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Original Article

Fractional Explicit Iterative Method to Solve Fractional Differential Equations

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Abstract -*This paper presents the Fractional Explicit iterative Method (FEIM), a new numerical technique for solving fractional differential equations of order* $0 < \tau < 1$. *In order to increase accuracy and stability, the method incorporates an explicit iterative correction, augmenting the classical Euler Method (FEM) and the Modified Euler Method (MFEM) fractional type. Numerical experiments show that, particularly for larger* x, *IFEM yields solutions that are closer to the exact values than FEM and MFEM. The effectiveness of the proposed strategy in reducing truncation errors is confirmed by theoretical error analysis, which validates these results.*

Keywords - *Fractional differential equations (FDEs), Fractional Explicit iteration method, Fractional derivative operator, Caputo fractional derivative.*

1. Introduction

Fractional calculus provides a strong mathematical foundation for simulating memory and hereditary characteristics present in physical and engineering systems by extending the traditional concepts of integration and differentiation to arbitrary (real or complex) orders. Because of this, fractional differential equations (FDEs) have been widely used in a variety of domains, including control theory, biomedical sciences, viscoelastic material modelling, and anomalous diffusion processes.

Because FDEs are nonlocal and complex, analytical solutions are frequently impossible, despite their increasing significance. As a result, numerous numerical techniques have been put forth to efficiently approximate solutions. Among the most popular methods are the Homotopy Perturbation Method (HPM) [12], Variational Iteration Method (VIM) [15], Adomian Decomposition Method (ADM) [10], and Homotopy Analysis Method (HAM) [11]. Although these approaches have shown promise in specific problem classes, they are occasionally constrained by problems with convergence, high computational costs, or challenges with nonlinear fractional-order systems. Extending classical schemes into the fractional domain has been the focus of more recent developments.

The discretization of Caputo derivatives for FDEs was first accomplished by the Fractional Euler Method (FEM) [7], which offered a straightforward but efficient method. By expanding on FEM, Batiha et al. [1] developed the Modified Euler Method Fractional type (MFEM) and used it to model the progression of breast cancer in healthcare systems, showing increased computational efficiency and accuracy. Khader [2] further validated the applicability of MFEM in epidemiological studies by using it to solve a fractional smoking model. In order to demonstrate how iterative corrections can improve solution accuracy, Qureshi et al. [3] created and contrasted an explicit iterative algorithm with nonstandard finite difference schemes. This concept was furthered by Meghwar et al. [4], who developed an explicit iterative numerical scheme over the Modified Euler's Method and demonstrated its efficacy using a number of test problems. Although the numerical treatment of FDEs has greatly improved as a result of these contributions, there are still issues, especially when it comes to finding a balance between computational efficiency and accuracy for fractional initial value problems (FIVPs) of order $0 < \tau < 1$. When applied to stiff or highly nonlinear systems, existing techniques, such as MFEM and explicit iterative schemes, may still experience reduced stability or accumulated truncation errors.

The Fractional Explicit Iterative Method (FEIM), a novel numerical technique created to get around these restrictions, is presented in this paper. FEIM improves local accuracy without compromising computational simplicity by using a generalized Taylor series expansion and an explicit iterative correction strategy. In contrast to FEM and MFEM, FEIM improves convergence

characteristics by methodically lowering local truncation errors. An error analysis supports the theoretical underpinnings of FEIM, and comparative numerical experiments validate its performance.
The rest of this paper is structured as follows: The basic definitions and foundational ideas of fractional calculus are described in Section 2. Section 3 provides a theoretical error bound and describes the FEIM formulation in detail. Numerical experiments contrasting FEIM with other approaches, such as MFEM and FEM, under various test conditions are presented in Section 4. A summary of the results and possible avenues for further research are provided at the end of Section 5.

2. Preliminaries

Basic definitions and characteristics that will be used in the following sections are presented in this section.

Consider the fractional differential equations in Caputo sense [9]:

$$D^{\tau} z(x) = h\big(x, z(x)\big), \tag{1}$$

subject to initial condition

$$z(x_0) = z_0, \tag{2}$$

where $0<\tau<1$

Definition 2.1. For a fractional Integral of order $\tau (0 < \tau < 1)$, the Riemann-Liouville integral is defined as:

$$I^{\tau} h(x) = \frac{1}{\Gamma(\tau)} \int_0^x (x-t)^{\tau}\, h(t)\, dt,$$

where a well-defined integral is supplied τ is a complex number where $R(\tau)>0$ and $x>0$ and $h(x)$ are a locally integrable function on $[0, c]$.

Similarly, The Reimann-Liouville fractional derivative of order τ is defined as:

$$D_x^{\tau} h(x) = \frac{1}{\Gamma(n-\tau)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\tau-1} h(t)\, dt,$$

which is called the Reimann-Liouville fractional derivative of order τ where $n-1 < \tau < n$ $(n \in \mathbb{N})$ and $x > 0$.

Definition 2.2. The derivative of $h(x)$ of fractional order in the Caputo sense is defined as:

$$D^{\tau}\, h(x) = \frac{1}{\Gamma(n-\tau)} \int_0^x \frac{h^{(n)}(t)}{(x-t)^{\tau-n+1}} dt,$$

for $n-1 < \tau < n$ $(n \in \mathbb{N})$ and $x > 0$, throughout this paper, consider D^{τ} as a Caputo fractional derivative.

Lemma 1. Assuming that $h \in \mathbb{C}^p[0, c], x > 0$ and $p-1 < \tau < p$ $(p \in \mathbb{N})$, it follows

$$D^{\tau} I^{\tau} h(x) = h(x),$$

and

$$I^{\tau} D^{\tau}\, h(x) = h(x) - \sum_{k=1}^{p-1} \frac{h^{(k)}(0^+)}{k!} x^k.$$

Lemma 2. Suppose that $p, s \in \mathbb{R}$ $(t > 0, s > 0)$ and $\{a_m\}_{m=0}^{k}$ be a sequence such that $a_0 \geq \frac{p}{s}$ and $a_{m+1} \leq (1+s)a_m + p, \forall m \in 0,1,2, \dots, k$. Then

$$a_{m+1} \leq e^{(m+1)s} \left(a_0 + \frac{p}{s}\right) - \frac{p}{s}. \tag{3}$$

Definition 2.3. Mittag –Leffler function is defined by the power series as:

$$E_{\alpha,\beta}(x) = \sum_{j=0}^{\infty} \frac{x^j}{\Gamma(\alpha k + \beta)},$$

where $\mathbb{R}(\alpha) > 0, \mathbb{R}(\beta) > 0$ and $x \in \mathbb{C}$.

Theorem 1. (Generalized Taylor's Expansion formula [7]). Suppose that $D^{k\tau}h(x) \in \mathbb{C}(0,c]$ for $k = 0,1,2,\ldots,n+1$ where $0 < \tau \leq 1$ and $D^{k\tau}$ denotes the Caputo fractional derivative. Then it can expand the function $h(x)$ about the point x_0 is given:

$$h(x) = \sum_{m=0}^{n} \frac{(x - x_0)^{m\tau}}{\Gamma(m\tau + 1)} \tau h(x_0) + \frac{(x - x_0)^{(n+1)\tau}}{\Gamma((n+1)\tau + 1)} D^{(n+1)\tau} h(\xi), \tag{4}$$

with $0 < \xi < x, \forall x \in (0, c]$.

Now for $k = 2$, it has been found

$$h(x) = h(x_0) + \frac{(x - x_0)^{\tau}}{\Gamma(\tau + 1)} D^{\tau} h(x_0) + \frac{(x - x_0)^{2\tau}}{\Gamma(2\tau + 1)} D^{2\tau} h(\xi). \tag{5}$$

3. Fractional Explicit Iterative Method (FEIM)

The objective of this section is to establish a novel numerical scheme to solve the fractional differential equation.

Consider the IVP

$$D(z(x)) = h(x, z(x)), \qquad z(x_0) = z_0. \tag{6}$$

Qureshi et.al.[3] developed an algorithm to solve IVP of the form Equation (6) as:

$$z_{k+1} = z_k + \delta h\left[x_k + \frac{\delta}{2}, z_i + \frac{\delta}{4}\Big(h(x_k, z_k) + h\big(x_k + \delta, z_k + \delta h(x_k, z_k)\big)\Big)\right], \tag{7}$$

Where $x_0, x_1, x_2, \ldots, x_n$ are distinct points on closed interval $[a, c]$, $a = x_0 < x_1 < x_2 < \cdots < x_n = c$ and $x_k - x_{k-1} = \delta \ (\delta > 0) \ \ \forall k = 0,1,2,3 \ldots$.

The Fractional Euler's Method (FEIM), a numerical scheme to solve the fractional initial value problem defined by equation (6), is proposed in this section based on this perspective and Formula equation (7). Consider a uniform partition of the interval $[a, c]$ such that $0 = x_0 < x_1 = x_0 + \delta < x_2 = x_0 + 2\delta < \cdots < x_n = x_0 + n\delta = c$ where the mesh point denoted by x_m, and the step size represented by δ such that $\delta = (c - a)/n$, for $m = 1,2,\ldots,n$. The following expression is obtained by expanding the function $z(x)$ about the point $x = x_m$ employing the initial three terms from the generalized Taylor expansion as stated in Theorem 1.

$$z(x) = z(x_m) + \frac{D^{\tau} z(x_i)}{\Gamma(\tau+1)} (x - x_m)^{\tau} + \frac{D^{2\tau} z(\xi)}{\Gamma(2\tau+1)} (x - x_m)^{2\tau}. \tag{8}$$

Let substitute $x = x_{m+1}$ in Equation (8)

$$z(x_{m+1}) = z(x_m) + \frac{D^{\tau} z(x_m)}{\Gamma(\tau + 1)} (x_{m+1} - x_m)^{\tau} + \frac{D^{2\tau} z(\xi)}{\Gamma(2\tau + 1)} (x_{m+1} - x_m)^{2\tau}, \tag{9}$$

let $x_{m+1} - x_m = \delta$, then

$$z(x_{m+1}) = z(x_m) + \frac{\delta^{\tau}}{\Gamma(\tau+1)} D^{\tau} z(x_m) + \frac{\delta^{2\tau}}{\Gamma(2\tau+1)} D^{2\tau} z(\xi), \tag{10}$$

The following outcome is achieved by combining equations (7) and (10).

$$\begin{aligned} z(x_{m+1}) &= z(x_m) \\ &+ \frac{\delta^{\tau}}{\Gamma(\tau + 1)}\, h\left[x_m + \frac{\delta^{\tau}}{2\Gamma(\tau + 1)}, z(x_m)\right. \\ &\left. + \frac{\delta^{\tau}}{4\Gamma(\tau + 1)}\left(h(x_m, z(x_m)) + h\left(x_m + \frac{\delta^{\tau}}{\Gamma(\tau + 1)}, z(x_m) + \frac{\delta^{\tau}}{\Gamma(\tau + 1)} h(x_m, z(x_m))\right)\right)\right] \\ &+ \frac{\delta^{2\tau}}{\Gamma(2\tau + 1)} D^{2\tau} z(\xi). \end{aligned} \tag{11}$$

The exact solution of equation (6) at mesh point x_m is represented by the value $z(x_m)$, whereas the numerical approximation of the same problem at x_m is represented by n_m as:

$$n_{m+1} = n_m + \frac{\delta^\tau}{\Gamma(\tau+1)} h\left[x_m + \frac{\delta^\tau}{2\Gamma(\tau+1)}, n(x_m) + \frac{\delta^\tau}{4\Gamma(\tau+1)}\left(h(x_m, n(x_m)) + h\left(x_m + \frac{\delta^\tau}{\Gamma(\tau+1)}, n(x_m) + \frac{\delta^\tau}{\Gamma(\tau+1)} h(x_m, n(x_m))\right)\right)\right]. \quad (12)$$

Our new numerical scheme, called FEIM, is shown in Equation (12).

The FEIM Error Bound

The goal is to determine the suggested scheme's error bound, which is shown in equation (12). The following theorem is derived using Lemma 2 in order to prove this result.

Theorem 2. Suppose function h is a continuous real valued function defined on domain $D = [a, c] \times \mathbb{R}$, satisfying Lipchitz condition with constant L $(L > 0)$, i.e.,

$$|h(x, e_1) - h(x, e_2)| \leq L|e_1 - e_2|.$$

Suppose there exist a constat M with

$$|D^{2\tau} z(x)| \leq M \quad \forall x \in [a, c].$$

The expression that results is as follows:

$$|z(x_m) - n_m| \leq \frac{\mu}{\sigma}(e^{\sigma m} - 1), \qquad \forall m = 0,1,2, \dots, n,$$

where

$$\mu = \frac{\delta^{2\tau} M}{\Gamma(2\tau + 1)},$$

and

$$\sigma = \sum_{j=1}^{3} \frac{\delta^{\tau j} L^j}{2^{j-1}\Gamma(\tau+1)^j}.$$

Proof: Equation (11) is deducted from equation (10), which produces the following result to illustrate this point:

$$\begin{aligned} z(x_{m+1}) - n_{m+1} = z(x_m) - n_m &+ \frac{\delta^\tau}{\Gamma(\tau+1)} h\left[x_m + \frac{\delta^\tau}{2\Gamma(\tau+1)}, z(x_m) + \frac{\delta^\tau}{4\Gamma(\tau+1)}\left(h(x_m, z(x_m)) + h\left(x_m + \frac{\delta^\tau}{\Gamma(\tau+1)}, z(x_m) + \frac{\delta^\tau}{\Gamma(\tau+1)} h(x_m, z(x_m))\right)\right)\right] \\ &- \frac{\delta^\tau}{\Gamma(\tau+1)} h\left[x_m + \frac{\delta^\tau}{2\Gamma(\tau+1)}, n_m + \frac{\delta^\tau}{4\Gamma(\tau+1)}\left(h(x_m, n_m) + h\left(x_m + \frac{\delta^\tau}{\Gamma(\tau+1)}, n_m + \frac{\delta^\tau}{\Gamma(\tau+1)} h(x_m, n_m)\right)\right)\right] + \frac{\delta^{2\tau}}{\Gamma(2\tau+1)} D^{2\tau} z(\xi). \end{aligned}$$

The Lipschitz condition is used to get:

$$\begin{aligned}|z(x_{m+1})-n_{m+1}| \\ &\le |z(x_m-n_m)| \\ &+ \frac{\delta^\tau L}{\Gamma(\tau+1)}\left|\left(z(x_m)\right.\right. \\ &+\frac{\delta^\tau}{4\Gamma(\tau+1)}\left(h(x_m,z(x_m))+h\left(x_m+\frac{\delta^\tau}{\Gamma(\tau+1)},z(x_m)+\frac{\delta^\tau}{\Gamma(\tau+1)}h(x_m,z(x_m))\right)\right)\Bigg) \\ &-\left.\left(n_\mathrm{m}+\frac{\delta^\tau}{4\Gamma(\tau+1)}\left(h(x_m,n_\mathrm{m})+h\left(x_m+\frac{\delta^\tau}{\Gamma(\tau+1)},n_m+\frac{\delta^\tau}{\Gamma(\tau+1)}h(x_m,n_m)\right)\right)\right)\right|+\frac{M\delta^{2\tau}}{\Gamma(2\tau+1)},\end{aligned}$$

which results in the inequality that follows:

$$\begin{aligned}|z(x_{m+1})-n_{m+1}| \\ &\le |z(x_m)-n_m|+\frac{L\delta^\tau}{\Gamma(\tau+1)}|z(x_m)-n_m|+\frac{\delta^\tau L}{\Gamma(\tau+1)}\frac{\delta^\tau}{4\Gamma(\tau+1)}\left|h(x_m,z(x_m))-h(x_m,n_m)\right| \\ &+\frac{L\delta^\tau}{\Gamma(\tau+1)}\frac{\delta^\tau}{4\Gamma(\tau+1)}\left|h\left(x_m+\frac{\delta^\tau}{\Gamma(\tau+1)},z(x_m)+\frac{\delta^\tau}{\Gamma(\tau+1)}h(x_m,z(x_m))\right)\right. \\ &-\left.h\left(x_m+\frac{\delta^\tau}{\Gamma(\tau+1)},n_m+\frac{\delta^\tau}{\Gamma(\tau+1)}h(x_m,n_m)\right)\right|+\frac{M\delta^{2\tau}}{\Gamma(2\tau+1)}.\end{aligned}$$

Therefore, the following result is obtained:

$$\begin{aligned}|z(x_{m+1})-n_{m+1}| \\ &\le |z(x_m)-n_m|+\frac{L\delta^\tau}{\Gamma(\tau+1)}|z(x_m)-n_m|+\frac{\delta^\tau L}{\Gamma(\tau+1)}\frac{L\delta^\tau}{4\Gamma(\tau+1)}|z(x_m)-n_m| \\ &+\frac{L\delta^\tau}{\Gamma(\tau+1)}\frac{L\delta^\tau}{4\Gamma(\tau+1)}|z(x_m)-n_m|+\frac{L\delta^\tau}{\Gamma(\tau+1)}\frac{L\delta^\tau}{4\Gamma(\tau+1)}\frac{L\delta^\tau}{\Gamma(\tau+1)}|z(x_m)-n_m|+\frac{M\delta^{2\tau}}{\Gamma(2\tau+1)}.\end{aligned}$$

Thus, it follows as

$$|z(x_{m+1})-n_{m+1}|\le\left(1+\frac{L\delta^\tau}{\Gamma(\tau+1)}+\frac{L^2\delta^{2\tau}}{2\Gamma(\tau+1)^2}+\frac{L^3\delta^{3\tau}}{4\Gamma(\tau+1)^3}\right)|z(x_m)-n_m|+\frac{M\delta^{2\tau}}{\Gamma(2\tau+1)}.$$

That is to say, the following expression is obtained:

$$|z(x_{m+1})-n_{m+1}|\le\left(1+\sum_{j=1}^{3}\frac{L^j\delta^{\tau j}}{2^{j-1}\Gamma(\tau+1)^j}\right)|z(x_m)-n_m|+\frac{M\delta^{2\tau}}{\Gamma(2\tau+1)}.$$

Currently, by letting

$$\mu=\textstyle\sum_{j=1}^{3}\frac{L^j\delta^{\tau j}}{2^{j-1}\Gamma(\tau+1)^j},\qquad \sigma=\frac{M\delta^{2\tau}}{\Gamma(2\tau+1)}\qquad\text{and}\qquad a_m=|z(x_m)-n_m|,$$

it follows that

$$a_{m+1} \leq (1+\mu)a_m + \sigma,$$

for $m = 1,2,3,\dots,k$. Hence, applying Lemma 2 yields the following result:

$$|z(x_{m+1}) - n_{m+1}| \leq e^{(m+1)\mu}\left(a_0 + \frac{\sigma}{\mu}\right) - \frac{\sigma}{\mu},$$

which implies that:

$$|z(x_{m+1}) - n_{m+1}| \leq e^{(m+1)\mu}\left(|z_0 - n_0| + \frac{\sigma}{\mu}\right) - \frac{\sigma}{\mu},$$

however, since $|z_0 - n_0| = 0$, it follows that

$$|z(x_{m+1}) - n_{m+1}| \leq e^{(m+1)\mu}\ \frac{\sigma}{\mu} - \frac{\sigma}{\mu},$$

which provides:

$$|z(x_{m+1}) - n_{m+1}| \ \leq \frac{\sigma}{\mu}\left(e^{(m+1)\mu} - 1\right).$$

This completes the proof for $m = 1,2,3,\dots,k$.

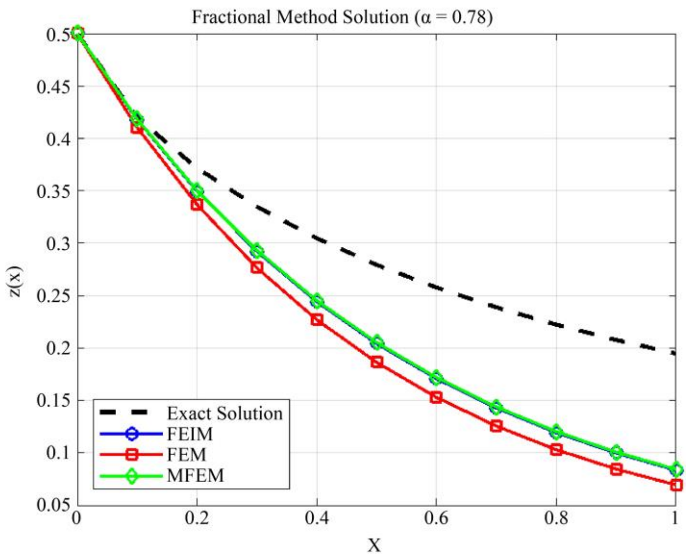
4. Results and Discussion

Consider the FIVP as follows [7]

$$\mathrm{D}^{\alpha}\mathrm{z}(\mathrm{x}) = -\mathrm{z}(\mathrm{x}), \qquad \mathrm{z}(0) = 1,$$

Where D^{α} denotes the Caputo fractional derivative, $0 < \alpha \leq 1$ and $x > 0$. Note that $z(x) = E_{\alpha,1}(-x^{\alpha})$ is the exact solution to the a forementioned problem. To solve this issue, however, formula (12) is used. With $\delta = 0.1$ and $\alpha = 0.78$, Figure 1 shows a numerical comparison of the solutions found for the given problem using FEIM, MFEM, and FEM.

x	Exact	FEIM	FEM	MFEM
0.0	0.50000	0.50000	0.50000	0.50000
0.1	0.41957	0.41804	0.41041	0.41844
0.2	0.37172	0.34951	0.33688	0.35018
0.3	0.33480	0.29221	0.27651	0.29306
0.4	0.30468	0.24431	0.22697	0.24525
0.5	0.27936	0.20426	0.18630	0.20525
0.6	0.25767	0.17078	0.15292	0.17176
0.7	0.23884	0.14278	0.12552	0.14375
0.8	0.22232	0.11938	0.10303	0.12030
0.9	0.20770	0.09981	0.08457	0.10067
1	0.19468	0.08345	0.06942	0.08425



5. Conclusion

The numerical results for $\alpha = 0.78$ indicate that all methods approximate the exact solution well at initial steps, but differences grow as x increases. The Euler Method Fractional type (FEM) underestimates the solution, while the Modified Fractional Euler Method (MFEM) provides improved accuracy. The Iterative Fractional Explicit Method (IFEM) demonstrates the best performance, closely matching the exact solution across the interval. At $x = 1$, IFEM achieves a smaller error compared to MFEM and FEM, validating its superior convergence and stability as predicted by the theoretical error bounds.

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