $S_2(0) = 3.1$, $E_1(0) = 60$, $E_2(0) = 50$, $E_3(0) = 50$, $I_1(0) = 71$, $I_2(0) = 50$, $I_3(0) = 0.1$ and $R = 1$ which was estimated based on the percentage output as of May 29, 2023 when the Fuel subsidy was total removed in Nigeria. Figure 3 below shows the dynamics of the variables in the SEIR FS-removal model when the parameters remain constant, as shown in Table 4. The dynamics were found to reduce faster than expected, indicating that the oil and gas sector is more vulnerable to fuel subsidy removal than the banking sector. A faster effect of the FS removal is observed in the $I_1(t)$ sector than in the $I_2(t)$ other $I_3(t)$ sectors.

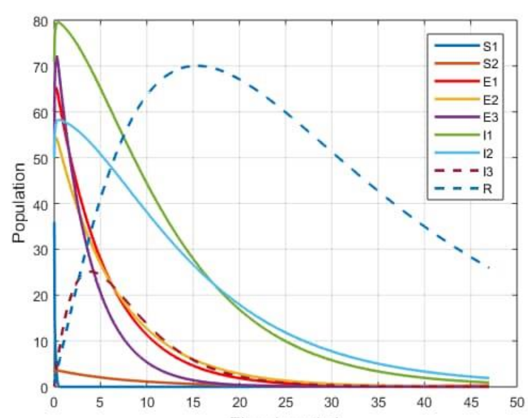

Fig. 3 Plot showing the dynamics of the variables in the SEIR FS-removal model when the parameters remain constant

In this work, more emphasis is placed on the infected and the recovered compartments, particularly $I_1(t)$ $R(t)$ classes; hence, the sensitive parameters are varied in order to ascertain the one that gives better results in terms of reduced number of infected cases as a result of the FS removal and increased number of recovered cases. The following plots show the dynamics of the SEIR FS removal model with varied parameters, specifically δ, ε, and μ

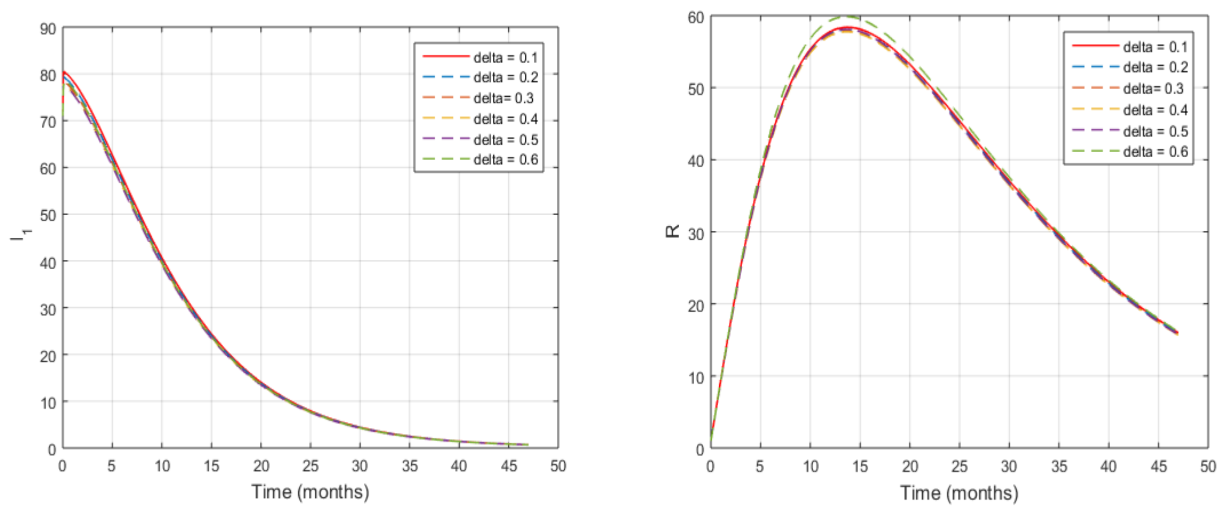

Fig. 4 Simulation of the SEIR FS –removal model with varying effects δ on $I_1(t)$ $R(t)$ firms

In Figure 4, it is observed that the number of households that consume foodstuff which have been affected by hike in fuel price, cannot be significantly reduced by the variability of δ, while a significant slight change will occur to the number of individuals that recovers from the effect of the FS removal based on the effect of the varied parameter δ.

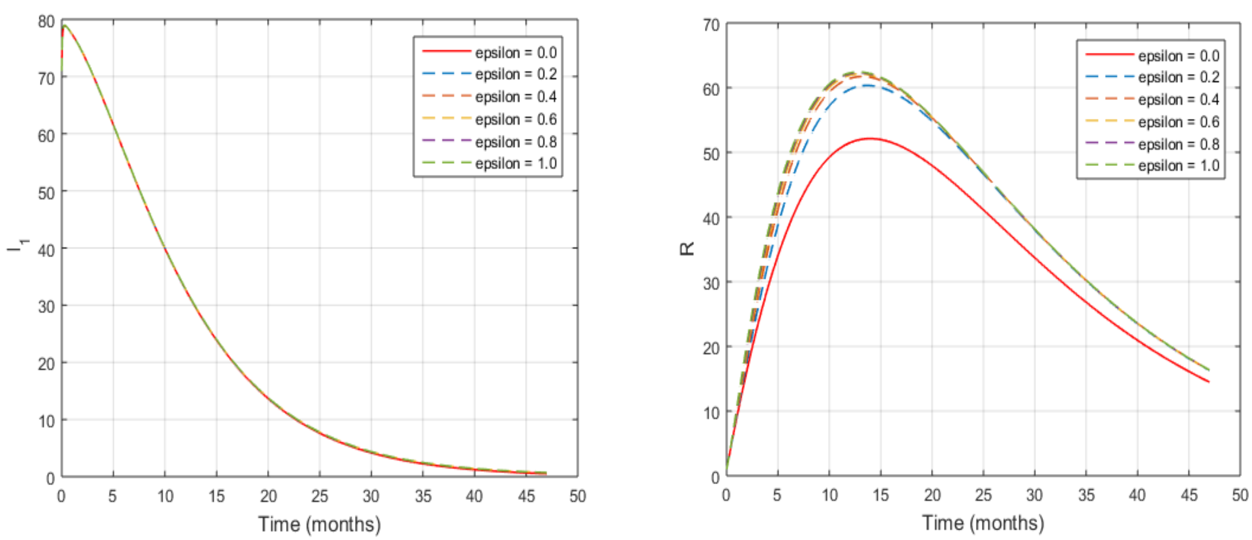

Fig. 5 Simulation of the SEIR FS –removal model with varying effects ε on I_1 R firms

The plots given in Figure 5 depict a non-significant effect of the varied parameter ε on the number of households that consume foodstuffs that have been affected by a hike in fuel price over time. A contrary result is observed in the case of the number of individuals that recover from the effect of the FS removal based on the effect of the varied parameter ε.

Figure 6 presents the plot of the effects of the varied parameter μon the number of households that consume foodstuffs that have been affected by a hike in fuel price over time, and the number of individuals that recover from the effect of the FS removal. A drastic reduction occurs R when μ it is increased.

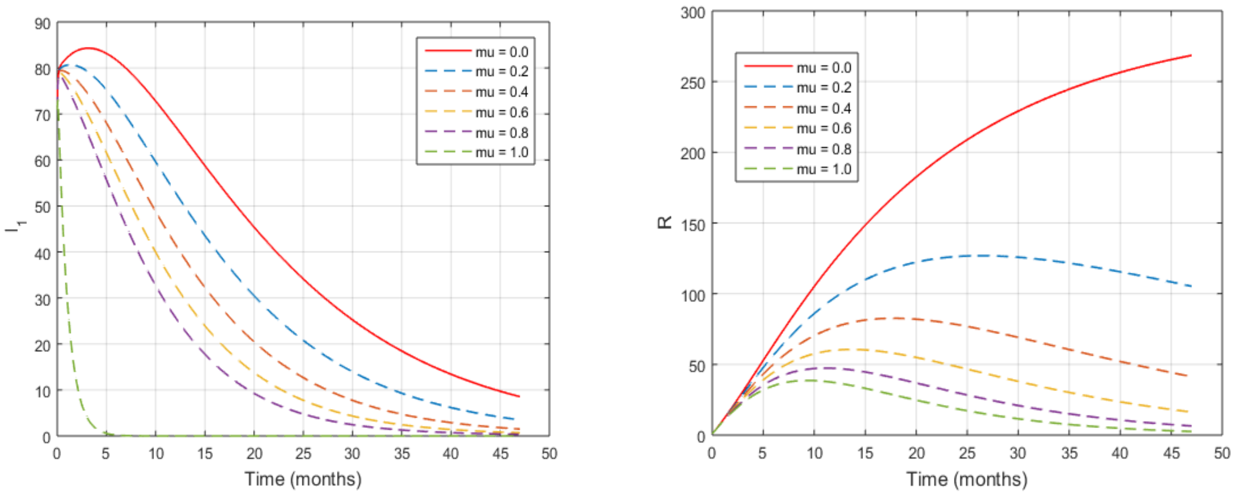

Fig. 6 Simulation of the SEIR FS –removal model with varying effects μ on I_1 R firms

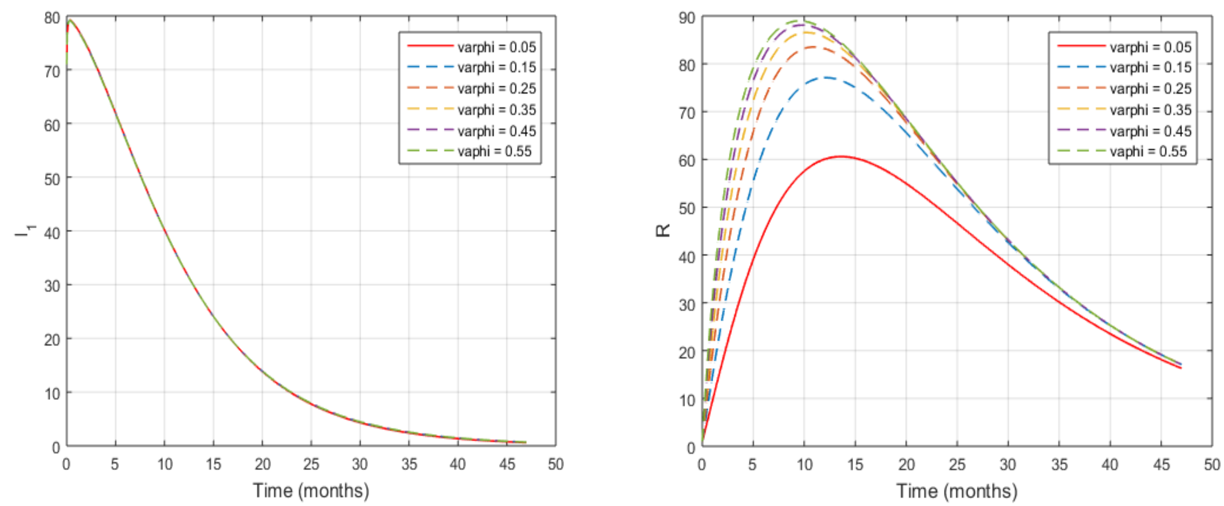

Fig. 7 Simulation of the SEIR FS –removal model with varying effects φ on I_1 R firms

The plots in Figure 7 show that the varied parameter φ does not affect the number of households that consume foodstuffs that have been affected by a hike in fuel price over time, while the effect is vividly seen on the recovered class.

4. Discussion, Conclusion and Recommendation

This section provides the concluding remark and recommendation for further research.

4.1. Discussion and Conclusion

The study has developed a deterministic SEIR FS removal model to mitigate the adverse effects of fuel subsidy removal, based on specific assumptions in the Nigerian context. A system of nine non-linear differential equations was derived from the assumed interaction among the nine sub-compartments that constitute the entire Nigerian economic sectors, within the framework of the SEIR FS removal model. The system of model equations was simultaneously solved at equilibrium to obtain both the fuel subsidy removal effect-free equilibrium and the endemic equilibrium states. To characterize the threshold condition for the eradication of the economic shock induced by FS removal, the Economic Distress reproduction number was analytically derived. This threshold parameter measures the expected number of secondary economically distressed firms across all sectors that arise from a single distressed firm within the oil and gas sector, assuming that firms in this sector are initially and directly susceptible to the fluctuations induced by fuel subsidy removal.

Due to the FS removal, the descriptive statistics of the PMS price monthly sampled data for approximately four years present relatively high values of mean, price range, standard deviation, and sample variance after the FS removal period, as compared to the period before the FS removal. The minimum price of the PMS after the FS removal was found to be 546 naira, while the maximum price during the period was 1262 naira per litre.

The economic distress reproductive number obtained in this work was found to be strictly greater than one, indicating a case of economic distress due to FS removal. A sensitivity analysis carried out on the parameters showed that the reproductive number of people with economic distress can be reduced to the greatest extent if η, θ, δ it is increased.

The numerical solutions of the derived system of equations (1) show a fast and steep increase in the dynamics of the exposed and Infected sectors; this was followed by a slow recovery rate, indicating the effects of the sudden removal of FS. More so, it I_1 was found to be greatly distressed than I_2 I_3. The recovery sector R dynamics showed partial and unstable recovery, indicating structural impacts in the long run.

In conclusion, the FS removal had an immediate effect on all sectors of the Nigerian economy, with varying rates, which has caused destabilisation throughout the entire system. The recovery rates are found not to be uniform, indicating the need for special interventions for the most vulnerable firms in the sector.

4.2. Recommendation

The declining rates $I_1(t)$ in the face of increased values μ do not indicate recovery; instead, they indicate a fast-declining economy. Hence, further research will focus on developing strategies that reduce firm exit by introducing recovery rate parameters to mitigate the devastating effect of μ.

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