Original Article

The Riemann Hypotheses: A Lesson in Universal Paradoxes that Lead to the Axiom of Choice – A Path of **Unexpected Outcomes**

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Abstract - The Riemann Hypothesis, in the article "A view on how to solve it", challenges the schema of Russell's Paradoxes through its result. This paper presents, in three segments, how the paradoxes can be viewed. In each segment, paradoxes are introduced to force a deeper look into the area of discourse. This main issue embraces the entire essay as it progresses to different aspects of this work. The first segment outlines the original naïve set concept and schema. The second segment submits the naïve set in the context of functional reasoning, which is a derivation of common sense and reason. Common sense will show that the empty set is irrelevant when functional reasoning, in its core derivation, is considered. Core derivation imposes the Gödel arguments for the existence of God. Thus, these sets of axioms and theorems strengthen the axiom of choice set theory and expose mathematics as the transport mechanism to touch upon the state of consciousness in its understanding of the universe as an evolving entity. Furthermore, the eloquent design of the universe in the quantum sphere of interaction is partly intrinsic to internal awareness of the self. Notwithstanding, the external makeup of human beings is influenced deeply by the classical knowledge of the world. Finally, the third segment, the functional reasoning, introduces Artificial Intellig ence and sets the stage for the Peano Axiom-Theory. The latter sets the stage for limits and quantifiers, which will open the door for omega, the recursive schema associated with a limit point.

Keywords - Riemann Hypothesis, Quantum, Artificial Intelligence, Universe, Axiom.

1. Introduction

This essay revolves around four main principles: Functional Reasoning, Artificial Intelligence, the existence of God, and the universe with its modus operands, the consciousness. The first issue introduces common sense and reasoning in the analysis of the result in its generic form. The second issue delineates the Riemann Hypothesis through a constructive process followed by a detachment principle. The second principle concentrates on the concept of knowledge and information. The third issue introduces the Gödel existence of God arguments, which permeate the essay and expand the views of the universe concept. The fourth issue explains the mechanics of consciousness and its interaction with the universe.

The significance of this work lies in the intuitive arguments exposed in this work. The latter can be framed into constructive principles that are laced with historical works, which can be the stepping stone for the deductive argument of a proof that addresses the core needs of the Riemann Hypothesis, as well as some of our defined unsolved problems that we must address in the future. Absence of a logistic argument will limit our ability to satisfy the solution sets that will eventually be found. The culprit in all of that is the native sets for now. Finally, every time a new problem is attempted to be solved, the core knowledge base may need to be revisited in order to learn and adjust to the new truths that will surface.

The following section revisits the knowledge base surrounding the logistic framework of this work by defining the schema and the naïve set proposal of Russell's Paradox.

2. Defining a Naïve Set

A naïve set is a collection of objects that are without restriction. The initial rules of naïve sets are called the schema. The order of elements is immaterial. $\{1,2\}$, $\{2,1\}$. Repetition of elements is irrelevant, i.e., $\{1,2,2\} = \{1,1,1,2\} = \{1,2\}$.



Set builder notation or set comprehension, particularly in the context of functional programming $\{x \in A/P(x)\}$, denotes the set of all x values that are already members of A, such that the condition P holds for x. The axiom of specification $\{F(x) \in A\}$ denotes the set of all objects obtained by putting members of the set A into the formula F. The axiom of replacement F(x)/P(x)x's owner/x/x is a dog) is a set of all dog owners.

2. Naïve Set Schema

The naïve set could be conceived objectively or subjectively. Given the nature of these sets, a Paradox is bound to emerge. However, since this is a scientific world, the collection of sets is required to be purely objective.

Objective sets must be constructed through an axiom that delineates the truthfulness of well-defined ideas that are accepted universally through a consensus. Subjective sets imply no consensus. Mathematics is defined by a single axiom – life itself. Being in the presence of reality and the changing world, mathematics will respond to its needs and its rigor.

Furthermore, Naïve Set Theory introduces the Empty Set concept and the self-referential sets, which form the basis of the Russel Paradox. However, as the Riemann Hypotheses are introduced, potential issues are encountered. When dealing with an objective set, the rules cannot be arbitrarily imagined subjectively. The latter must be constructive; thus, the following becomes essential: First, through a Venn Diagram, it is known that an expression defines the empty set. This implies the expression (A \cap B) \cap (A \cup B) is the empty set. If the formula ((S(A)+S(B)-S(A \cap B)) \cap S(A \cap B) is applied, the result is set (A \cap B), the empty set graphically. To see this, it must be assumed that the empty set is not empty. Thus, through an illustration, the following can be seen:

$$U = \{1,2,3,4\}; A = \{2,3\}; B = \{3,4\}$$

It can be seen that $S(A \cap B) = 3$; $S(A \cup B) = \{2,3,4\}$. Now, apply the expression $S(A \cap B) \cap (A \cup B) = 3$. Since the intersection is empty, 3 can be replaced by \emptyset . Furthermore, it can be illustrated through the Venn diagram the reason why the empty set is an element of all the sets. This means that 3 can simply be replaced in all the set enumerations with the \emptyset set, and it will illustrate this fact. Replacing 3 with \emptyset :

U=
$$\{1,2, \emptyset,4\}$$
; S (A \cap B $\} = \emptyset$;
A= $\{2, \emptyset\}$; S (A \cup B) = $\{2, \emptyset,4\}$.

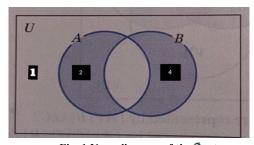


Fig. 1 Venn diagram of the Ø set

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B = \{\emptyset,4\}; S(A \cap B) \cap (A \cup B) = \emptyset(A \cap B) \cap (A \cup B)
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Another common issue is to represent the set in the Venn diagram to illustrate some properties of equivalences.

De Morgan's laws:

If there are two states, A and B, the number of outcomes is 4. Thus, if p=A and q=B, then the following equivalence occurs:

$$\sim (p \cap q) \equiv \sim p \cup \sim q$$

 $\sim (p \cup q) \equiv \sim p \cap q$

The truth table is the format that will be used to test the validity of the equivalence of an argument. This is done in the arena of first-order logic. For the first order logic to occur, the statement must be defined to represent one idea that could be evaluated as true or false. Under this scenario, all these tables and diagrams can be built to extrapolate their consequences. First-order logic overview:

Name	Symbol form	English Translation.
Conditional	$p \rightarrow q$	if p then q
Converse	$q \rightarrow p$	if q then p
Inverse	~p→~q	if not p then not q
Contrapositive	~q > ~p	if not q then not p

Here, the structure of a first-order argument can proceed before embarking on the main concern of this essay. The form of the argument $P_1 \cap P_2 \dots \rightarrow C$. These stand for premise₁ and premise₂ ...and lead to C. These set the stage for an argument analysis:

Valid Arguments				
Direct Reasoning	Contrapositive Reasoning	Disjunctive Reasoning	Transitive Reasoning	
$\begin{array}{c} p \to q \\ p \\ \therefore q \end{array}$	$ \begin{array}{c} p \to q \\ \sim q \\ \vdots \sim p \end{array} $	$ \begin{array}{ccc} p \lor q & p \lor q \\ & \sim p & \sim q \\ & \therefore q & \therefore p \end{array} $	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$ $\therefore \sim r \rightarrow$	
Invalid Argumen	ts			
Fallacy of the Converse	Fallacy of the Inverse	Misuse of Disjunctive Reasoning	Misuse of Transitive Reasoning	
p → q <u>q</u> ∴ p	$ \begin{array}{c} p \to q \\ \stackrel{\sim}{\sim} p \\ \vdots \sim q \end{array} $	$\begin{array}{ccc} p \lor q & p \lor q \\ \underline{p} & \underline{q} \\ \vdots \sim q & \vdots \sim p \end{array}.$	$p \rightarrow q$ $q \rightarrow r$ $\therefore r \rightarrow p$ $\therefore \sim p \rightarrow q$	

Fig. 2 Arguments and Analyses

These arguments are handled with truth tables – At least 3 states or more are preferred. A, B, C mean simply p,q,r. These set the stage for argument analysis by truth table.

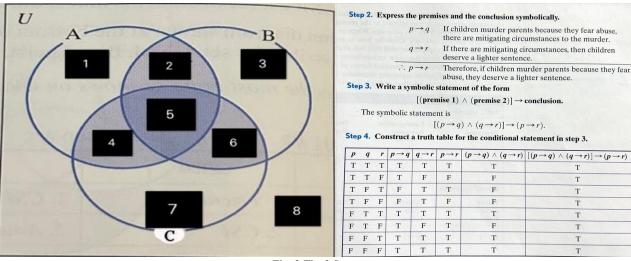


Fig. 3 The 3 States

3. Useful Quantifier Expressions

A quantifier is an operator that specifies how many individuals in the domain of discourse satisfy an open formula. For instance, the universal quantifier \forall in the first order formula $\forall_x P(x)$ expresses everything in the domain and satisfies the property denoted by P. On the other hand, the existential quantifier \exists in the formula $\exists_x P(x)$) expresses that something in the domain satisfies that property.

A formula where a quantifier takes a wider scope is called a Quantifier Formula. A Quantifier Formula must contain a bound variable and a subformula specifying a property of the reference of that variable.

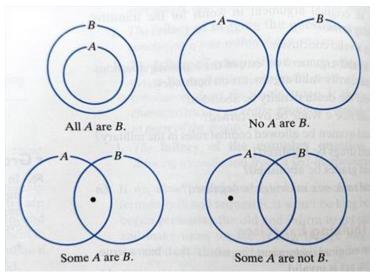


Fig. 4 Quantifying Diagram Examples

Knowing this, and going back to the original concern, the *area of disclosure* can be presented. Functionally, the construction of the empty set violates the natural senses; it is an illusion, for the latter becomes a schema that is defined and added conceptually. Second, the self-reference set below can be replaced by an artificial intelligence set.

4. Further Illustrating Aspects of Russel Bertrand's Paradox

Y $\{X/X \notin X\}$, i.e., the set of all sets that do not contain themselves as elements, is the core concept of Russell's Paradox in 1902. If the axiom schema of unrestricted comprehension is weakened to the axiom schema of specification, it can only create a subset of an existing set based on a specific property. However, if P is a property, then for any set X, there exists a set Y $\{x \in X: P(x), and all the above paradoxes disappear.$

The next step is to situate the concept of unrestricted comprehension and restrictive comprehension. Due to the nature of the naïve set, sets are usually envisioned suggestively; however, this might open a Pandora's box of Paradoxes. This could be avoided through the specification schema. Before this is discussed, however, it must be known that this issue is deeper. First, the stage must be set for philosophical functional reasoning.

The concept of the empty set is the issue in our narration that evolves the most due to its sensitive nature. Axiomatic conception becomes a nonstarter since the latter exposed the concept of Vacuous truth. By this, the following is meant:

In mathematics and logic, a vacuous truth is a true conditional statement because the antecedent cannot be satisfied. It is sometimes said that a statement is vacuously true because it does not really say anything.

5. Common Sense Ideas About Sets and Reasoning

The set of all sets does not exist. This concept of a universal set will be elaborated on later in the context of functional reasoning because it contains itself. Something must create nothing. If the empty set is the nothing set, then it cannot be created. This is in the concept of interpretation. The set will be analyzed further in this essay; however, an essential concern must be tackled now.

If God is present in every conceivable time and space, then today is an extension of yesterday and tomorrow. Humans are only living in the present. The night is a simple boundary of the day, and during sleep, dreams are just echoes of the day. This is significant as a value is assigned to it in relation to the present state. Reasoning is simply truth talking about a particular state of being. This falls into the context of what reality is being confronted at a specific moment.

These sets of statements are analytic philosophy characterized by clarity of prose and rigor in arguments, using formal logic and mathematics, and to a lesser degree, natural science. It also takes things piecemeal, to focus philosophical reflection on small problems that lead to the answers of bigger questions. Due to the elusive nature of artificial intelligence, morality must be concerned with two paradigms of realities:

The first is global; do not introduce hard to the universe. The second is to do for others what you will do for yourself. These are well-known teachings. In this season of discomfort, it would do well to adhere to these principles.

To reiterate and delineate some truth about sets:

The set of all sets does not exist. There is no universe. Suppose proof exists and call it U. Now apply the axiom schema of separation with X=U and for P(x), use $x \notin x$. This leads to Russell's paradox again. Hence, U cannot exist in this theory.

$$Y=\{x/(x \notin x) \rightarrow \{\} \notin \{\}\}$$

Where the statement following the empty set is false.

It follows from the definition of Y, using the usual inference rules.

The issue we seldom acknowledge is the existence of God.

Does God exist following the Big Bang fiasco? One cannot start with nothing. Who created nothing? The only conclusive response is God. Nothing else fits this axiom. However, instead of stating facts, further elaboration is necessary. If God is present in every conceivable time and space, then today is an extension of yesterday and tomorrow. One can only live in the present. Using dimensional analysis, a set of axioms is presented to delineate the existence of God as a prime entity. These axioms, theorems, and definitions are attributed to Gödel, the father of modern logic.

6. Artificial Intelligence Consists of Two Ideas: Information and Knowledge

Information can be viewed as a sequence, and knowledge can be viewed as a limit; thus, a correlation can be formed to delineate this relation, where a decision can occur. This sets a dynamic that can be refined and becomes the root of another sequence of information. This introduces the world of functional logic and artificial intelligence into the equation.

Functional logic adds common sense and reasoning to the process, and artificial intelligence adds the concept of sequence and limits to the conversation, as discussed earlier. Given these new concepts, common sense parameters are established in the Riemann Hypotheses for the purpose of constructing numbers through the process of detachment. This allows the formation of extension fields that will lead to the zeta zero location for each specific number under study. A known limit point was introduced as part of this inherent process. These concepts open the way to institutional logic, which is the bedrock of the constructive logic of programing. Finding the zeta location was an elusive search. Identifying the exact number that must be attached to the zeta location extension was a challenge. Hence, for the Riemann Hypotheses, this was a departure from the original way a limit is taken from a function. In fact, what has been done is called *Completion*.

7. Completion

For any metric space M, it is possible to construct a complete metric space M' that contains M as a dense subspace. It has the following universal property: If N is any metric space, and f is any continuous function from M to N, then there exists a unique uniformly continuous function f' from M' to N that extends f. This property (among all metric spaces isometrically containing M) determines the space M' up to isometry and is called the Completion of M.

The Completion of M can be constructed as a set of equivalent classes of Cauchy sequences in M. For any two Cauchy sequences $x_c = (x_n)$ and $y_c = (y_n)$ in M, the distance may be defined as:

$$d(x_c, y_c) = \lim_n d(x_{n, '}y_n)$$

The original space is embedded in this space via the identification of an element x of M' with equivalence classes of sequences M converging to x.

Calling the zeta numbers a Cauchy sequence requires further analysis. Indeed, more can be said; the latter is a subset of the real number dense and closed in the metric space. A natural mapping of the integer field exists. Meaning an extension field f. Going forward, these zeta numbers will be treated as a subset of the real number set and proceed accordingly.

Zeta Numbers: A Subset of the Real
Key factors of this essay
All zeta numbers are real numbers.
An elementary description is provided in the overview.

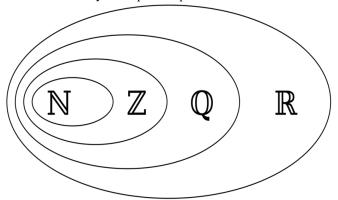


Fig. 5 Quantified Diagram of the Number System

Definitions

A Cauchy sequence: A sequence $x_1, x_2, x_3...$ in a metric space (X, d) is called Cauchy if, for every positive real number r>0, there exists a positive integer m, n > N, d ($x_m, x_n > r$) < r

Complete space: A metric space (X, d) is complete if any of the following equivalent conditions are satisfied:

- Every Cauchy sequence of points in X has an X limit.
- Every Cauchy sequence in X converges in X (to some point of X).
- Every decreasing sequence of non-empty closed subsets of X, with diameters tending to 0, has a non-empty intersection: if F_n is closed and non-empty, $F_{n+1} \subseteq F_n$ for every n, and diam $(F_n) \to 0$, then there is a point $x \in X$ common to all sets F_n .

This is the method by which the zeta location is extended to its real counterpart. The latter, due to its unique structure, forces us to conclude that the concept of the empty set could not be constructed in the same manner as with natural numbers. This means that the empty set could be defined, but not constructed. Using part of the article published earlier this year, the Torus Reality is introduced next. A look back at the results will facilitate comprehension of the different concepts of higher logic used to bring about a solution to the Riemann Hypotheses problem. All conceivable concerns need to be addressed forcefully.

8. The Calculation: The Torus Reality

A torus is defined as two non-intersecting circles embedded in the same plane. Since a torus is a set of parallel circles that are not in the same plane, they are, by structure, looking at concentric circles. Thus, the last circle needs to be reduced to a point. This can be done using Cauchy's Integral Theorem of Imaginary Fields. The π result after integration is 2. Now, using the latter as a limit point, the outer circle can be reduced to establish the area of Cauchy's expectation. In doing so, the delineate can be found, i.e., the last prime in the knapsack volume that was searched for.

A function f(z) has a period C if and only if: F(z+C)=f(z) for all z. In the complex plane, it is fine to consider a value for C.

$$e^{2\pi i} = 1_{\text{by IF}}$$

$$_{\rho}z+2\pi i_{=\rho}z_{\rho}2\pi i_{=\rho}z$$

So, the exponent function is periodic with imaginary period $C=2\pi i$. The method used is analogous to dealing with an angle greater than 360 degrees, by dividing by 360° to make the domain smaller by imposing a limit 1-L into the domain of the $\ln(x)$. The working assumption is that the zeta location and the x value, counting the number of zeros, are close. To delineate this reality, the formula is presented here. However, examples can be found in the original article Riemann Hypotheses: A Vision on How to Solve It."

Linear πk .. n Vs e^L where L or 1-L is the neighborhood of Υ "the zeta zero" for a specific x under study.

We can say more, expanding the left side of Euler's product, we get:

Linear πk .. n Vs e^{L} , where L or 1-L is the neighborhood of Υ "the zeta zero" for a specific x under study

We can say more, expanding the left side of the Euler product, we get

$$\frac{1}{1-Y}\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n} s = \zeta(s) = \prod_{p} (1 - \frac{1}{p})^{-1}$$

For a specific x

$$\frac{D}{1-Y}\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n} s = \zeta(s) = \prod_{p} (1 - \frac{1}{p})^{-1}$$

Positioning D closer to the center by reduction

$$\frac{D}{\frac{1-Y}{2\pi k}}\zeta(s) = \sum_{n} 1/n^{s} = \zeta(s) = \prod_{p} (1-\frac{1}{p})^{r}$$

Making a change of base and Taylor approximation to get the final high of L, i.e., the saddle point

$$\frac{D}{\frac{1-Y}{pL}}\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n} s = \zeta(s) = \prod_{p=1}^{\infty} (1 - \frac{1}{p})^{-1}$$

A complete Excel table is available at the following location on OneDrive: https://bit.ly/3F2wap9

Here, there is the structure of a torus not centered at its origin. The effect of applying multiples of $1\pi,2\pi$, $k\pi$ reduces the outer layer of a torus structure to the limit of the knapsack volume – only if a base change is performed to e. Under this scenario, the desired limit will be reached for any input value of x under study. This implies an embedded torus. If L is the center of the torus, 1-L is the neighborhood of L. This geometric series is related to the zeta location, i.e., 1-L.

9. John Nash, the Father of Embedded Geometry

John Nash shows that the distance in a dimensional plane is equivalent to any embedded sphere of an n-dimensional plane or higher. Surely enough, a torus was one of the classical ways to prove it. The PAC-MAN game outlines the complexity of the problem since it is a visual paradox.

10. Introducing the Peano Axiom-Theory

To address artificial intelligence fully, the Peano axiom theory must be introduced, which will lead to omega, the recursive schema. In mathematical logic, the piano axioms are axioms for natural numbers.

The first axiom states:

- 1. 0 is a natural number.
- 2. For every natural number x, x=x. That is, equality is reflexive.
- 3. For all natural numbers x and y, if x=y, then y=x. That is, equality is symmetrical.
- 4. For all natural numbers x, y, and z. That is, equality is transitive.

5. For all a and b, if b is a natural number and a=b, then a is also a natural number. That is, natural numbers are closed under equality.

The remaining axioms define the arithmetic properties of the natural numbers. The latter are closed under S. The successor function S:

- 6. For every natural number n, S(n) is a natural number. That is, the natural numbers are closed under S.
- 7. For all natural numbers m and n, if S(m)=S(n), then m=n. That is, S is an injection.
- 8. For every natural number n, S(n)=0 is false. That is, there is no natural number whose successor is 0. The intuitive notion that each natural number can be obtained by its successor is called induction.
- 9. If k is a set such that:
 - 0 is in k, and for every natural number n, being in k implies that S(n) is in k; then k contains all natural numbers.

The same axioms can be expressed with predicates. Then, the hierarchy of arithmetic can be presented. The latter will specifically introduce recursion theory (i.e., an algorithm).

The Tarski-Kuratowski algorithm for the Arithmetic hierarchy consists of the following steps:

- 1. Convert the formula to prenex Normal form. (This is the non-deterministic part of the algorithm, as there may be more than one valid prenex normal form for the given formula.)
- 2. If the formula is quantifier-free, it is in $\sum_{i=1}^{n} \prod_{j=1}^{n}$
- 3. Otherwise, count the number of alternations of the quantifiers; call this k.
- 4. If the first quantifier is \exists , the formula is in $\sum_{k=1}^{0}$
- 5. If the first quantifier is \forall , the formula is in $\prod_{k=1}^0$

Convergence sequence and limit are some of the natural applications that establish the recursion principle.

11. Relativized Arithmetic Hierarchies

Just as it can define what it means for a set X to be recursive relative to another set Y by allowing the computation defining X to consult Y as an oracle, this notion can be extended to the whole arithmetic hierarchy and define what it means for X to be Σ_0^0 , Π_o^0 : Δ_0^0 in Y, denoted respectively: $\Sigma_0^{0,y}$, $\Pi_o^{0,y}$, $\Delta_0^{0,y}$. To do so, fix a set of natural numbers Y and add a predicate for membership of Y to the language of Peano arithmetic. It can then be said that X is in $\Sigma_0^{0,y}$, if it is defined by Σ_n^0 , a formula in this expanded language.

According to Gottlob Frege, the meaning of a predicate is exactly a function from the domain of objects to the truth-values "true" and "false"; it can be viewed as a relation or interpretation R (a, b).

Consequently, artificial intelligence can be viewed using common sense as a sequence and limit with an added recursive nature (limit is quantifier-free convergence to a known or unknown value). This is why an oracle must manage the database expectations or set control. This is where the "caring" takes place in the arena of artificial intelligence (this takes the form of monitoring). Finally, the section in the Riemann Hypotheses, "A Vision on How to Solve It," is the segment where the number is treated as an Egyptian numeral and, through the detachment principle, arrives at the zeta location of a given number under study. This is the main reason why the concept of the empty set and its derivation are questioned.

Furthermore, the natural correspondence relation that the latter plays as the center of the derived formula leads to the complementary solution of the prime conjecture of $n \approx x/\ln(x)$: the number of primes up to a given value of x. In addition, the saddle point is only achieved through artificial intelligence, which refines the limit point so that the calculation of the location of the last prime can be found.

The latter is simply the error term of the Lagrange error calculation. In doing so, the factorization of numbers into their prime elements can be appreciated further, as well as identifying whether a given number is prime or not. Now, using the artificial intelligence treatment in the arena of the Trisecting of Angles article, the versatile nature of the latter can be demonstrated. So far, the identity of the actual angle that will zero out the polynomial of $\cos (3\Theta)$ has been found, instead of a close estimate. Venturing further, a pattern could have been derived to show the strength of artificial intelligence, if and only if the latter is used with care. In the absence of genuine care, people run the risk of hurting themselves.

12. Conclusion

The logical framework established by the lesson of the Riemann hypotheses allows the detachment principle to be the transport mechanism to explore the symmetrical nature of the golden ratio. A gauge point can be found that can put to rest the error term associated with the Lagrange Method. This is where the recursive nature of the data-driven algorithm of Artificial Intelligence (A.I), could be useful. This is because the information lies only on the data itself. This does not withstand the universal components and paradoxes that will be associated with the result.

Finally, this article illustrates the beauty and truth of mathematics. For the studies of prime numbers play a critical role in the understanding of the arithmetic system. During this adventure, liberties were taken to address some of the concerns about human beings as well as the universe. This essay did not shy away from controversial issues such as the unification paradigm of the universe or the existence of God. Considering these truths, this essay ends by borrowing from the wisdom of an elder.

Science without religion is lame.
Religion without science is blind.
Albert Einstein

References

- [1] Alfred V. Aho et al., Data Structures and Algorithms, 1983. [Google Scholar] [Publisher Link]
- [2] John C. Amazigo, Advanced Calculus and Its Applications to the Engineering and Physical Sciences, Wiley, 1st Edition, 1980. [Google Scholar] [Publisher Link]
- [3] Frank Ayres, and Elliot Mendelson, Schaum's Outline of Calculus (Shaum's Outlines), McGraw-Hill Education Europe, 2000. [Publisher Link]
- [4] Thomas Becker, and Volker Weispfenning, "Gröbner Bases A Computational Approach to Commutative Algebra," *Graduate Texts in Mathematics*, vol. 141, 1993. [CrossRef] [Google Scholar] [Publisher Link]
- [5] Thomas Becker, Volker Weispfenning, and Heinz Kredel, *Grobner Bases A Computational Approach to Commutative Algebra*, Springer, Corrected Edition, 1993. [Publisher Link]
- [6] Robert Blitzer, *Thinking Mathematically*, 4th Edition, 2018. [Google Scholar]
- [7] Folkmar Bornemann, "PRIMES is in P: Breakthrough for Everyman," Notices of the American Mathematical Society, pp. 545-552, 2003. [Google Scholar] [Publisher Link]
- [8] Arne Broman, Introduction to Partial Differential Equations from Fourier Series to Boundary-Value Problems, 2012. [Google Scholar] [Publisher Link]
- [9] Samuel Buss, Bounded Arithmetic, Department of Mathematics, University of California, Berkley, 1997. [Google Scholar]
- [10] P.J. Cameron, and J.H. Van Lint, Designs, Graphs, Codes, and their Links, Cambridge, Great Britain, 1991. [Google Scholar] [Publisher Link]
- [11] Georg Cantor, Contributions to the Founding of the Theory of Transfinite Numbers, Dover Publications, 1955. [Google Scholar] [Publisher Link]
- [12] Judith N. Cederberg, A Course in Modern Geometries, Second Edition, 2001. [Google Scholar] [Publisher Link]
- [13] Harvey Cohn, Advanced Number Theory, Dover Publications, 1980. [Google Scholar] [Publisher Link]
- [14] Harvey Cohn, Conformal Mapping on Riemann Surfaces, Dover Publications, 1967. [Google Scholar] [Publisher Link]
- [15] Richard E. Crandall, Projects in Scientific Computation, Springer-Verlag New York, 1994. [Google Scholar] [Publisher Link]
- [16] Nell B. Dale, and Chip Weems, Introduction to PASCAL and Structured Design, D.C. Heath, 1983. [Google Scholar] [Publisher Link]
- [17] Rene Descartes, The Geometry of Rene Descartes, David Eugene Smith, 1925. [Google Scholar] [Publisher Link]
- [18] John Derbyshire, *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics*, Plume, 2004. [Google Scholar] [Publisher Link]
- [19] Reinhard B. Diestel, *Graph Decompositions A Study in Infinite Graph Theory*, Clarendon Press, 1990. [CrossRef] [Google Scholar] [Publisher Link]
- [20] A. Evyatar, and P. Rosenbloom, Motivated Mathematics, Cambridge University Press, 1981. [Google Scholar] [Publisher Link]
- [21] Paul C. Evyatar, and Paul Rosenbloom, Motivated Mathematics, CUP Archive, 1981. [Google Scholar] [Publisher Link]
- [22] Gill Williamson, Combinatorics for Computer Science, 1985. [Publisher Link]
- [23] Frisk Gustafson, Elementary Geometry, Third Edition, 1991. [Google Scholar] [Publisher Link]
- [24] David W. Henderson, and Daina Taimina, Experiencing Geometry, Third Edition, 2005. [Google Scholar]
- [25] Klaus Jänich, Vector Analysis, Springer, 2000. [CrossRef] [Google Scholar] [Publisher Link]
- [26] John W. Keese, Elementary Abstract Algebra, DC Heath/Order HM College, 1965. [Google Scholar] [Publisher Link]
- [27] Donald E. Knuth, The Art of Computer Programming, Fundamental Algorithms, 2nd Edition, Addison-Wesley, 1973. [Google Scholar]

- [28] Neal Koblitz, A Course in Number Theory and Cryptography, Springer-Verlag, Second Edition, 1994. [Google Scholar] [Publisher Link]
- [29] Neal Koblitz, A Course in Number Theory and Cryptography, Springer, 1994. [Google Scholar] [Publisher Link]
- [30] Bernard Kolman, and Robert C. Busby, Discrete Mathematical Structures for Computer Sciences, 1984. [Google Scholar]
- [31] Roland E. Larson, Robert P. Hostetler, Bruce H. Edwards, Calculus, Sixth Edition, Boston, New York, 1998. [Publisher Link]
- [32] Jean Louis Lassez, and Gordon Plotkin, Computational Logic: Essays in Honor of Alan Ronbinson, The MIT Press, 1991. [Google Scholar] [Publisher Link]
- [33] Harry R. Lewis, and Christos H. Papadimitriou, *Element of the Theory of Computation*, Prentice-Hall, 2nd Edition, 1997. [Google Scholar] [Publisher Link]
- [34] Melvin J. Maron, and Robert J. Lopez, *Numerical Analysis: A Practical Approach*, International Thomson Publishing, 1991. [Publisher Link]
- [35] The American Mathematical Monthly, Mathematical Association of America, 1894. [Online]. Available: https://maa.org/publication/the-american-mathematical-monthly/
- [36] Walter Meyer, Geometry and its Applications, Academic Press, 1999. [Google Scholar] [Publisher Link]