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Original Article

Fourier Series of Incomplete H-Function

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Abstract *- In this paper, we present an approach to establish some integrals associated with the Incomplete H-Function and engage them to derive Fourier Series for the Incomplete H-Function. Various Fourier Series are derived for the Incomplete Meijer G-function, the Incomplete Fox-Wright function. The results presented here have a wide applicability in science and engineering.*

Keywords - *Fourier Series, Incomplete H-Function, Incomplete G-Function.*

1. Introduction

In this segment, a concise recapitulation of vital definitions and explanations has been investigated in specific earlier studies [1-7] related to incomplete function, which are used for the whole of this work.

1.1. Incomplete Gamma Function (IGF)

The lower incomplete gamma function $\gamma(\mu, x)$ and the incomplete upper gamma function $\Gamma(\mu, x)$indicate by

$$\gamma(\mu, x) = \int_0^x t^{\mu-1} e^{-t} dt; \qquad (\Re(\mu) > 0; x \geq 0) \tag{1}$$

$$\Gamma(\mu, x) = \int_x^\infty t^{\mu-1} e^{-t} dt; \qquad (x \geq 0; \Re(\mu) > 0 \text{ when } x = 0) \tag{2}$$

The sum of equations (1) and (2) gives the complete gamma function:

$$\gamma(\mu, x) + \Gamma(\mu, x) = \Gamma(\mu); \quad (\Re(\mu) > 0) \tag{3}$$

1.2. Incomplete H-function

The incomplete H-function defined by Srivastava et al. [7] [equation (2.1) -(2.4)] as follows:

$$\Gamma_{p,q}^{m,n}(z) = \Gamma_{p,q}^{m,n}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right]$$

$$= \Gamma_{p,q}^{m,n}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_2, U_2), \dots, (u_p, U_p) \\ (v_1, V_1), (v_2, V_2), \dots, (v_q, V_q) \end{matrix} \right.\right] = \frac{1}{2\pi i} \int_L \theta(\xi, x) z^\xi d\xi, \tag{4}$$

where

$$\theta(\xi, x) = \frac{\Gamma(1-u_1+U_1\xi, x) \prod_{j=1}^m \Gamma(v_j - V_j\xi) \prod_{j=2}^n \Gamma(1-u_j+U_j\xi)}{\prod_{j=m+1}^q (1-v_j+V_j\xi) \prod_{j=n+1}^p \Gamma(u_j - U_j\xi)} \tag{5}$$

and

$$\gamma_{p,q}^{m,n}(z) = \gamma_{p,q}^{m,n}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right]$$

$$= \gamma_{p,q}^{m,n}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_2, U_2), \dots, (u_p, U_p) \\ (v_1, V_1), (v_2, V_2), \dots, (v_q, V_q) \end{matrix} \right.\right] = \frac{1}{2\pi i} \int_L \theta(\xi, x) z^\xi d\xi, \tag{6}$$

where $$\theta(\xi,x)=\frac{\gamma(1-u_1+U_1\xi,x)\prod_{j=1}^{m}\Gamma(v_j-V_j\xi)\prod_{j=2}^{n}\Gamma(1-u_j+U_j\xi)}{\prod_{j=m+1}^{q}(1-v_j+V_j\xi)\prod_{j=n+1}^{p}\Gamma(u_j-U_j\xi)} \tag{7}$$

Where $i=(-1)^{\frac{1}{2}}$ and

$\begin{cases}u_j,[j=1,2,\ldots,p]\\ v_j,[j=1,2,\ldots,q]\end{cases}$ are complex numbers with their corresponding coefficients. $\begin{cases}U_j,[j=1,2,\ldots,p]\\ V_j,[j=1,2,\ldots,q]\end{cases}$ belong to R^+, and Lstand for the contours that are taken up at the point$\varsigma-i\infty$and expand to the point $\varsigma+i\infty$with $\varsigma\in R$. The integrals in (4) and (6) are convergent subject to the conditions provided by Srivastava et al. [21].

If $|arg\, z|<\frac{\pi}{2}\Omega$

Where$\varphi\equiv\sum_{j=1}^{p}U_j+\sum_{j=1}^{q}V_j$ (8)

and $\Omega=\sum_{j=1}^{n}U_j-\sum_{j=n+1}^{p}U_j+\sum_{j=1}^{m}V_j-\sum_{j=m+1}^{q}V_j\le 0$ (9)

Where m,n,p,q it belongs to I^+ and is limited by the $0\le n\le pq\ge m\ge 0$ inequalities in (8) impose restrictions on the acceptable values of the complex variables z. The points $z=0$ and other inconsistent cases are being excluded. As shown by Srivastava and Panda [13], we get

$$\Gamma[z\xi]=O\left(|z|^{\partial}\right)\left(\lim_{1\le j\le m}\ \|z_j\|\to 0\right)$$

Where $\partial=\lim_{1\le j\le m}\mathrm{Re}\left(\frac{v_j}{V_j}\right)$

Here L designate a Mellin-Barnes contour from $\varphi-i\infty$to $\varphi+i\infty$with$(\xi\in\Re)$, and suitable indented, as required to separate poles of the integrand.

The incomplete H-function in (4) and (6), respectively, is valid for all $x\ge 0$subject to the same set of admissibility conditions and contour requirement as reported in Srivastava et al. [7], Mathai and Saxena [8], and Kilab et al. [11].

2. Preliminaries

As listed I. S. Gradshteyn, M. I. Ryzhik ([12], p. 397 Equation (5.12)), the following Integral representation is given as:

$$\int_0^{\pi}(\sin\phi)^{2\alpha}\sin(2n+1)\,\phi d\phi=\frac{(-1)^n\sqrt{\pi}\Gamma\left(\frac{1}{2}+\alpha\right)\Gamma(\alpha+1)}{\Gamma\left(\frac{1}{2}+\alpha-n\right)\Gamma\left(\frac{3}{2}+\alpha+n\right)},\ \text{for}R(\alpha)>-\frac{1}{2} \tag{10}$$

$$\int_0^{\pi}(\sin\phi)^{2\alpha}\cos 2\,n\phi\, d\phi=\frac{(-1)^n\sqrt{\pi}\Gamma\left(\frac{1}{2}+\alpha\right)\Gamma(\alpha+1)}{\Gamma(1+\alpha-n)\Gamma(1+\alpha+n)},\ \text{for}R(\alpha)>-\frac{1}{2} \tag{11}$$

3. Main Result

In this section, we derive certain integrals by using (10) and (11).

3.1. First Integral

$$\int_0^{\pi}(\sin\phi)^{2\alpha}\sin(2n+1)\phi\,\Gamma_{p,q}^{m,n}\left[z\sin^{2\delta}\phi\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)}\\(v_j,V_j)_{(1,q)}\end{matrix}\right.\right]d\phi$$
$$=(-1)^n\sqrt{\pi}\,\Gamma_{p+2,q+2}^{m,n+2}\left[z\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)};\left(-\alpha+\frac{1}{2};\delta\right);(-\alpha;\delta)\\(v_j,V_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm n;\delta\right)\end{matrix}\right.\right] \tag{12}$$

and

$$\int_0^{\pi}(\sin\phi)^{2\alpha}\sin(2n+1)\phi\,\gamma_{p,q}^{m,n}\left[z\sin^{2\delta}\phi\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)}\\(v_j,V_j)_{(1,q)}\end{matrix}\right.\right]d\phi$$

$$=(-1)^n \sqrt{\pi}\, \gamma_{p+2,q+2}^{m,n+2}\left[z\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)};\left(-\alpha+\frac{1}{2};\delta\right);(-\alpha;\delta)\\(v_j,V_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm n;\delta\right)\end{matrix}\right.\right] \tag{13}$$

3.2. Second Integral

$$\int_0^{\pi}(\sin\phi)^{2\alpha}\cos 2n\phi\, \Gamma_{p,q}^{m,n}\left[z\sin^{2\delta}\phi\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)}\\(v_j,V_j)_{(1,q)}\end{matrix}\right.\right]d\phi$$

$$=(-1)^n \sqrt{\pi}\, \Gamma_{p+2,q+2}^{m,n+2}\left[z\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)};\left(-\alpha+\frac{1}{2};\delta\right);(-\alpha;\delta)\\(v_j,V_j)_{(1,q)};(-\alpha\pm n;\delta)\end{matrix}\right.\right] \tag{14}$$

and

$$\int_0^{\pi}(\sin\phi)^{2\alpha}\cos 2n\phi\, \gamma_{p,q}^{m,n}\left[z\sin^{2\delta}\phi\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)}\\(v_j,V_j)_{(1,q)}\end{matrix}\right.\right]d\phi$$

$$=(-1)^n \sqrt{\pi}\, \gamma_{p+2,q+2}^{m,n+2}\left[z\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)};\left(-\alpha+\frac{1}{2};\delta\right);(-\alpha;\delta)\\(v_j,V_j)_{(1,q)};(-\alpha\pm n;\delta)\end{matrix}\right.\right] \tag{15}$$

3.3. Proof of First Integral

The integrand, which includes the incomplete H-Function, is describable as a Mellin-Barnes type Integral. We have the LHS of equation (12)

$$\Rightarrow\int_0^{\pi}(\sin\phi)^{2\alpha}\sin(2n+1)\phi\, \Gamma_{p,q}^{m,n}\left[z\sin^{2\delta}\phi\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)}\\(v_j,V_j)_{(1,q)}\end{matrix}\right.\right]d\phi$$

$$=\int_0^{\pi}(\sin\phi)^{2\alpha}\sin(2n+1)\phi\left[\frac{1}{2\pi i}\int_L\theta(\xi,x)z^{\xi}d\xi\right]d\phi$$

The absolute convergence of the integrals explains the interchange of the order of integration.

$$=\frac{1}{2\pi i}\int_L\theta(\xi,x)z^{\xi}\left[\int_0^{\pi}(\sin\phi)^{2(\alpha+\delta\xi)}\sin(2n+1)\phi d\phi\right]d\xi$$

Now, by using (10), we have

$$=\frac{(-1)^n\sqrt{\pi}}{2\pi i}\int_L\theta(\xi,x)\frac{\Gamma\left(\frac{1}{2}+\alpha+\delta\xi\right)\Gamma(\alpha+\delta\xi+1)}{\Gamma\left(\frac{1}{2}+\alpha+\delta\xi-n\right)\Gamma\left(\frac{3}{2}+\alpha+\delta\xi+n\right)}z^{\xi}\,d\xi$$

By using (4), we get the RHS of (12)

$$=(-1)^n \sqrt{\pi}\, \Gamma_{p+2,q+2}^{m,n+2}\left[z\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)};\left(-\alpha+\frac{1}{2};\delta\right);(-\alpha;\delta)\\(v_j,V_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm n;\delta\right)\end{matrix}\right.\right]$$

Similarly, we get proof of equation (13) by using (10) and (6)

3.4 Proof of the Second integral

The integrand, which includes the incomplete H-Function, is describable as a Mellin-Barnes type Integral. We have the LHS of equation (14)

$$\Rightarrow\int_0^{\pi}(\sin\phi)^{2\alpha}\cos 2n\phi\, \Gamma_{p,q}^{m,n}\left[z\sin^{2\delta}\phi\left|\begin{matrix}(u_1,U_1,x);(u_j,U_j)_{(2,p)}\\(v_j,V_j)_{(1,q)}\end{matrix}\right.\right]d\phi$$

$$=\int_0^{\pi}(\sin\phi)^{2\alpha}\cos 2n\phi\left[\frac{1}{2\pi i}\int_L\theta(\xi,x)z^{\xi}d\xi\right]d\phi$$

The absolute convergence of the integrals explains the interchange of the order of integration.

$$= \frac{1}{2\pi i}\int_{L} \theta(\xi, x) z^{\xi} \left[\int_0^{\pi} (sin\,\phi)^{2(\alpha+\delta\xi)}\, cos\, 2\, n\phi d\phi\right] d\xi$$

Now, by using (11), we have

$$= \frac{(-1)^n \sqrt{\pi}}{2\pi i} \int_{\mathfrak{J}} \theta(\xi, x) \frac{\Gamma\left(\frac{1}{2}+\alpha+\delta\xi\right)\Gamma(\alpha+\delta\xi+1)}{\Gamma(1+\alpha+\delta\xi-n)\Gamma(1+\alpha+\delta\xi+n)} z^{\xi}\, d\xi$$

By using (4), we get the RHS of (14)

$$= (-1)^n \sqrt{\pi}\, \Gamma_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha+\frac{1}{2}; \delta\right); (-\alpha; \delta) \\ (v_j, V_j)_{(1,q)}; (-\alpha \pm n; \delta) \end{matrix} \right.\right]$$

Similarly, we get proof of equation (15) by using (11) and (6)

4. Specific Cases

In (4), supposing $\xi \in z^{+}$ (positive integer), and putting $U_j = V_j = 1: (j = 1,2,\dots,p: j = 1,2,\dots,q)$, and $\delta = 1$, then

$$\Gamma_{p,q}^{m,n}(z) = {}^{\Gamma}G_{p,q}^{m,n}\left[z \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right.\right]$$

Clarify with the help of (12)

$$\int_0^{\pi} (sin\,\phi)^{2\alpha}\, sin(2n+1)\, \phi \Gamma G_{p,q}^{m,n}\left[z\, sin^2\phi \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= (-1)^n \sqrt{\pi} \Gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)}; \left(-\alpha+\frac{1}{2}\right); (-\alpha) \\ (v_j)_{(1,q)}; \left(-\alpha \pm \frac{1}{2} \pm n\right) \end{matrix} \right.\right] \quad (16)$$

And clarify with the help of (13)

$$\int_0^{\pi} (\sin\phi)^{2\alpha} \sin(2n+1)\phi\, \gamma_{p,q}^{m,n}\left[z\, sin^{2\delta}\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= (-1)^n \sqrt{\pi} \gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)}; \left(-\alpha+\frac{1}{2}\right); (-\alpha) \\ (v_j)_{(1,q)}; \left(-\alpha \pm \frac{1}{2} \pm n\right) \end{matrix} \right.\right] \quad (17)$$

Similarly, with the help of (16)

$$\int_0^{\pi} (sin\,\phi)^{2\alpha}\, cos\, 2\, n\phi\, \Gamma G_{p,q}^{m,n}\left[z\, sin^2\phi \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= (-1)^n \sqrt{\pi} \Gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)}; \left(-\alpha+\frac{1}{2}\right); (-\alpha) \\ (v_j)_{(1,q)}; (-\alpha \pm n) \end{matrix} \right.\right] \quad (18)$$

And clarify with the help of (17)

$$\int_0^{\pi} (sin\,\phi)^{2\alpha}\, cos\, 2\, n\phi\, \gamma G_{p,q}^{m,n}\left[z\, sin^2\phi \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= (-1)^n \sqrt{\pi}\gamma G^{m,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}\right); (-\alpha) \\ (v_j)_{(1,q)}; (-\alpha \pm n) \end{matrix} \right.\right] \tag{19}$$

5. The Fourier Series of Incomplete H-Function

5.1 Fourier Sine Series of Incomplete H-Function

$$(\sin\phi)^{2\alpha}\,\Gamma^{m,n}_{p,q}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= \sum_{\kappa=0}^{\infty} \frac{2(-1)^\kappa}{\sqrt{\pi}} \sin(2\kappa+1)\phi\, \Gamma^{0,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}; \delta\right); (-\alpha; \delta) \\ (v_j, V_j)_{(1,q)}; \left(-\alpha \pm \frac{1}{2} \pm \kappa; \delta\right) \end{matrix} \right.\right] \tag{20}$$

$$R(2\alpha) \geq 0, 0 \leq \phi \leq \pi$$

and

$$(\sin\phi)^{2\alpha}\,\gamma^{m,n}_{p,q}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= \sum_{\kappa=0}^{\infty} \frac{2(-1)^\kappa}{\sqrt{\pi}} \sin(2\kappa+1)\phi\, \gamma^{0,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}; \delta\right); (-\alpha; \delta) \\ (v_j, V_j)_{(1,q)}; \left(-\alpha \pm \frac{1}{2} \pm \kappa; \delta\right) \end{matrix} \right.\right] \tag{21}$$

$$R(2\alpha) \geq 0, 0 \leq \phi \leq \pi$$

5.2. Fourier Cosine Series of Incomplete H-Function

$$(\sin\phi)^{2\alpha}\,\Gamma^{m,n}_{p,q}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= \frac{(-1)^\kappa}{\sqrt{\pi}}\, \Gamma^{m,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}; \delta\right) \\ (v_j, V_j)_{(1,q)}; (-\alpha; \delta) \end{matrix} \right.\right]$$

$$+ \sum_{\kappa=1}^{\infty} \frac{2(-1)^\kappa}{\sqrt{\pi}} \cos\kappa\phi\, \Gamma^{0,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}; \delta\right); (-\alpha; \delta) \\ (v_j, V_j)_{(1,q)}; (-\alpha \pm \kappa; \delta) \end{matrix} \right.\right] \tag{22}$$

$$R(2\alpha) \geq 0, 0 \leq \phi \leq \pi$$

and

$$(\sin\phi)^{2\alpha}\,\gamma^{m,n}_{p,q}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= \frac{(-1)^\kappa}{\sqrt{\pi}}\, \gamma^{m,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}; \delta\right) \\ (v_j, V_j)_{(1,q)}; (-\alpha; \delta) \end{matrix} \right.\right]$$

$$+ \sum_{\kappa=1}^{\infty} \frac{2(-1)^\kappa}{\sqrt{\pi}} \cos\kappa\phi\, \Gamma^{m,n+2}_{p+2,q+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha + \frac{1}{2}; \delta\right); (-\alpha; \delta) \\ (v_j, V_j)_{(1,q)}; (-\alpha \pm \kappa; \delta) \end{matrix} \right.\right] \tag{23}$$

$$R(2\alpha) \geq 0, 0 \leq \phi \leq \pi$$

where $\varphi \equiv \sum_{j=1}^{p} U_j + \sum_{j=1}^{q} V_j$
and $\Omega = \sum_{j=1}^{n} U_j - \sum_{j=n+1}^{p} U_j + \sum_{j=1}^{m} V_j - \sum_{j=m+1}^{q} V_j > 0, \quad |arg\, z| < \frac{\pi}{2}\Omega$

Proof:
To prove (20), let

$$F(\phi) = (\sin\phi)^{2\alpha}\, \Gamma_{p,q}^{0,n}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi = \sum_{\kappa=0}^{\infty} A_\kappa\, sin(2\kappa+1)\,\phi \tag{24}$$

Equation (20) holds under the assumption that the function $F(\phi)$is continuous and of bounded variation on $(0,\pi)$, when $R(2\alpha) \geq 0$.

Multiplying by$sin(2n+1)\,\phi$in both sides of (20) and integrating from $0 to \pi$w. r. t.ϕ, we get

$$\int_0^{\pi} (\sin\phi)^{2\alpha} \sin(2n+1)\phi\, \Gamma_{p,q}^{0,n}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi = \sum_{\kappa=0}^{\infty} A_\kappa \int_0^{\pi} sin(2n+1)\,\phi\, sin(2\kappa+1)\,\phi$$

Employing the orthogonality of the trigonometric sine function along with (12), we derive.

$$A_\kappa = \frac{2(-1)^\kappa}{\sqrt{\pi}}\, \Gamma_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha+\frac{1}{2};\delta\right); (-\alpha;\delta) \\ (v_j, V_j)_{(1,q)}; \left(-\alpha \pm \frac{1}{2} \pm \kappa; \delta\right) \end{matrix} \right.\right] \tag{25}$$

From equations (23) and (24), the result (20) is derived.

Similarly, to prove (21), we assume that the conditions for the Incomplete H-function $\gamma_{p,q}^{m,n}$ in equation (6) are fulfilled.
To prove (22), let

$$F(\phi) = (\sin\phi)^{2\alpha}\, \Gamma_{p,q}^{m,n}\left[z\, sin^{2\delta}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi = \frac{B_0}{2} + \sum_{\kappa=1}^{\infty} B_\kappa\, cos\,\kappa\,\phi \tag{26}$$

Multiplying by $cos\, n\,\phi$in both sides of equation (26) and integrating from $0 to \pi$w. r. t. θwe find

$$\int_0^{\pi} \cos n\phi (\sin\phi)^{2\alpha}\, \Gamma_{p,q}^{0,n}\left[z\, sin^{2\alpha}\,\phi \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)} \\ (v_j, V_j)_{(1,q)} \end{matrix} \right.\right] d\phi = \int_0^{\pi} cos\, n\,\phi \left[\frac{B_0}{2} + \sum_{\kappa=1}^{\infty} B_\kappa\, cos\,\kappa\,\phi\right] d\phi \tag{27}$$

Employing the orthogonality of the trigonometric sine function along with (15), we derive.

$$B_\kappa = \frac{2(-1)^\kappa}{\sqrt{\pi}}\, \Gamma_{p+2,q+2}^{0,n+2}\left[z \left| \begin{matrix} (u_1, U_1, x); (u_j, U_j)_{(2,p)}; \left(-\alpha+\frac{1}{2};\delta\right); (-\alpha;\delta) \\ (v_j, V_j)_{(1,q)}; (-\alpha \pm \kappa; \delta) \end{matrix} \right.\right]$$

From equations (26) and (27), the result (22) is derived.

Similarly, to prove (23), we assume that the conditions for the Incomplete H-function $\gamma_{p,q}^{m,n}$ in equation (6) are fulfilled.

6. Specific Case of Fourier Series of Incomplete H-Function:

In equations (20) to (23), assuming $\xi \in z^+$ (positive integer), putting $\alpha_j = 1, (j = 1,2,\ldots,q): B_j = 1, (j = 1,2,\ldots,q)$ and $u = 1$, two Fourier series for the Incomplete G-function is derived.

$$(sin\,\phi)^{2\alpha}\Gamma G_{p,q}^{m,n}\left[z\, sin^2\,\phi \left| \begin{matrix} (u_1, x); (u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right.\right] d\phi$$

$$= \sum_{\kappa=0}^{\infty} \frac{2(-1)^{\kappa}}{\sqrt{\pi}} sin(2\kappa+1)\,\phi \Gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)};\left(-\alpha+\frac{1}{2}\right);(-\alpha) \\ (v_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm\kappa\right) \end{matrix} \right. \right] \tag{28}$$

and the other is

$$(sin\,\phi)^{2\alpha} \Gamma G_{p,q}^{m,n}\left[z\,sin^2\phi \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right. \right] d\phi$$

$$= \frac{B_0}{2} + \sum_{\kappa=1}^{\infty} \frac{2(-1)^{\kappa}}{\sqrt{\pi}} cos\,\kappa\,\phi \Gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)};\left(-\alpha+\frac{1}{2}\right);(-\delta) \\ (v_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm\kappa\right) \end{matrix} \right. \right] \tag{29}$$

Similarly, we get

$$(sin\,\phi)^{2\alpha} \gamma G_{p,q}^{m,n}\left[z\,sin^2\phi \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right. \right] d\phi$$

$$= \sum_{\kappa=0}^{\infty} \frac{2(-1)^{\kappa}}{\sqrt{\pi}} sin(2\kappa+1)\,\phi \gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)};\left(-\alpha+\frac{1}{2}\right);(-\alpha) \\ (v_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm\kappa\right) \end{matrix} \right. \right] \tag{30}$$

and the other is

$$(sin\,\phi)^{2\alpha} \gamma G_{p,q}^{m,n}\left[z\,sin^2\phi \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)} \\ (v_j)_{(1,q)} \end{matrix} \right. \right] d\phi$$

$$= \frac{B_0}{2} + \sum_{\kappa=1}^{\infty} \frac{2(-1)^{\kappa}}{\sqrt{\pi}} cos\,\kappa\,\phi \gamma G_{p+2,q+2}^{m,n+2}\left[z \left| \begin{matrix} (u_1,x);(u_j)_{(2,p)};\left(-\alpha+\frac{1}{2}\right);(-\delta) \\ (v_j)_{(1,q)};\left(-\alpha\pm\frac{1}{2}\pm\kappa\right) \end{matrix} \right. \right] \tag{31}$$

7. Conclusion

In this paper, we have expected sine and cosine Fourier series involving the Incomplete H-function. These results are then applied to evaluate a Fourier series expansion of the Incomplete Meijer G-function. The Fourier series derived in this analysis is of a general form and may work as a basis for expanding several results relevant to practical and applied contexts.

Conflicts of Interest

The first and second authors declare no conflict of interest regarding the publication of this paper.

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